



GEOMETRICAL DRAWING

FOR

ART STUDENTS

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# GEOMETRICAL DRAWING

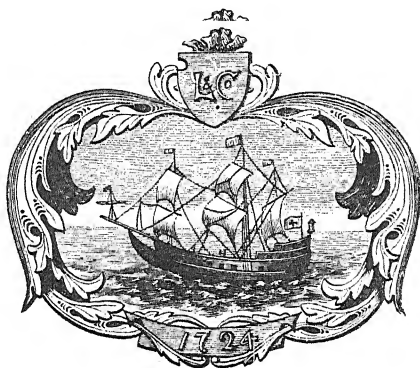
FOR

## ART STUDENTS

BY

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AN INSPECTOR OF SCHOOLS TO THE SHEFFIELD EDUCATION COMMITTEE  
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IN LONGMANS' ELEMENTARY SCIENCE MANUALS  
'THE TEACHING OF DRAWING'  
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## PREFACE TO THE FIRST EDITION

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THIS little book has been prepared to meet the wants of those students who only require the Geometry necessary for the Art Student's course. The work necessary for the Second and Third Grade Art Certificates is fully covered, and the Student who has thoroughly mastered the contents of the book will find himself well equipped either for examination or for taking up a more advanced course of the subject.

The book contains over seven hundred figures arranged in a convenient form, and a very complete and exhaustive collection of exercises, and covers rather more ground than is absolutely necessary for the South Kensington Examination in Geometrical Drawing. The chapter on Solid Geometry has been made unusually full, as the Author's experience is that one of the student's chief difficulties is the want of sufficient variety of examples in this important branch of the subject.

The Author is indebted to Bradley's 'Practical Geometry' (Library of Useful Knowledge), and his later work published for the Committee of Council on Education, Winter's 'Geometrical Drawing,' Carroll's 'Practical Geometry for Science and Art Students,' and Meyer's 'Handbook of Ornament,' for much useful information.

I H M.

SHEFFIELD, *August* 1890.

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## PREFACE TO THE THIRTEENTH EDITION

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THIS Edition has been carefully revised. New figures have been added, and the chapter on Solid Geometry rewritten.

I H M

*Sept.* 1912.

THE Publishers desire to acknowledge the kind permission of the Controller of H M Stationery Office to reprint the Examination Papers at the end of this book.

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# SYLLABUS

OF

## ART EXAMINATION

### GEOMETRICAL DRAWING (ART)

NOTE —The portions of this book dealing with the present syllabus are indicated in *italics*.

This examination is intended to test —

(A) The students' ability to use compasses, T square, set squares, protractor, and scales, in showing their knowledge of ordinary geometrical figures, and

(B) Their power of applying these figures as bases for ornamental and decorative work, which they may be required to do by freehand drawing or by means of instruments.

Candidates will be required to qualify in :—

(A) 1. Constructions of triangles (*Chap V.*), quadrilaterals (*Chap VI.*), and polygons from given data (*Chap. VIII.* and *Chap. X. probs. 110-116*)

2. Describing circles to satisfy given conditions—passing through given points (*Chap. VII. probs 64-70*), touching lines and circles (*Chap. XI. probs 121-126 and 146-163*) Drawing straight lines touching circles (*Chap. VII. probs 71-76*)

3. Construction of figures similar to given figures (*Chap. X. probs 117-119*)

4. Proportional division of lines, including third, fourth, and mean proportionals, extreme and mean ratio (*Chap. IV.*). Plain and diagonal scales. Scale of Chords (*Chap IX.*).

5. Construction of the ellipse, drawing its tangents and normals. Drawing curves defined by simple conditions (*Chap. XIV.*).

6. Inscribing and describing rectilinear figures and circles within and about others (*Chap. XI. probs 127-145, 164-177, and Chap. XII.*)

7. Plans, elevations, and sections of simple geometrical solids, singly or in combination, in simple positions (*Chap XV.*).

(B) The application of geometrical constructions to setting out schemes of ornamental patterns, construction of units of patterns, spacing of wall and other surfaces for decorative purposes, and construction of arch forms, tracery, and mouldings, etc (*Chap. XVI.*).

# GEOMETRICAL DRAWING

## FOR ART STUDENTS

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### CHAPTER I

#### INTRODUCTION

IN Practical Geometry we apply the principles of Theoretical Geometry to construction by the aid of instruments. The student who has a knowledge of Euclid will find it of considerable service in understanding the principles used, and also in remembering the methods of construction adopted.

It is of the highest importance that the problems should be worked with the greatest possible accuracy and neatness, and in a variety of positions, as problems frequently present fresh difficulties when the position of the points or lines is altered.

The instruments used need not be numerous, but should be of the best make and finish that the student can obtain, as inferior instruments frequently cause much trouble and vexation, and render the accuracy so indispensable in a geometrical drawing an impossibility. The following are absolutely essential —

1. **A Drawing-board** with a perfectly level surface, and with its corners true right angles. Half-imperial is a very convenient size.

2. **A T square.**—This is used to draw lines parallel to the edge of the board. It is not advisable to use it for the purpose of drawing *vertical* as well as horizontal lines, as, if the board be not true, inaccuracies will be caused in the drawing. The *vertical* lines are best obtained by using the set square. It is advantageous to have the edge of the T square bound with hard wood, and bevelled. The blade should be screwed on to the head, as this arrangement allows the set squares to be used more freely.

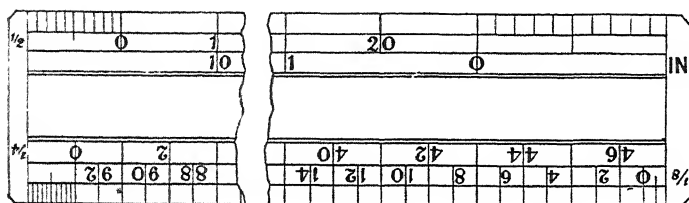
3 **Two set-squares**, having angles of  $45^{\circ}$  and  $60^{\circ}$  respectively. Hollow-framed ones with bevelled edges are the best. These are used to obtain perpendiculars and parallels. Skilful manipulation of these useful instruments will enable the student to construct many simple figures by their aid alone.

4 **Pencils.**—These should be HH for the construction lines, and H or HB for darkening the lines of the constructed figure. They

are best sharpened like the edge of a *chisel* for geometrical drawing, as the point lasts longer.

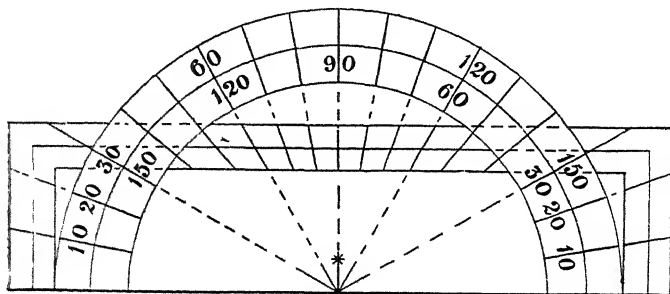
**5 Mathematical instruments**—These should include :—A compass with movable pen and pencil legs (those with needle-points are preferable, as they do not make so large a hole in the paper) : a pair of dividers, for measuring, a mathematical pen, for ruling lines in ink. In addition to these, a set of *spring bow compasses*, for describing small arcs and circles, are of great assistance. **Indian ink** must always be used with the instruments, as it does not corrode them, that sold in a liquid form is very convenient. After the pens have been used they should be carefully wiped, to prevent rust.

**6 Scale.**—This should be 12 inches long and bevelled at both edges as in the illustration. It should show the inch, half-inch,



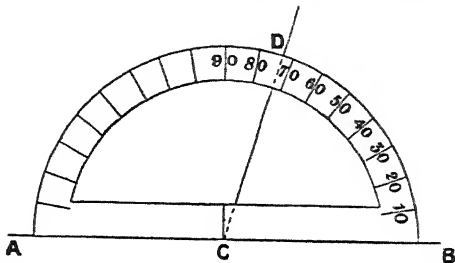
and quarter-inch, subdivided decimally, the ordinary subdivisions of eighths, twelfths, etc., and a scale of centimetres.

**7. Protractor**—This is made either circular, semicircular, or rectangular. The illustration shows how the markings on the



rectangular protractor are obtained from the semicircular one. The scale is figured from each end from 0° to 180°, so that it is equally convenient for setting out angles either in a right or left hand direction.

The instrument is used as follows.—Suppose it is required to set out an angle of  $73^\circ$  at point *C* in the line *A B*. Place the straight edge of the protractor exactly on line *A B*, with the index on the point *C*. Mark a point *D* with a sharp pencil at  $73^\circ$  on the protractor. Remove the protractor and join *C* and *D*. Then *D C B* will be the required angle.



**8 Paper and pins**—For ordinary pencil-work cartridge-paper is the most suitable, when the drawings have to be inked in, a better quality is desirable, such as Whatman's smooth papers. Pins with heads soldered on are the most convenient.

**Parallels and perpendiculars.**—Remember that horizontal lines are drawn with the T-square, and vertical lines with the set-square. The following instruction is given on the examination papers of the Board of Education: "Lines parallel or perpendicular to others may be drawn mechanically, without showing any construction. Lines may be bisected by trial."



## CHAPTER II

## DEFINITIONS, TERMS USED, &amp;c.

1. A point has no magnitude. It merely indicates *position*, and is marked either by a dot or as in Fig. 1.

## LINES.

2. A line has *length* and *position*, but neither breadth nor thickness. It is indicated by the letters placed at its extremities, as A B, Fig. 2. Various methods of drawing lines are used in practice, as thick, thin, dotted, and chain lines. Fig 2.

3. A straight line is the shortest distance between two given points. It is also called a *right* line

4. A curved line is nowhere straight. There are endless varieties of curved lines. Fig. 3 is a *simple* curve. Fig. 4 is a *compound* curve.

5. A horizontal line is perfectly level, like the surface of still water. Fig 5.

6. A vertical line is perfectly upright, like a plumb-line. Fig 6.

7. An oblique line is neither horizontal nor vertical Fig. 7.

8. Parallel lines are the same distance apart, and cannot meet, however far they may be produced. Fig 8.

## ANGLES.

9 An angle is the inclination to each other of two straight lines which meet at a point.

10 A right angle.—When one straight line meets another straight line so as to make the adjacent angles (those on each side of the line) equal to one another, each of these angles is a right angle, and the lines are perpendicular to each other In Fig. 9, the angles A B C and A B D are each right angles.

11. An obtuse angle is greater than a right angle. Fig. 10.


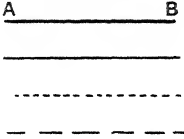
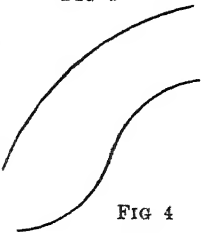
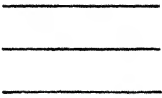

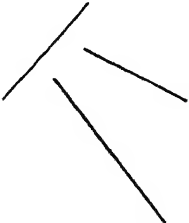
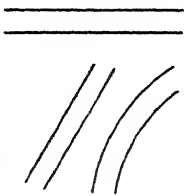
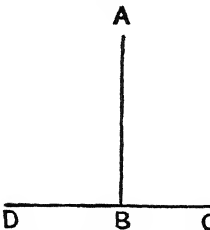
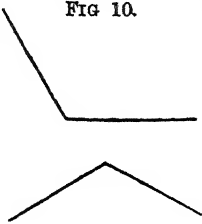
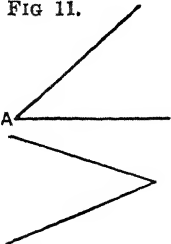
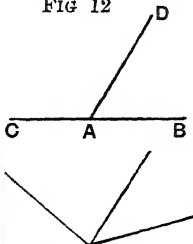
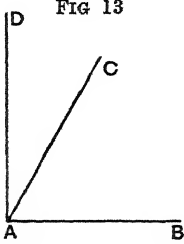
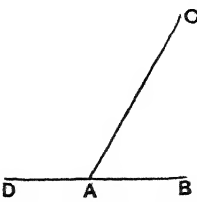
12. An acute angle is less than a right angle. Fig. 11.

13. The vertex is the point where the two lines forming an angle meet, as at A, Fig 11.

14. Adjacent angles have a common vertex and one common arm. In Fig. 12, the angle D A B is adjacent to the angle D A C.

15. The complement of an angle is the difference between it and a right angle In Fig 13, the angle C A B is the complement of the angle C A D, and C A D is the complement of C A B.

16. The supplement of an angle is the difference between it and two right angles. In Fig. 14, the angle C A B is the supplement of the angle C A D, and *vice versâ*.

<p>FIG. 1</p>  <p>FIG. 2.</p> 	<p>FIG. 3</p>  <p>FIG. 4</p>	<p>FIG. 5.</p> 
<p>FIG. 6</p> 	<p>FIG. 7.</p> 	<p>FIG. 8</p> 
<p>FIG. 9</p> 	<p>FIG. 10.</p> 	<p>FIG. 11.</p> 
<p>FIG. 12</p> 	<p>FIG. 13</p> 	<p>FIG. 14</p> 

## TRIANGLES.

17. A triangle is a figure contained by three straight lines. If it be supposed to stand upon one of its sides, that side is termed its **base**; the point where the other two sides meet is its **vertex** or **apex**, the angle at the vertex is the **vertical angle**, and the perpendicular from the apex to the base or base produced is the **altitude**. Thus, in Fig 15, if  $AB$  be the base, then  $C$  is the vertex or apex,  $ACB$  the vertical angle, and  $CD$  the altitude.

Triangles are classified either from the comparative lengths of their sides, or from the magnitudes of their angles.

1st *With reference to their sides* —

18. An equilateral triangle has three equal sides. Fig 16.

19. An isosceles triangle has two of its sides equal. Fig 17.

20. A scalene triangle has three unequal sides. Fig 18.

2nd. *With reference to the angles* :—

21. A right-angled triangle has one of its angles a right angle. The side opposite to the right angle is called the **hypotenuse**. In Fig 19,  $AB$  is the hypotenuse.

22. An obtuse-angled triangle has one of its angles greater than a right angle. Fig. 20.

23. An acute-angled triangle has each of its angles less than a right angle. Fig 21.

**Note**—All the angles of any triangle equal two right angles, or  $180^\circ$ , thus, if two of the angles of a triangle are respectively  $60^\circ$  and  $50^\circ$ , then the remaining angle must be  $70^\circ$ , to make up the  $180^\circ$ .

## QUADRILATERALS.

24. A quadrilateral figure is bounded by four straight lines. It is also termed a **quadrangle**, from having four angles.

**Note**.—The four angles of any quadrilateral figure must always equal four right angles, or  $360^\circ$ .

25. A parallelogram is a quadrilateral figure in which the opposite sides are parallel. The straight line joining the opposite angles of the parallelogram is called the **diagonal**. In Fig. 22,  $AB$  is the diagonal. There are four parallelograms—viz, the square, the rectangle or oblong, the rhombus, and the rhomboid.

26. A square has all its sides equal and all its angles right angles. Fig. 22.

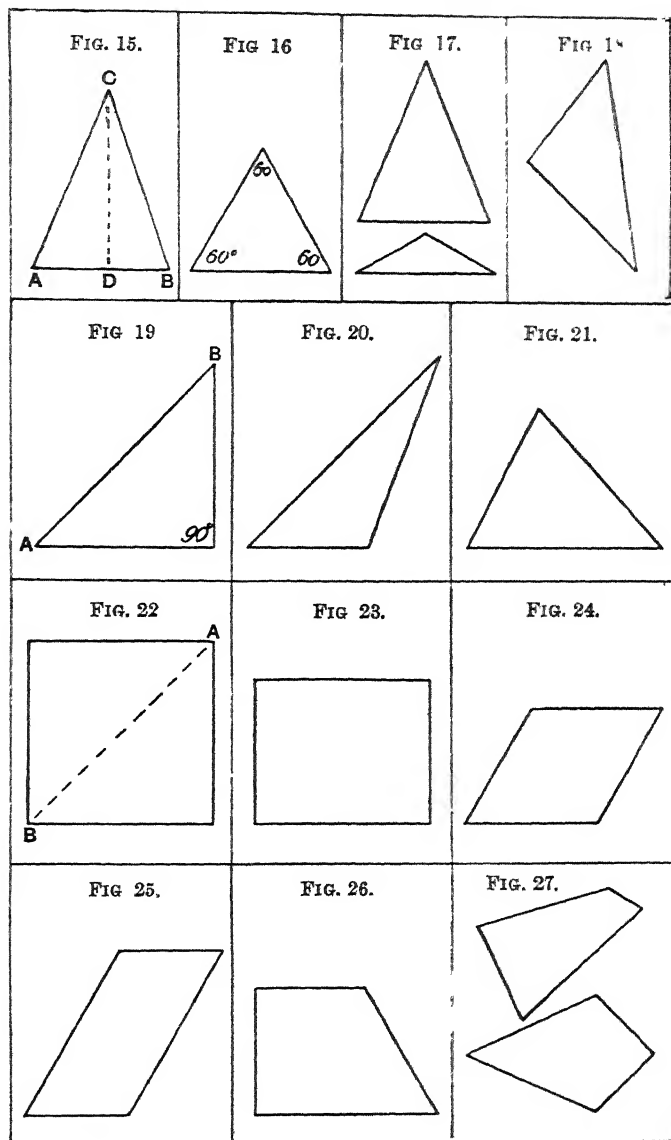
27. A rectangle or oblong has its opposite sides equal and all its angles right angles. Fig. 23.

28. A rhombus has all its sides equal, but its angles are not right angles. Fig. 24.

29. A rhomboid has its opposite sides equal, but its angles are not right angles. Fig. 25.

30. A trapezoid has only two sides parallel. Fig. 26.

31. A trapezium has none of its sides parallel, but may have two of its sides equal. Fig. 27. When two of the sides are equal, the figure is sometimes called a **trapezion**, or **kite**.



## POLYGONS.

32. A polygon is a plane figure bounded by straight lines.

Note.—In some works a polygon is defined as a figure bounded by more than four straight lines

If the sides of the figure are equal, it is termed a regular polygon  
Fig. 28. If the sides are unequal, it is called an irregular polygon  
Fig. 29.

Polygons are named according to the number of their sides :—

A pentagon has five sides.

A nonagon has nine sides.

A hexagon has six sides

A decagon has ten sides.

A heptagon has seven sides.

An undecagon has eleven sides.

An octagon has eight sides.

A duodecagon has twelve sides.

## THE CIRCLE

33 A circle is a plane figure contained by one curved line, called the circumference, every point of which is equally distant from a point within the circle, called the centre (A, Fig. 30).

Note —The circumference is sometimes spoken of as the circle.

34. The diameter of a circle is the straight line passing through its centre, and is terminated at both ends by the circumference, as B C, Fig 30 It divides the circle into two semicircles. If two diameters be drawn at right angles to each other, the circle is divided into quadrants Fig. 30

35. The radius of a circle is the distance from the centre to the circumference, as A B, Fig. 30.

36. An arc of a circle is the portion between any two points in the circumference, as A B, Fig. 31.

37. A chord is the straight line joining the ends of an arc, as the straight line C D, Fig. 31.

38. A segment is the part of a circle bounded by an arc and its chord. Fig. 32.

39 A sector is the part of a circle contained by two radii and the arc between them. Fig. 33.

40. A tangent is a straight line which touches a circle, but does not cut it when produced, as A B, Fig. 34.

41. Concentric circles have the same centre but different radii  
Fig. 35

FIG 28

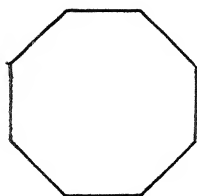


FIG. 29.

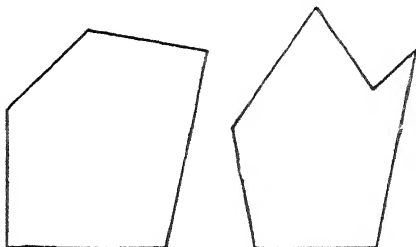


FIG 30.

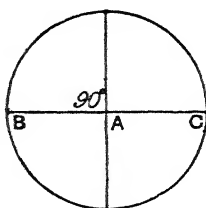


FIG. 31.

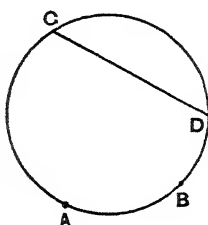


FIG 32.



FIG 33



FIG 34

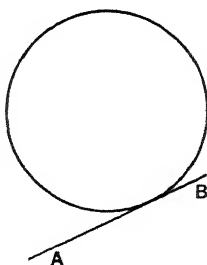
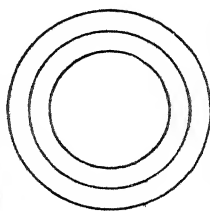


FIG 35



## CHAPTER III

### LINES AND ANGLES

**PROBLEM 1.**—To bisect a given line.

With centre *A*, and any distance greater than half the line, describe an arc. With centre *B*, and the same radius, intersect it in *C* and *D*. Draw the line *CD*. Then the line *CD* bisects the given line at right angles, or is perpendicular to it (Euc. I. 10.)

The arc of circle *EF* may be bisected in a similar manner. By this problem a line may be divided into 4 equal parts by again bisecting each half as shown. If those parts be again bisected, the line would be divided into 8 equal parts, and in a similar manner into 16, &c.

**PROBLEM 2.**—To draw a perpendicular to a given line, from a given point in the line.

Let *C* be the given point. With *C* as centre, and any radius, set off equal distances *C1* and *C2*. With 1 and 2 as centres, and any radius greater than half *12*, describe arcs intersecting at *D*. Draw *CD*, the required perpendicular.

**PROBLEM 3.**—To draw a perpendicular when the given point is at or near the end of the line.

Let *B* be the given point. With centre *B* describe an arc. With the same radius cut this arc in points 2 and 3. From 2 and 3 describe arcs intersecting at *C*. Draw *BC*, the required perpendicular.

Note.—Keep the same radius for describing all the arcs

**PROBLEM 4.**—The same. (Another method.)

Take any point *C* not in the given line *AB*. Draw *CB*. With centre *C* and radius *CB* describe an arc cutting *AB* in *D*. Join *C* and *D*, and produce until it cuts the arc in *E*. Draw *BE* the required perpendicular.

Note.—All angles in which the square on one side equals the sum of the squares on the other two sides is a right-angled triangle.  $AC^2 = AB^2 + BC^2$ , or  $5^2 = 3^2 + 4^2$ .

**PROBLEM 5.**—The same (For large work when instruments are not available)

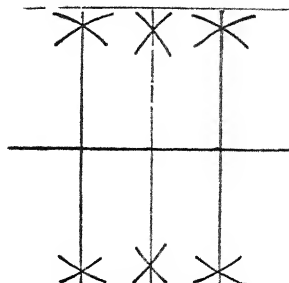
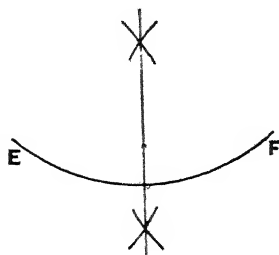
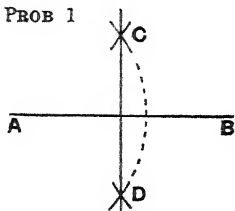
From *B* set off 3 equal divisions towards *A* to any convenient unit. With centre *B* and 4 of the same units describe an arc at *C*. With centre *A* and 5 units cut the arc at *C*. Draw *BC* the required perpendicular. (Euc. I. 48)

Note.—Any triangle in which the square on one side equals the sum of the squares on the other two sides is a right-angled triangle.  $AC^2 = AB^2 + BC^2$ , or  $5^2 = 3^2 + 4^2$ .

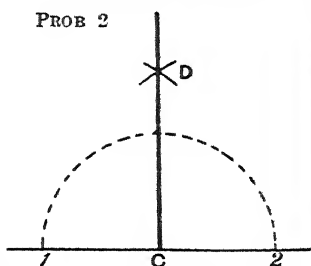
**PROBLEM 6.**—To draw a perpendicular to a line from a given point without it.

Let *A* be the given point. With centre *A* describe an arc cutting the given line *BC* in points 1 and 2. With centres 1 and 2 describe arcs intersecting at *D*. Draw *AD*, the required perpendicular. (Euc. I. 12.)

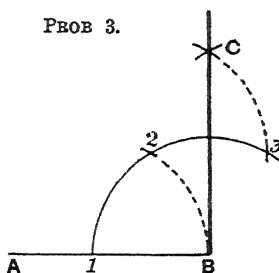
PROB 1



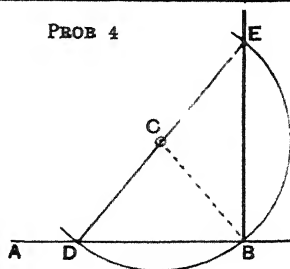
PROB 2



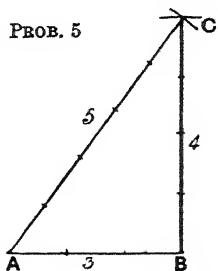
PROB 3.



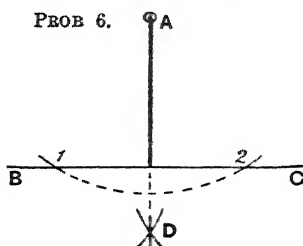
PROB 4



PROB. 5



PROB 6.





**PROBLEM 7**—To draw a perpendicular when the given point is nearly over the end of the given line.

Let *A* be the given point With centre *B* and radius *BA* describe an arc With centre *C* and radius *CA* describe an arc cutting the first arc in *A* and *D* Draw the perpendicular *AD*.

**Note**—This method may be adopted for all positions of the point, two of which are shown.

**PROBLEM 8**—The same. (Another method)

Let *A* be the given point Take any point *D* in the line *BC* Join *AD*, and bisect it in *E* With centre *E* describe the semi-circle *AFD*, cutting *BC* in *F*. Draw the perpendicular *AF*. (Eucl. III 31)

**PROBLEM 9**—To draw a line parallel to a given line, at a given distance from it.

Let *AB* be the given line, and *CD* the given distance Take any points 1 and 2, and with radius *CD* describe two arcs, *E* and *F* Then the line touching the arcs will be the required parallel.

**PROBLEM 10**—To draw a line parallel to a given line through a given point

Let *A* be the given point, and *BC* the given line With any point 1 as centre, and radius 1 *A*, describe the arc *A 2*. With *A* as centre and the same radius describe another arc Cut off 1 *D* equal to *A 2* Draw *AD*, the required parallel.

**Note**—Problems 2-10 would in practical work be done with *T* and set squares.

**PROBLEM 11**—To bisect a given angle.

With centre *A* and any radius describe the arc *BC* With centres *B* and *C* describe arcs intersecting in *D*. Then the line *AD* bisects the angle

By bisecting each half again, the angle will be divided into 4 equal parts, as shown In a similar manner, it may be divided into 8, 16, &c., equal parts

**PROBLEM 12**—To trisect a right angle.

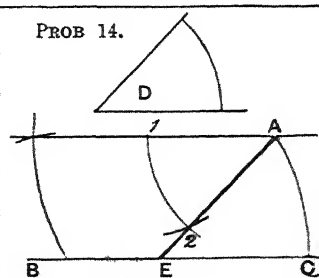
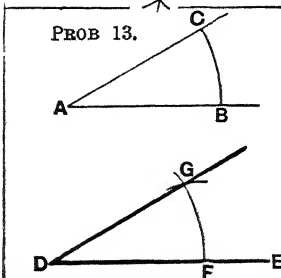
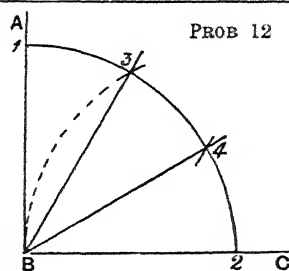
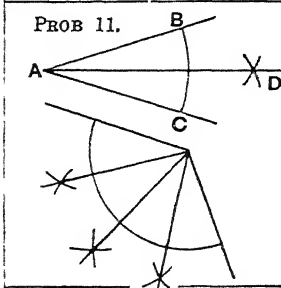
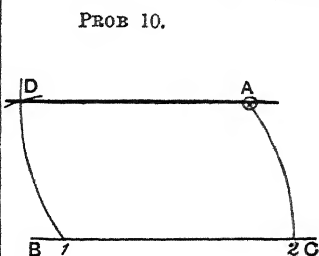
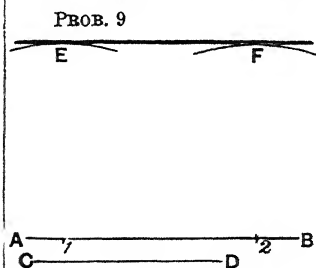
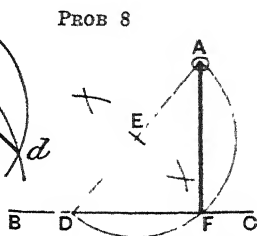
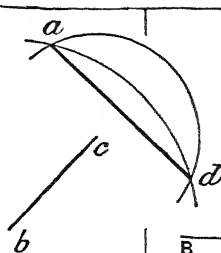
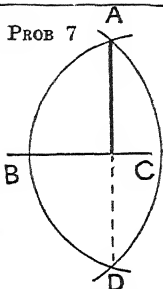
Let *ABC* be the given right angle With centre *B* and any radius describe the arc 1 2 With centres 1 and 2 and the same radius cut the arc in 3 and 4. The lines drawn from *B* through these points will trisect the right angle

**PROBLEM 13**.—To make an angle equal to a given angle.

Let *BAC* be the given angle Draw any line, *DE* With centres *A* and *D* describe arcs *BC* and *FG* Cut off *FG* equal to *BC* (Eucl. III 27.)

**PROBLEM 14**—Through a given point to draw a line meeting another line at an angle equal to a given angle

Let *A* be the given point, *BC* the given line, and *D* the given angle. Through *A* draw a line parallel to *BC*. (*Prob.* 10) At the point *A* make an angle 1 *A 2* equal to the angle *D* (*Prob.* 13) Produce *A 2* to *E*. Then the line *AE* meets *BC*, and makes the angle *AEC* equal to *D*. (Eucl. I 29)



**PROBLEM 15.**—To bisect the angle made by two converging lines without producing them.

Let  $AB$  and  $CD$  be the two lines. Draw a line parallel to  $AB$  at any convenient distance, and at the same distance draw another line parallel to  $CD$  intersecting the first parallel at  $E$ . Bisect the angle thus obtained. Then  $EF$  bisects the angle at which  $AB$  and  $CD$  are inclined.

Notes.—1 The angle of inclination might be found by drawing one parallel only to meet the opposite line. 2 All circles required to touch the lines  $AB$  and  $CD$  would have their centres in  $EF$ .

**PROBLEM 16.**—From any given point to draw a line which should meet at the same point as two converging lines would meet if produced.

Let  $A$  be the given point, and  $BC, DE$  the two converging lines. Draw any line  $A2$ . Mark a point 1 in the line  $BC$  not in the same straight line as  $A2$ . Join  $A1$  and  $21$ , forming the triangle  $A21$ . Take any point 3 in  $DE$ . From point 3 draw  $34$  parallel to  $21$ , and  $3F$  parallel to  $2A$ . From 4 draw  $4F$  parallel to  $1A$ , forming the triangle  $34F$ . The line  $AF$  passing through the corresponding angle of each triangle is the line required.

Notes.—1 Two positions of the point  $A$  are given, one between and one outside the lines. 2 Use the set square in obtaining the parallel lines in this and all complicated figures, as the describing of a number of arcs would cause confusion.

**PROBLEM 17.**—To find a point in a line equally distant from two given points without it.

Let  $AB$  be the given line, and  $C$  and  $D$  the given points. Join  $C$  and  $D$ , and bisect by a perpendicular meeting  $AB$  in  $E$ . Then  $E$  is the required point, and  $EC$  equals  $ED$ .

Note.—From  $E$  a circle could be described passing through  $C$  and  $D$ .

**PROBLEM 18.**—From two given points without a straight line to draw two straight lines to meet the given line and make equal angles with it.

Let  $A$  and  $B$  be the two points, and  $CD$  the given line. Draw  $AE$  perpendicular to  $CD$ , by *Prob. 7*. Draw  $EB$ , cutting  $CD$  in  $F$ . Join  $A$  and  $F$ . Then  $AF$  and  $BF$  are the required lines.

**PROBLEM 19.**—Through a given point between two converging lines to draw a straight line which shall be terminated by the given lines and bisected in the given point.

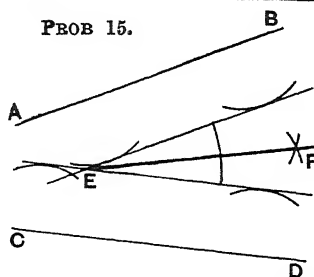
Let  $A$  be the given point, and  $CD, EF$  the given lines. Draw  $AB$  perpendicular to  $EF$ , and produce it, making  $AG$  equal to  $AB$ . From  $G$  draw  $GD$  parallel to  $EF$ . From  $D$  draw  $DE$  through the point  $A$ . Then  $DE$  is bisected in the point  $A$ . (EUC. I 26.)

**PROBLEM 20.**—To draw a line from a given point which shall make equal angles with two converging lines.

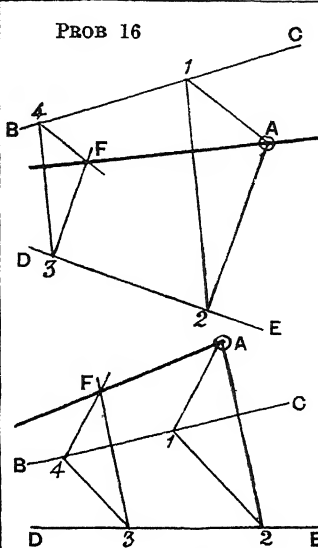
Let  $A$  be the given point, and  $BC, BD$  the converging lines. Produce  $BD$ , and bisect the *supplemental* angle thus formed. From  $A$  draw  $AF$  parallel to the bisecting line,  $BE$ .

If the converging lines do not meet, then obtain the *supplemental* angle by drawing a line parallel to  $BD$ , as shown in the second figure. Bisect the angle thus formed, and proceed as in the previous figure.

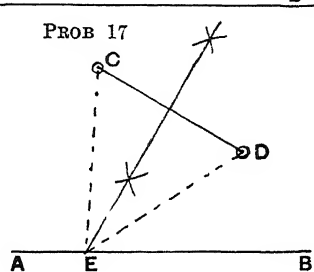
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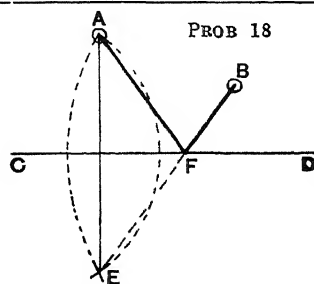
PROB 16



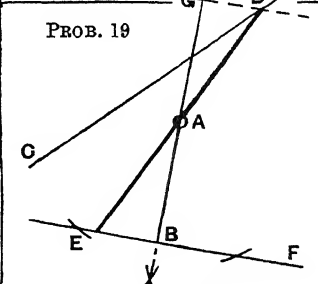
PROB 17



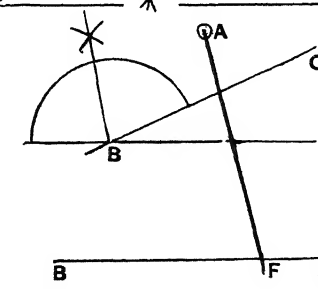
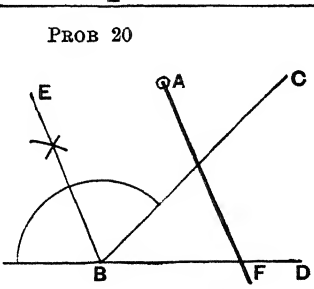
PROB 18



PROB. 19



PROB 20



**PROBLEM 21** —From a given point to draw a line so that the part intercepted between two given parallel lines shall be equal to a given distance

Let *A* be the given point, *BC* and *DE* the two parallel lines, and *F* the given distance.

Take any point *G* in *BC*. With centre *G* and radius equal to *F* describe an arc cutting *DE* in *H*. Draw *GH*.

From *A* draw a line, *AK*, parallel to *GH*. Then the distance *JK* is equal to the given distance *F*.

The solution when the given point is between the two parallel lines is exactly similar. Both positions are shown.

**PROBLEM 22.**—To divide a line into any number of equal parts (say 5).

Let *AB* be the given line. Draw a line at any angle, and set off any convenient distance five times. Draw *5B*, and from the points 4, 3, 2, 1, rule parallels to *5B* with the set square. These parallels will divide the line as required

**PROBLEM 23.**—The same. (Without using the set square.)

Draw *A5* at any angle to *AB*. At *B* make the angle *A B 5* equal to the angle *B A 5*. (*Prob. 13*.) From *A* and *B*, on the lines *A 5* and *B 5*, set off 5 equal distances. Join *5B*, 4 1, 3 2, 2 3, 1 4, and *A 5*, thus cutting the line *AB* into 5 equal parts.

**Note**—This method should never be used in preference to that in *Prob. 22*.

**PROBLEM 24** —To construct an angle containing a given number of degrees

The circumference of a circle is supposed to be divided into 360 equal parts, called *degrees*. The radius of a circle may be set off exactly six times round the circumference, hence, if an arc be described, and a portion cut off equal to the radius of the arc, an angle containing  $60^\circ$  will be obtained. With a knowledge of this principle, a variety of angles may be constructed. Constructions for angles of  $60^\circ$ ,  $120^\circ$ ,  $30^\circ$ ,  $15^\circ$ ,  $45^\circ$ , and  $75^\circ$  are shown on the opposite page

For  $60^\circ$ , describe an arc, and cut it with the same radius.

For  $120^\circ$ , describe an arc, and set off twice the radius.

For  $30^\circ$ , obtain an angle of  $60^\circ$ , and bisect it.

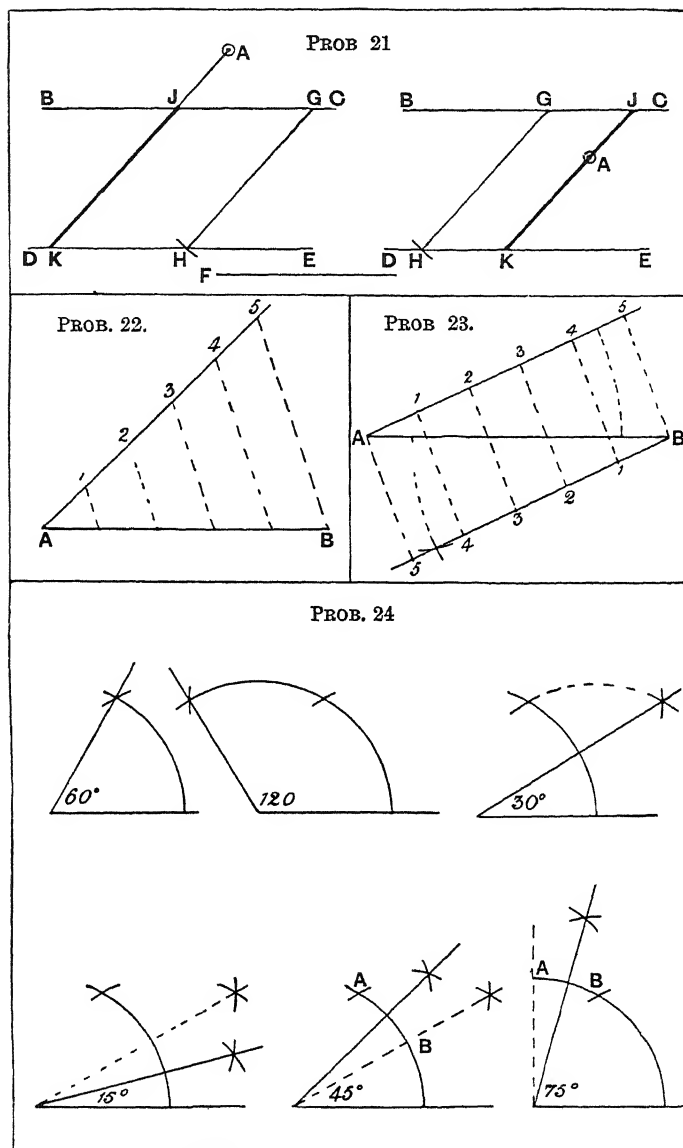
For  $15^\circ$ , obtain an angle of  $30^\circ$  as above, and bisect it

For  $45^\circ$ , obtain an angle of  $30^\circ$ , and bisect the arc *AB* ( $30^\circ + 15^\circ = 45^\circ$ .)

For  $75^\circ$ , draw a right angle, trisect it, and bisect the top division, *AB*.

**Note**—These angles may also be set out by using the T-square, with set-squares of  $45^\circ$  and  $60^\circ$ .

Other angles also may be easily made  $150^\circ = 120^\circ + 30^\circ$   
 $105^\circ = 90^\circ + 15^\circ$ .  $135^\circ = 90^\circ + 45^\circ$ .  $22\frac{1}{2}^\circ = \text{half } 45^\circ$ .  $67\frac{1}{2}^\circ = 45^\circ + 22\frac{1}{2}^\circ$ ; &c.



## CHAPTER IV

## PROPORTIONALS

If we compare two numbers with respect to the number of times one contains the other, a ratio is formed. Thus, as  $9 \cdot 3$  is a ratio, and means the same as the fractional expression  $\frac{9}{3}$ , or that 9 contains 3 three times. If we take two equal ratios, we have a proportion; for example,  $\frac{9}{3} = \frac{6}{2}$ ; or  $9 \cdot 3$  as  $6 \cdot 2$ .

The first term of a ratio is called the antecedent, and the second term, the consequent.

If the numbers are in true proportion, the product of the end terms, or extremes, equals the product of the middle terms, or means.

*'If a straight line be drawn parallel to one side of a triangle, it cuts the other two sides, or those produced, proportionally.'* (EUC. VI. 2.)

**PROBLEM 25.**—To divide a line in the same manner as another given divided line.

Let  $AB$  be the line. It is to be divided similarly to  $CD$ .

Draw a line at any angle to  $AB$ , and set off  $C1$ ,  $C2$ ,  $C3$ , and  $CD$  on it.

Join  $D'$  and  $B$ . Draw parallels to  $D'B$  from  $3'$ ,  $2'$ , and  $1'$ . Then  $AB$  will be divided similarly to  $CD$ .

**PROBLEM 26**—To divide a line into three parts in the same proportion as the numbers 2, 3, and 4.

Let  $AB$  be the given line. From  $A$  draw a line at any angle. Set off  $2 + 3 + 4$  equal parts. Join 9 and  $B$ . From the points 5 and 2 rule parallels cutting  $AB$ , so that  $AC : CD : DB$  as  $2 : 3 : 4$ .

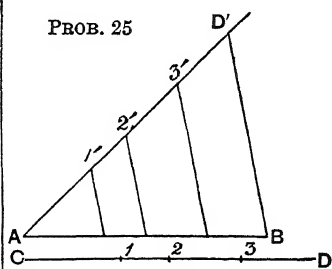
**PROBLEM 27**—To find a fourth proportional to three given lines.

Let  $A$ ,  $B$ , and  $C$  be the three given lines. Draw  $DJ$  and  $DK$ , of indefinite length, and at any angle to each other. Set off  $DE$  equal to  $A$ ,  $DF$  equal to  $B$ , and  $EG$  equal to  $C$ . Join  $E$  and  $F$ . From  $G$  rule a parallel to  $EF$ , cutting off  $FH$ , the fourth proportional required.  $DE : DF :: EG : FH$ .

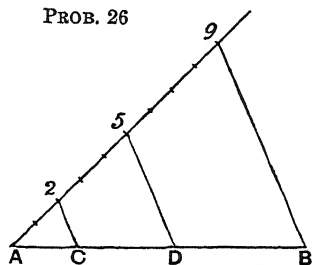
**Notes.**—1 This construction answers for all cases. It should be worked to scale, and the result verified by the student. If the three lines be 3,  $2\frac{1}{2}$ , and 2 inches long, then the fourth proportional will be found thus —  $3 : 2\frac{1}{2} :: 2 : FH$ . But  $3 FH = 2\frac{1}{2} \times 2$ . Therefore,  $FH = \frac{2\frac{1}{2} \times 2}{3} = 1\frac{2}{3}$  inches.

2 When the lines are long, it is sometimes more convenient to proceed as in **Problem 27a**. — Draw two lines at any angle as before, and set off  $DE$  equal to  $A$ , and  $DF$  equal to  $B$ . Make  $DG$  equal to  $C$ , and from  $G$  draw  $GH$  parallel to  $EF$ . Then  $DH$  will be the fourth proportional less than any of the given lines. The same method of construction is applicable when a fourth proportional greater than any of the given lines is required, but it is necessary to commence with the shortest line. See **Problem 27b**. Draw two lines at any angle, and set off  $DE$  equal to  $C$ ,  $DF$  equal to  $B$ , and  $DG$  equal to  $A$ . Join  $E$  and  $F$ , and from  $G$  draw  $GH$  parallel to  $EF$ . Then  $DH$  will be the fourth proportional greater.

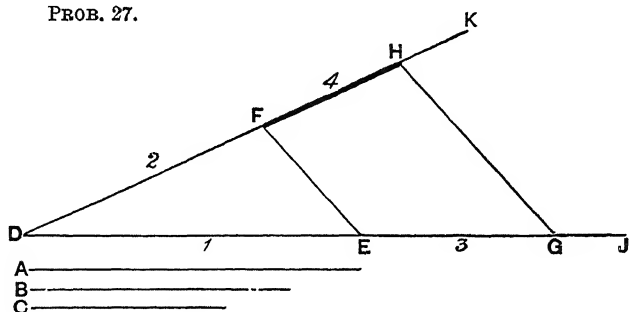
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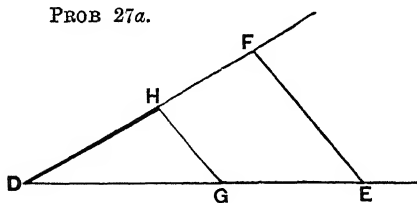
PROB. 26



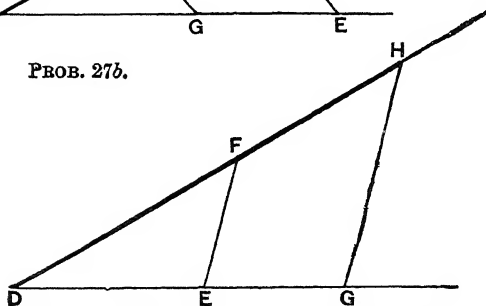
PROB. 27.



PROB 27a.



PROB. 27b.





**PROBLEM 28.**—To find a third proportional to two given lines.  
Let A and B be the given lines.

This problem is exactly the same as finding a fourth proportional to three magnitudes, the last two of which are equal. If we take numbers, for example, and require a third proportional to 8 and 4, the statement would read thus—8 4 as 4 2. Here 2 is manifestly the third proportional, because it bears the same relation to 4 that 4 does to 8.

In the case of the given lines, proceed in a similar manner, remembering that the line B is used as both the second and third term.

Draw two lines at any angle. Set off CD equal to A, and CE equal to B. Join D and E. Now set off DF, also equal to B. Draw FG parallel to DE. Then EG is the required third proportional.  $A : B$  as  $B : EG$ .

Note—The line B may be set off from C as in *Problem 27a* the parallel *fg* cutting off the third proportional *Cg*, which will be found to be of exactly the same length as *EG*.

**PROBLEM 29.**—To find a mean proportional to two given lines.

Let A B and C D be the given lines.

Produce A B, and set off BE equal to C D. Bisect A E, and describe a semicircle. At B draw BF perpendicular to A E. Then B F is the mean proportional required.  $AB : BF$  as  $BF : BE$ . (Euc. vi. 13)

Note—The square constructed on B F equals the rectangle constructed with sides equal to A B and C D.

**PROBLEM 30.**—To divide a line into an extreme and mean ratio—that is, so that one part shall be a mean proportional between the whole line and the other part.

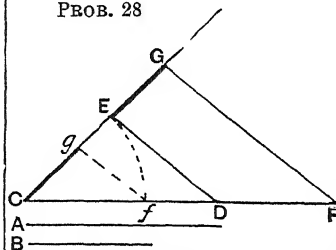
Let A B be the given line. Draw A C perpendicular to A B, and equal to half of it. Join B and C. With centre C and radius C A cut off C D. With centre B and radius B D cut off B E. Then the line is divided at E so that  $AE : EB$  as  $EB : AB$ , or so that the rectangle contained by A E and A B equals the square on E B. (Euc. vi. 30 and ii. 11.)

**PROBLEM 31.**—To make a proportional copy of any irregular figure, such as a plan, map, or picture.

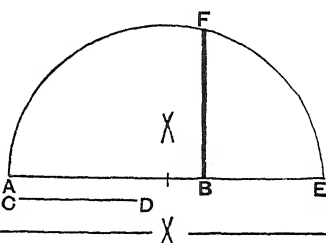
It is required to copy the given map so that the line A B shall be enlarged to the line X Y. Draw A C at right angles to A B. Set off equal spaces on A B and A C, and draw parallels forming a network of squares as shown. We now require the proportionate width of the new drawing—that is, the fourth proportional to the lines A B, X Y, and A C. Set off X Y at any angle to A B. Join A Y. Set off B C equal to A C. Draw C Z parallel to A Y. Then B Z is the required width, and  $AB : XY$  as  $AC : BZ$ .

The parallel E F cuts off B F equal to the side of a square in the new drawing; or X Y and X Z may be divided into the same number of equal parts as A B and A C are divided into. Draw parallels. With the pencil draw the figure carefully, taking care that each portion of the figure occupies a corresponding square to the original. For example, the point A is at the junction of vertical 3 and horizontal 5 on the original drawing, therefore it must occupy a similar position on the copy.

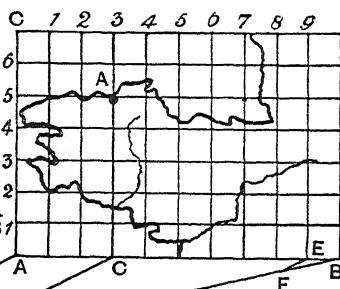
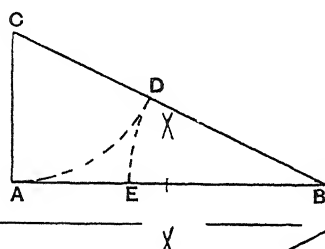
PROB. 28



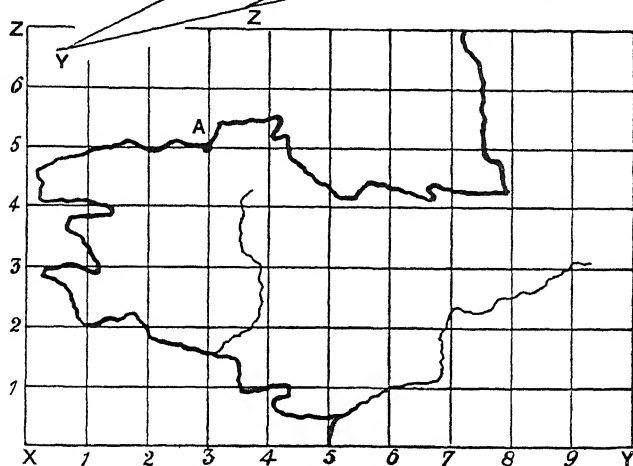
PROB 29



PROB 30



PROB. 31.



## CHAPTER V

## TRIANGLES

**PROBLEM 32.**—To construct an equilateral triangle on a given straight line.

Let  $AB$  be the given line. With centres  $A$  and  $B$ , and the line  $AB$  as radius, describe arcs intersecting at  $C$ . Join  $AC$  and  $BC$ . Then  $ABC$  is the required triangle. (Euc. I 1)

**PROBLEM 33.**—To construct an equilateral triangle, the altitude being given.

Let  $AB$  be the given altitude. At  $B$  draw  $CD$  at right angles to  $AB$ . With centre  $A$  and any radius describe an arc. On each side of  $AB$  construct an angle of  $30^\circ$  by cutting the arc in points 2 and 3 from centre 1, and bisecting 12 and 13 by the lines  $AC$  and  $AD$ .  $ADC$  is the required triangle.

**PROBLEM 34.**—To construct an isosceles triangle, the base and altitude being given.

Let  $AB$  be the base, and  $C$  the altitude. Bisect the base by the perpendicular,  $DE$ . Make  $DE$  equal to  $C$ . Draw  $AE$ ,  $BE$ , forming the required triangle,  $AEB$ .

**PROBLEM 35.**—To construct an isosceles triangle, the base and one side being given.

Let  $AB$  be the given base, and  $C$  the given side. With centres  $A$  and  $B$ , and radius equal to  $C$ , describe arcs intersecting at  $D$ . Join  $D$  with  $A$  and  $B$ .  $ABD$  is the required triangle.

**PROBLEM 36.**—To construct an isosceles triangle, the base and vertical angle being given.

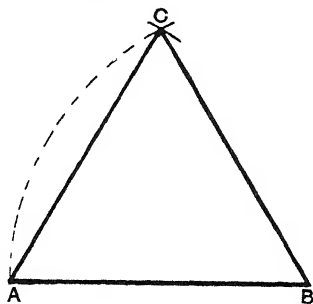
Let  $AB$  be the base, and  $C$  the given vertical angle. With centre  $C$  describe an arc cutting off equal distances  $C1$  and  $C2$ . Join 1 and 2, forming an isosceles triangle,  $C12$ . At  $A$  and  $B$  construct angles equal to the angle at 1. (Prob. 13)  $ABD$  will be the required triangle.

Note —  $ABD$  and  $12C$  are similar triangles. If the two base angles of the one equal the two base angles of the other, then the vertical angle  $D$  must equal the vertical angle  $C$ , because the three angles of any triangle must equal  $180^\circ$ . (Euc. I 32)

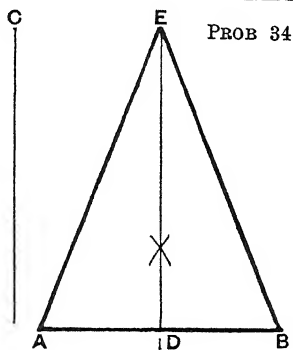
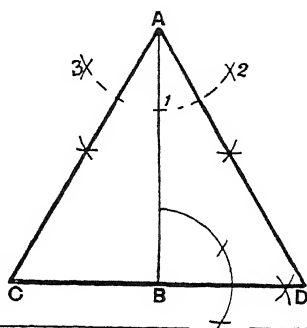
**PROBLEM 37.**—To construct an isosceles triangle, the altitude and the vertical angle being given.

Let  $AB$  be the altitude, and  $C$  the vertical angle. Draw  $DE$  perpendicular to  $AB$ . Bisect the angle  $C$ . At  $A$  construct angles  $BAE$  and  $BAD$ , each equal to half the angle  $C$ .  $DEA$  is the required triangle.

PROB. 32.

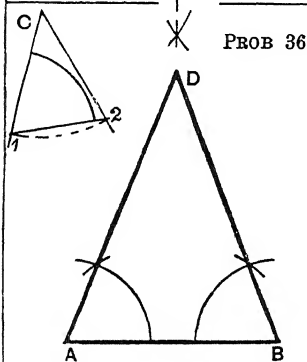
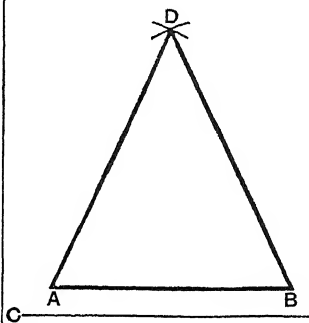


PROB. 33

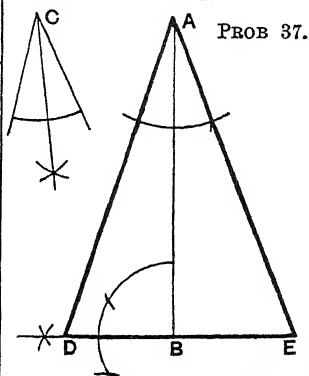


PROB. 34

PROB. 35.



PROB. 36



PROB. 37.

**PROBLEM 38.**—To construct an isosceles triangle, one of the equal sides and an angle at the base being given.

Let  $AB$  be the side, and  $C$  the given angle

Draw any line  $DE$ . At  $D$  make an angle equal to  $C$ , and cut off  $DF$  equal to  $AB$ . With centre  $F$  and radius  $FD$  describe an arc cutting  $DE$  in  $G$ . Draw  $FG$ . Then  $FDG$  is the required triangle

**PROBLEM 39.**—To construct a right-angled triangle, the base and hypotenuse being given.

Let  $AB$  be the base, and  $C$  the hypotenuse

At  $A$  erect a perpendicular. With centre  $B$  and radius equal to  $C$  cut the perpendicular in  $D$ . Join  $B$  and  $D$ .

**PROBLEM 40.**—To construct a right-angled triangle, the hypotenuse and an acute angle being given

Let  $AB$  be the hypotenuse, and  $C$  one of the acute angles

Bisect  $AB$  in  $D$ . With centre  $D$  describe a semicircle on  $AB$ . At  $A$  construct an angle,  $BAE$ , equal to  $C$ . Draw  $BE$ . Then  $BAE$  is the required triangle, the angle in a semicircle being a right angle. (EUC III 31.)

**PROBLEM 41.**—To construct a triangle, the three sides being given.

Let  $AB$ ,  $C$ , and  $D$  be the lengths of the three given sides.

With centre  $A$  and radius equal to  $C$  describe an arc. With centre  $B$  and radius equal to  $D$  intersect the arc in  $E$ . Draw  $EA$  and  $EB$ . Then  $ABE$  is the required triangle. (EUC I 22.)

**PROBLEM 42.**—To construct a triangle, the base and the two base angles being given.

Let  $AB$  be the base, and  $C$  and  $D$  the given angles.

At  $A$  and  $B$  make angles equal to the given angles  $C$  and  $D$ . Then  $ABE$  is the required triangle.

Note.—Take the same radius for all the arcs

**PROBLEM 43.**—On a given base to construct a triangle similar to a given triangle.

Let  $AB$  be the given base, and  $CDE$  the given triangle.

Make the angles at  $A$  and  $B$  equal to those at  $C$  and  $D$ , as in *Prob. 42*.

**PROBLEM 44.**—To construct a triangle, the altitude and two sides being given.

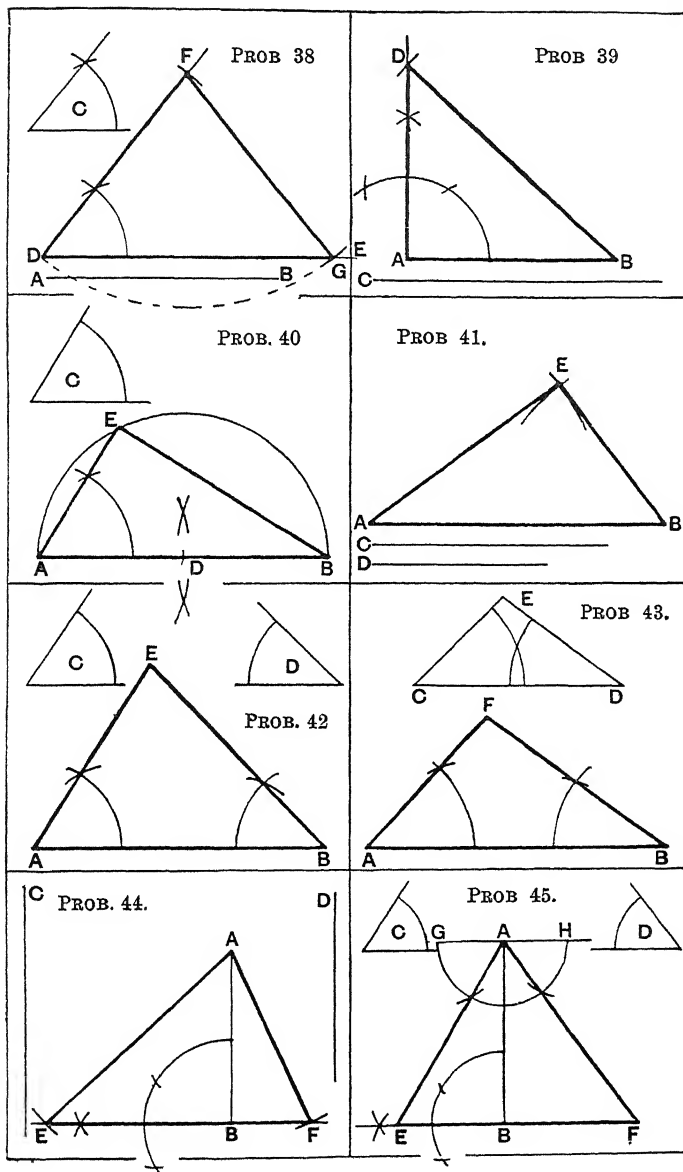
Let  $AB$  be the altitude, and  $C$  and  $D$  the lengths of the sides.

At  $B$  draw a base line at right angles to  $AB$ . With centre  $A$  and radius  $C$  cut the base in  $E$ . With centre  $A$  and radius  $D$  cut the base in  $F$ . Draw  $AE$ ,  $AF$ .

**PROBLEM 45.**—To construct a triangle, the altitude and base angles being given.

Let  $AB$  be the altitude, and  $C$  and  $D$  the base angles.

At  $A$  and  $B$  draw lines  $GH$  and  $EF$  perpendicular to  $AB$ . At  $A$  make the angle  $GAH$ , equal to  $C$ , and the angle  $FAH$ , equal to  $D$ .  $AEF$  is the required triangle.



**PROBLEM 46.**—To construct a triangle, the base, altitude, and one side being given.

Let  $AB$  be the base,  $C$  the altitude,  $D$  one of the sides.

Draw  $EF$  parallel to  $AB$ , at a distance equal to  $C$ .

With centre  $A$  and radius  $D$  cut  $EF$  in  $G$ .  $ABG$  is the required triangle.

**PROBLEM 47.**—To construct a triangle, the base, altitude, and vertical angle being given.

Let  $AB$  be the base,  $C$  the altitude, and  $D$  the vertical angle.

At  $A$  construct an angle,  $BAF$ , equal to  $D$ . Draw  $AG$  at right angles to  $AF$ . Bisect  $AB$  by the perpendicular  $EG$ . With centre  $G$  and radius  $GA$  describe the segment of a circle on  $AB$ .

Draw  $HJ$  parallel to  $AB$ , at a distance equal to  $C$ . Draw  $HA$  and  $HB$ . Then  $AHB$  is the required triangle.

Notes.—1. Any triangle on  $AB$  whose vertex lies in the arc  $AHJB$  will have its vertical angle equal to  $D$  (Euc III 33). Hence there will be two triangles satisfying the conditions,  $AHB$  and  $AJB$ .

2. This problem offers another solution to Problem 36. If an isosceles triangle were required on base  $AB$ , with its vertical angle equal to  $D$ , it would only be necessary to join  $A$  and  $B$  with  $K$  (See Prob. 47a).

**PROBLEM 48.**—To construct a triangle, the base, one side, and the angle opposite the base being given.

Let  $AB$  be the given base,  $C$  the side, and  $D$  the angle.

At  $A$  construct an angle,  $BAF$ , equal to the angle  $D$ .

Draw  $AG$  perpendicular to  $AF$ . Bisect  $AB$  by the perpendicular  $EG$ . With centre  $G$  and radius  $GA$  describe the segment  $AHB$ . With centre  $A$  and radius equal to  $C$  cut the segment in  $H$ . Then  $AHB$  is the required triangle.

**PROBLEM 49.**—To construct a triangle, the base, the sum of the other two sides, and one of the base angles being given.

Let  $AB$  be the base,  $C$  the sum of the other two sides, and  $D$  one of the base angles. At  $A$  make an angle equal to  $D$ .

Cut off  $AE$  equal to  $C$ . Draw  $BE$ , and bisect it by the perpendicular  $FG$ . Join  $G$  and  $B$ . Then  $AGB$  is the required triangle.

Note.—The triangle  $BGE$  is isosceles, therefore  $GE = GB$ , and  $AG, GB = AE$ .

**PROBLEM 50.**—To construct a triangle, the base and the ratio of the angles being given.

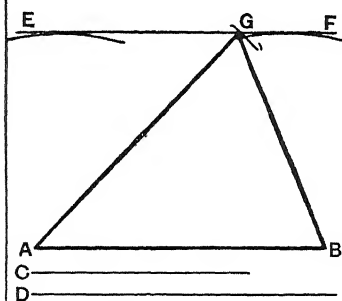
Let  $AB$  be the base, and let the angles be as  $2 : 3 : 4$ .

Produce  $AB$ , and describe a semicircle. Divide this semicircle in 9 parts ( $2 + 3 + 4$ ). Draw  $A2$  and  $A5$ , giving the three angles of the triangle. At  $B$  construct an angle,  $ABC$ , equal to the angle  $9A5$ , and produce  $A2$  to  $C$ , forming the required triangle,  $ABC$ .

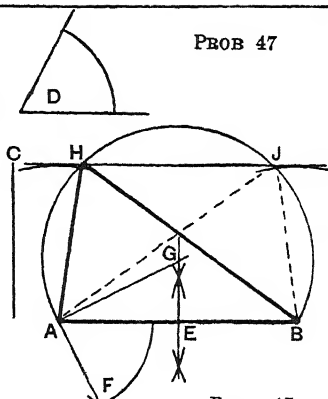
Notes.—1.  $BC$  may be drawn parallel to  $A5$  to form the angle  $ABC$ .

2. The three angles,  $9A5$ ,  $5A2$ , and  $2AB$ , are equal to two right angles, therefore they must equal the angles of the triangle  $ABC$ .

PROB. 46.

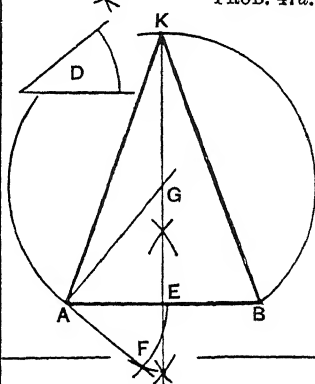
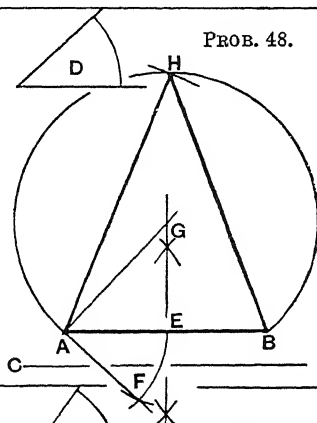


PROB 47



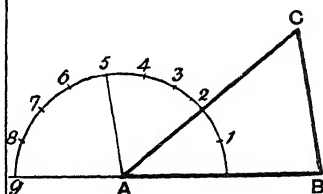
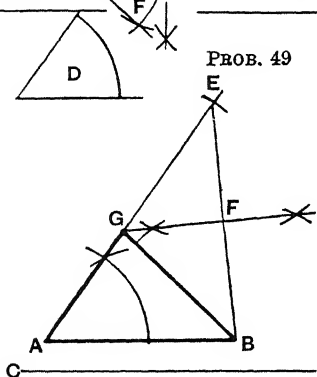
PROB. 47a.

PROB. 48.



PROB. 50

PROB. 49





**PROBLEM 51.**—To construct a triangle, the perimeter and two angles being given. (The *perimeter* of any plane figure equals the sum of its sides)

Let  $AB$  be the perimeter, and  $C$  and  $D$  the two angles.

On  $AB$  construct a triangle with its base angles equal to the angles  $C$  and  $D$ . Bisect the angles at  $A$  and  $B$  by lines meeting at  $F$ . From  $F$  draw  $FG$  parallel to  $AE$ , and  $FH$  parallel to  $EB$ , giving  $FGH$ , the required triangle.

Note— $AGF$  and  $BHF$  are isosceles triangles, therefore  $AG = GF$ , and  $BH = HF$ . The angle  $FGH =$  the angle  $EAB$ , and the angle  $FHG =$  the angle  $EBA$ . (Euc I 29)

**PROBLEM 52.**—To construct a triangle, the perimeter and the proportion of the sides being given.

Let  $AB$  be the perimeter, and let the sides be as  $3 : 4 : 5$ . Draw a line at any angle to  $AB$ , and set off  $3 + 4 + 5$  equal parts. Join  $B$  and 12. Divide  $AB$  so that  $AC$ ,  $CD$ , and  $DB$  shall equal 3, 4, and 5 parts respectively. With centre  $C$  and radius  $CA$  describe an arc, and with centre  $D$  and radius  $DB$  intersect the first arc in  $E$ . Draw  $DE$  and  $CE$ , forming the required triangle,  $CDE$ .

**PROBLEM 53.**—To construct a triangle, the base, the ratio of the other two sides, and the angle opposite to the base being given

Let  $AB$  be the given base,  $C$  the angle opposite to the base, and let the two sides be as  $5 : 6$ .

Draw any line,  $DE$ . At  $D$  make an angle equal to  $C$ . On  $DE$  set off five, and on  $DF$ , six, equal parts. Join 5 and 6. On line  $56$  cut off  $6G$  equal to  $AB$ . Draw  $GH$  parallel to  $D5$ . Then  $GH6$  is the required triangle, and is similar to the triangle  $65D$ .

Note—If  $AB$  be longer than  $56$ , then both  $56$  and  $6D$  must be produced as in Problem 53a.

## CHAPTER VI

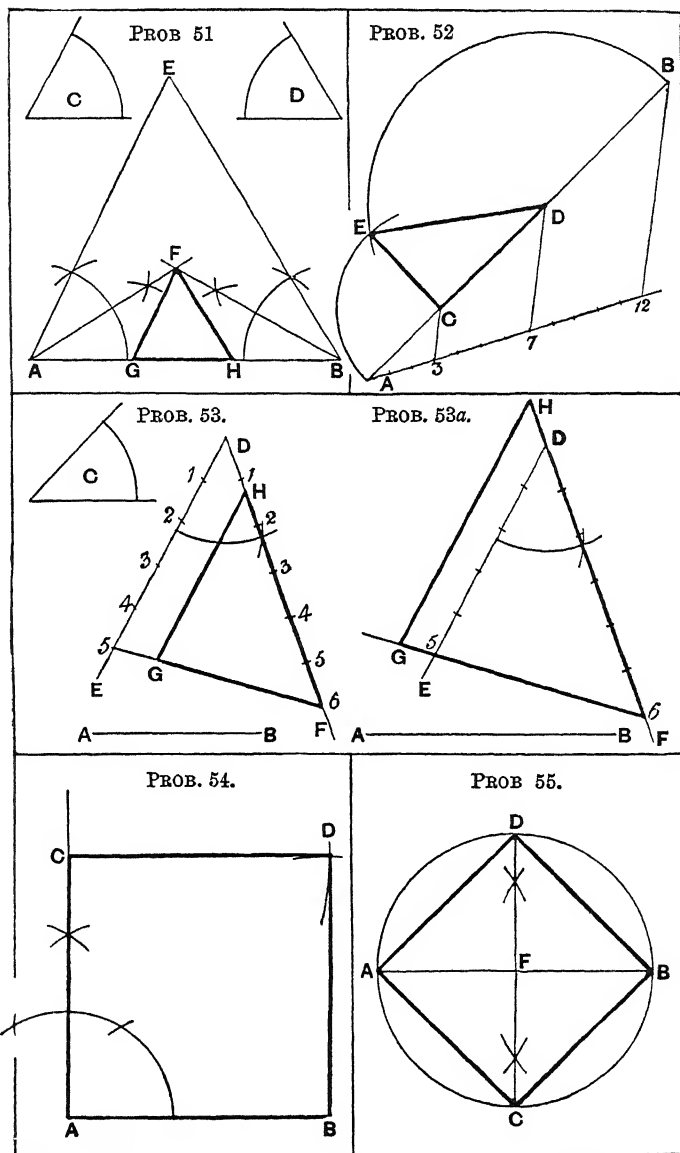
### QUADRILATERALS

**PROBLEM 54.**—To construct a square, the side being given.

Let  $AB$  be the given side. At  $A$  erect a perpendicular,  $AC$ , and make it equal to  $AB$ . With centres  $C$  and  $B$  and radius  $AB$  describe arcs intersecting at  $D$ . Draw  $CD$ ,  $BD$ .

**PROBLEM 55.**—To construct a square, the diagonal being given.

Let  $AB$  be the diagonal. Bisect  $AB$  by the perpendicular  $CD$ . With centre  $F$  and radius  $FA$  describe arcs cutting the perpendicular in  $C$  and  $D$ . Draw  $AD$ ,  $DB$ ,  $BC$ , and  $CA$ .



**PROBLEM 56.**—To construct a rectangle, the two sides being given.

Let  $AB$  and  $CD$  be the two sides. At  $A$  erect a perpendicular  $AE$ , and make it equal to  $CD$ . With centre  $E$  and radius  $AB$  describe an arc, and with centre  $B$  and radius  $CD$  intersect the arc in  $F$ . Draw  $EF$ ,  $FB$ .

**PROBLEM 57.**—To construct a rectangle, the diagonal and one side being given.

Let  $AB$  be the diagonal, and  $CD$  the side. Bisect  $AB$  in  $F$ .

With centre  $F$  and radius  $FB$  describe a circle. With radius  $CD$  and centres  $A$  and  $B$  cut the circle in  $G$  and  $H$ . Draw  $AG$ ,  $GB$ ,  $BH$ ,  $HA$ .

Note.—The angle in a semicircle is a right angle (Euc III 31.)

**PROBLEM 58.**—To construct a rhombus, the side and one of the angles being given.

Let  $AB$  be the given side, and  $C$  the given angle.

At  $A$  construct an angle equal to  $C$ , and make  $AD$  equal to  $AB$ .

With centres  $D$  and  $B$  and radius  $AB$  describe arcs intersecting at  $E$ . Draw  $DE$ ,  $BE$ .

**PROBLEM 59.**—To construct a rhombus, the diagonal and side being given.

Let  $AB$  be the diagonal, and  $C$  the side. With radius  $C$  and centres  $A$  and  $B$  describe arcs intersecting at  $D$  and  $E$ . Draw  $AD$ ,  $DB$ ,  $BE$ ,  $EA$ .

**PROBLEM 60.**—To construct a rhomboid, the two sides and one of the angles being given.

Let  $AB$  and  $C$  be the two sides, and  $D$  the given angle.

At  $A$  construct an angle equal to  $D$ , and make  $AE$  equal to  $C$ .

With centre  $E$  and radius  $AB$  describe an arc, and with centre  $B$  and radius  $C$  intersect the arc at  $F$ . Draw  $EF$ ,  $BF$ .

**PROBLEM 61.**—To construct a rhomboid, the diagonal and the two sides being given.

Let  $AB$  be the diagonal, and  $C$  and  $D$  the two sides.

With radius  $C$  and centres  $A$  and  $B$  describe two arcs.

With radius  $D$  and the same centres intersect the arcs in  $E$  and  $F$ . Draw  $AE$ ,  $EB$ ,  $BF$ ,  $FA$ .

**PROBLEM 62.**—To construct a trapezium, the diagonal and two pairs of equal sides being given.

Let  $AB$  be the diagonal, and  $C$  and  $D$  the sides.

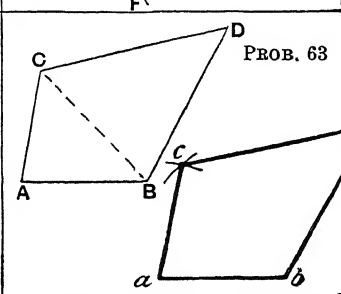
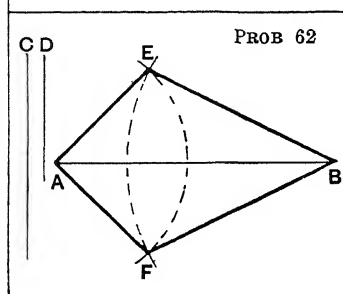
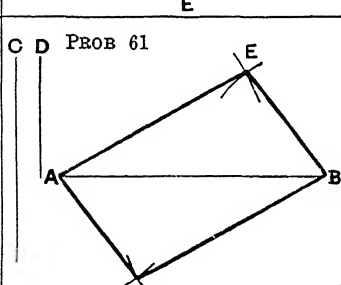
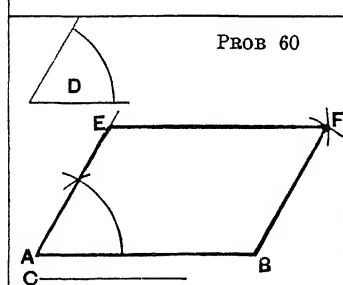
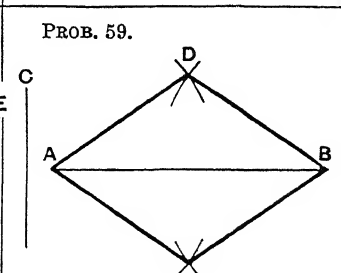
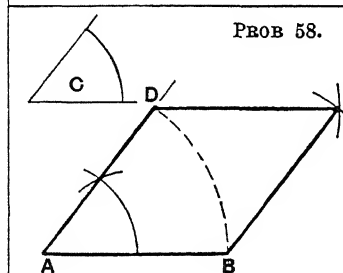
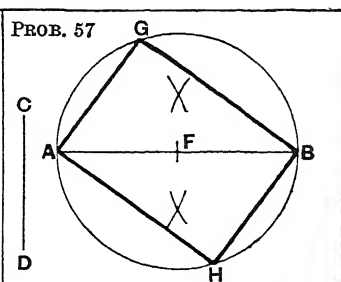
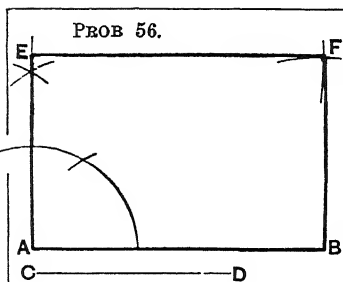
With centre  $B$  and radius  $C$  describe an arc. With centre  $A$  and radius  $D$  intersect this arc in  $E$  and  $F$ . Draw  $AF$ ,  $FB$ ,  $BE$ , and  $EA$ .

**PROBLEM 63.**—To construct a trapezium equal to another given trapezium.

Let  $ABCD$  be the given trapezium. Draw  $ab$  equal to  $AB$ .

With centres  $a$  and  $b$ , and radii equal to  $AC$  and  $BC$ , describe arcs intersecting in  $c$ . With centres  $b$  and  $c$ , and radii equal to  $CD$  and  $BD$ , describe arcs intersecting in  $d$ . Draw  $ac$ ,  $cd$ , and  $bd$ .

Note.—The same principle of cutting the figure into triangles may be applied to figures with any number of sides.



## EXERCISES.

Note—The exercises taken from the examination papers of the Science and Art Department in Subject I. are marked with the letters *Sc.*, and those from the Art examinations are marked *Art*. They should be worked in connection with the chapter to which they relate.

## CHAPTER III

1 Draw a line 3" long; mark a point  $1\frac{3}{4}$ " below it, and from this point draw a perpendicular to the line

2 Draw a line, *A B*, 2 5" long. From a point *C* in it, 5" from *A*, draw a line making  $45^\circ$  with *C B*, and at *B* draw a line at  $75^\circ$  with *B C* to meet it

3. Draw a line 3 5" long. Divide it into 7 equal parts, and on  $\frac{4}{7}$  of the line describe a semicircle

4 Draw a line 3" long, and erect a perpendicular from a point  $\frac{1}{2}$ " from one end, and without the line, without producing it

5 From a point *A* draw two lines, *A B*, *A C*, making with each other an angle of  $75^\circ$ , bisect this angle

6. Set off with your protractor an angle of  $57^\circ$ , and divide it into 4 equal parts.

7. At the extremities of a line 3" long erect perpendiculars 2" and  $2\frac{1}{2}$ " long. From the upper ends of these perpendiculars draw two equal lines meeting each other in the first line (*Prob* 17)

8 From *A* and *B*, two points 2" apart, draw two right lines to meet at an angle of  $70^\circ$  (*Art*)

9 Through a point *C*,  $1\frac{1}{2}$ " from a line *A B*, draw two right lines, one parallel and the other perpendicular to *A B* (*Art*)

10 Draw two lines intersecting at an angle of  $52^\circ$  and between them place a line 2 25" long, making  $58^\circ$  with one of them (*Sc.*)

Note—Draw any line 2 25" long, and making  $58^\circ$  with one line, and through its extremity draw a parallel to the one line until it meets the other. A line from the point of intersection will be the line required.

11. Draw a line 3 5" long, and at one extremity erect a perpendicular 1 75" long. From the top of this perpendicular draw a line to make an angle of  $80^\circ$  with the given line. (*Sc.*) (*Prob* 14.)

12. Draw any two parallel lines, *A* and *B*,  $1\frac{1}{2}$ " apart. Mark a point *P*,  $\frac{3}{4}$ " above *A*. Through *P* draw a line cutting the given lines in points  $1\frac{1}{8}$ " apart (*Sc.*) (*Prob* 21)

13. *ABC* is an obtuse angle, and *X* is a point within it. Determine *X*'s position when *AB* = 2 33", *BC* = 3 15", *ABC* =  $160^\circ$ , *ABX* =  $65^\circ$ , *BCX* =  $37^\circ$ . State the lengths of *AX* and *BX*. (*Sc.*)

## CHAPTER IV

1 Find a fourth proportional to three lines whose lengths are  $2\frac{1}{4}$ ",  $3\frac{3}{8}$ ",  $11\frac{3}{10}$ " respectively

2 Find a mean proportional between two lines, 2" and 1" long, and figure its length on the line

3 Draw a line 8 75" long, and divide it in the proportion of the numbers 2, 7, 3, 6. Figure the parts

4 Find a third proportional to two lines,  $1\frac{1}{2}$ " and  $1\frac{3}{4}$ " long

5 Find a line which shall have the same ratio to a line  $1\frac{3}{4}$ " long that  $2\frac{1}{2}$ " has to  $1\frac{1}{4}$ "

Note— $1\frac{1}{2} \cdot 2\frac{1}{2} = 1\frac{3}{4} \cdot \text{required line}$

6. Draw a line 2" long, and divide it into 4 unequal parts. Draw another line two-thirds of the length of the first line, and divide it proportionally to it. (*Prob* 25)

7. From the extremity *A*, of a line *A B*, obtain the  $\frac{1}{2}$ ,  $\frac{1}{3}$ , and  $\frac{1}{5}$  of the line

8 Divide a line 3 25" long into 4 parts, *A*, *B*, *C*, *D*, so that *B* is double of *A*, *C* three times *A*, and *D* four times *A*.

9. Produce a line,  $AB$ , 3" long, to a point  $P$ , so that  $BP : AB$  as 3 : 5. (Sc.)

Note.—Draw a line at any angle, and set off 5+3 parts. Join 5  $B$ , and from 3 draw 3  $P$  parallel to 5  $B$ .

10. Find a point,  $P$ , in a line  $AB$  (2" long), produced, so that  $AP : AB$  as 7 : 4. (Sc.)

Note.—Draw a line at any angle to  $AB$ , and set off 7 parts. Join 4  $B$ . From 7 draw 7  $P$  parallel to 4  $B$ .

11. Divide a line,  $AB$ , 3 5" long, in a point  $P$ , so that the rectangle contained by  $AB$ ,  $AP$  may be equal to the square on  $BP$ . (Sc.) (Prob. 30.)

12. Find a mean proportional to two lines, 2 5" and 1 5" long respectively. State any Problem that you know of which this is the solution. (Sc.) (Euc II 14.)

## CHAPTER V

1. Construct a triangle with sides  $AB=8''$ ,  $AC=2\frac{5}{8}''$ ,  $BC=1\frac{9}{10}''$ , and construct an angle equal to  $BAC$ .

2. Two lines,  $A$  and  $B$ , 4" long, contain an angle of  $45^\circ$ . From a point  $C$  in one line draw a line to make equal angles with the two converging lines.

3. Upon a base of  $2\frac{1}{2}''$  construct a triangle having two of its angles  $75^\circ$  and  $45^\circ$  respectively, and then construct a similar triangle on a base of 2". (Art.)

4. An isosceles triangle has a base of  $1\frac{1}{2}''$  and a vertical angle of  $42^\circ$ . Construct it by using Problem 47a.

5. Describe a segment of a circle which shall contain an angle equal to the angle of an equilateral triangle. (Prob 47.)

6. Construct an isosceles triangle with its equal sides  $2\frac{1}{2}''$  long and the included angle  $30^\circ$ . On the same base describe another isosceles triangle with its vertical angle double that of the first triangle. (Art.)

Note.—Bisect the sides, and find the centre for the circumscribing circle. The angle at the centre of a circle is double the angle at the circumference, standing upon the same base. (Euc III 20.)

7. Draw a triangle, two of whose sides are 2 5" and 3" respectively, the angle opposite the shorter side being  $40^\circ$ . (Sc.) (Prob 38.)

8. Draw a triangle having its vertical angle  $30^\circ$ , the base 1 7", and the sides as 4 : 5. (Prob 53.)

9. Construct a triangle of which the sides are as 1, 1 5, 2, the perimeter being 4". (Prob 52.)

10. Construct a triangle on a base of 2", altitude 1 75", and the angle opposite the base  $42^\circ$ . (Sc.) (Prob 47.)

11. Construct a triangle whose sides,  $ab$ ,  $bc$ ,  $ca$ , are  $3\frac{1}{4}''$ ,  $2\frac{3}{8}''$ , and 2" respectively. On  $ac$  construct a second triangle,  $adc$ , whose vertical angle,  $adc$ , is equal to the angle  $abc$ , and the side  $ad$   $1\frac{5}{8}''$ . (NB.—The angles upon the same base and in the same segment of a circle are equal.) (Sc.)

Note.—At  $a$  draw a line making with  $ac$  an angle equal to  $abc$ , and use Problem 48.

## CHAPTER VI

1. Construct a square of 4" sides, bisect the sides, and join the adjacent points of bisection, thus obtaining a second square, bisect the sides of this square, and obtain a third square. Continue the process until five squares have been drawn. (Sc.)

2. Construct a rhomboid, one diagonal being 2", and the adjacent sides  $1\frac{1}{2}''$  and 1" respectively. (Prob 61.)

3. Construct a rhombus having an angle of  $65^\circ$  and a base of 3". Measure its two diagonals accurately, and write down their lengths.

4. The adjacent sides of a trapezium are 2 3" and 1 8" long respectively. The included angle is  $60^\circ$ . The other sides are  $2\frac{3}{4}''$  and 3". Construct the figure, and give the length of the longest diagonal.

5. Upon a line  $1\frac{1}{2}''$  long describe a square, and divide it by parallel lines, alternately thin and dotted, into 5 equal rectangles.

6. The diagonals of a parallelogram 2 4" and 4 2" long contain an angle of  $61^\circ$ . Construct the parallelogram. (Sc.)

## CHAPTER VII

## THE CIRCLE AND TANGENTS

**PROBLEM 64.**—To find the centre of a circle

Draw any two chords,  $AB$ ,  $BC$ . Bisect them by lines at right angles. The point  $D$ , where the bisecting lines intersect, is the centre. (Euc. III. 1, Cor)

**PROBLEM 65.**—To describe a circle passing through three given points not in the same straight line.

Let  $A$ ,  $B$ , and  $C$  be the three points.

Join  $AB$ ,  $BC$ . Bisect both lines as above. From  $D$ , the point where the bisecting lines meet, describe the circle.

**PROBLEM 66.**—To describe a circle about a triangle

Bisect two sides, and proceed as in Problem 64.

**PROBLEM 67.**—To describe an arc equal to a given arc, and having the same radius

Draw any two chords,  $AB$  and  $CD$ , in the given arc. Bisect them, and find the centre  $E$ , from which the arc was described. With centre  $O$  and radius  $EF$  describe the arc  $GH$ , and make it equal in length to the given arc.

**PROBLEM 68.**—To draw a tangent to a circle through a given point in the circumference.

Let  $A$  be the given point. Find the centre,  $B$ . Draw  $BA$  and produce, making  $AC$  equal to  $AB$ . With centres  $B$  and  $C$  describe arcs intersecting at  $D$ . Draw  $DA$ , the required tangent.

Notes.—1 This method of drawing the perpendicular is preferable to the other methods when there is room to produce the radius, as it is likely to be more accurate

2 A tangent is always at right angles to the radius.

**PROBLEM 69.**—To draw a tangent to a circle from a given point without it.

Let  $A$  be the given point. Find the centre,  $B$ , and draw  $BA$ . Bisect  $BA$ , and describe a semicircle cutting the given circle in  $C$ . Draw  $AC$ , the required tangent.

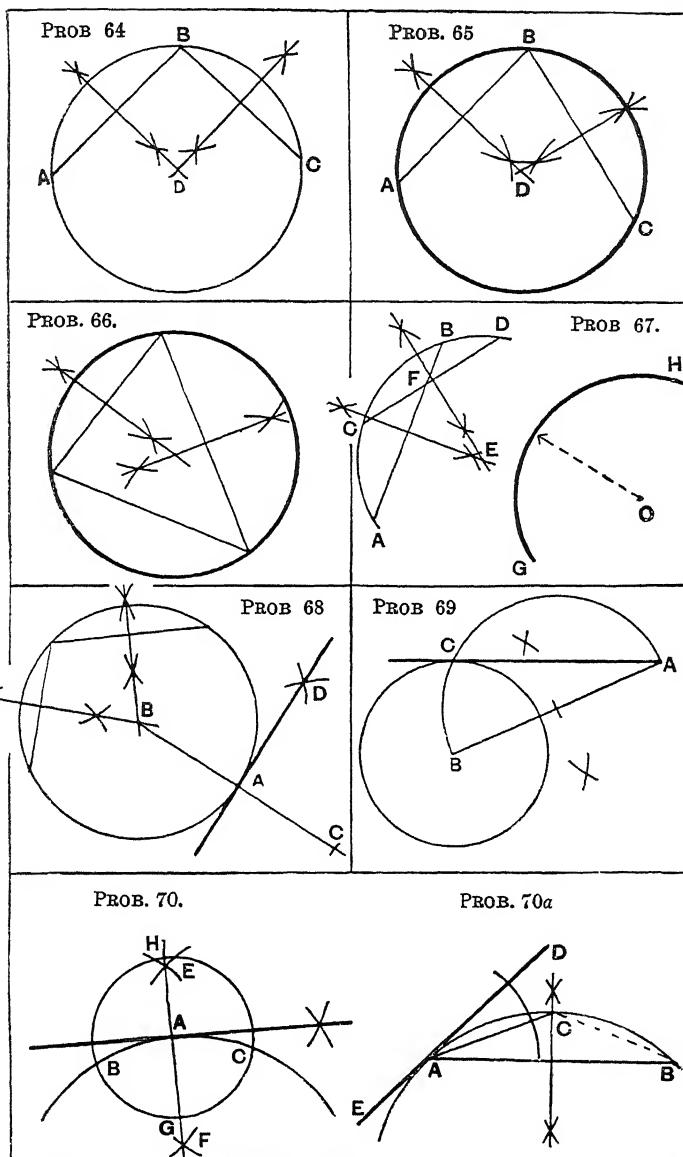
Note.—If the whole circle on  $AB$  were described, another tangent might be obtained

**PROBLEM 70.**—To draw a tangent to an arc from a point in it, when the centre is not accessible.

Let  $A$  be the given point.

With centre  $A$  describe a circle. With centres  $B$  and  $C$  describe arcs intersecting in  $E$  and  $F$ . Draw  $EF$ . At  $A$  draw the tangent at right angles to  $EF$ , using  $H$  and  $G$  as centres for the intersecting arcs.

*Another solution is shown in Problem 70a.* Draw a chord,  $AB$ , from the given point  $A$ . Bisect this chord. Draw  $CA$ . Make the angle  $CAD$  equal to  $CAB$ . Then  $DE$  is the required tangent, the angle  $DAC$  being equal to the angle  $ABC$ . (Euc. III. 32)





**PROBLEM 71.**—To draw a tangent to a circle which shall be parallel to a given straight line.

Let  $AB$  be the given straight line. Find the centre,  $C$ , of the circle. From  $C$  draw a perpendicular to  $AB$ . Through  $D$ , the point where the perpendicular cuts the circumference, draw  $EF$  at right angles to  $CD$ .

Note.—In all problems where the centre of the circle is not given, it must be found as shown in Problem 64

**PROBLEM 72.**—To draw two tangents to a circle to meet at a given angle.

Let the given angle in this case equal  $60^\circ$

From the centre,  $B$ , draw any line,  $BC$ . Take any point,  $C$ , and on each side of the line  $BC$  construct an angle equal to *half* the given angle—in this case  $30^\circ$ . Now apply Problem 71, and draw perpendiculars  $BE$ ,  $BD$ . Through  $F$  and  $G$  draw parallels to  $EC$  and  $DC$ .

**PROBLEM 73.**—To draw a common tangent to two equal circles which shall not cross the line joining their centres. (This is commonly called an exterior tangent)

Join the centres  $A$  and  $B$ . At  $A$  and  $B$  draw perpendiculars  $AC$ ,  $BD$ . Through the points  $C$  and  $D$  draw the exterior tangent.

**PROBLEM 74.**—To draw a common tangent to two equal circles which shall cross the line joining their centres. (Interior tangent)

Join the centres  $A$  and  $B$ . Bisect  $AB$  in  $C$ , and  $AC$  in  $D$ . With centre  $D$  and radius  $DA$  describe a semicircle. Draw  $AE$ , and from  $B$  draw  $BF$  parallel to  $AE$ . (Use set square.) Through  $E$  and  $F$  draw the interior tangent.

**PROBLEM 75.**—To draw an exterior tangent to two unequal circles.

Join the centres  $A$  and  $B$ . From  $C$  set off  $CD$  equal to  $BE$ , the radius of the smaller circle. With centre  $A$  and radius  $AD$  (the *difference* of the radii of the given circles) describe a circle. Bisect  $AB$ , and describe a semicircle. From  $B$  draw  $BF$ , a tangent to the small, described circle. Through  $F$  draw  $AG$ , and from  $B$  draw  $BH$  parallel to  $AG$ . Through the points  $G$  and  $H$  draw the exterior tangent.

**PROBLEM 76.**—To draw an interior tangent to two unequal circles.

Join the centres  $A$  and  $B$ . From  $C$  set off  $CD$  equal to  $BE$ . With centre  $A$  and radius  $AD$  (the *sum* of the radii of the given circles) describe a circle. Bisect  $AB$ , and describe a semicircle. From  $B$  draw  $BF$ , a tangent to the large, described circle. Draw  $AF$ , and from  $B$  draw  $BG$  parallel to  $AF$ . Through the points  $G$  and  $H$  draw the interior tangent.

Note.—In Problems 73, 74, 75, 76, two tangents might be drawn, if required, as shown in dotted line on Problem 74.



## CHAPTER VIII

### REGULAR POLYGONS

**PROBLEM 77.**—To construct ANY regular polygon on a given line, A B (in this case a pentagon).

Produce A B, and with centre A and radius A B describe a semicircle. Divide the semicircle into as many equal parts as the required figure has sides (in this case five) with the dividers. Join A with 2, giving another side of the polygon. (Always join A with the second division for any polygon.) Bisect the two sides, A B and A 2, by lines meeting at O, giving the centre of the polygon. With centre O and radius O B describe a circle. Mark off B E and E D equal to A B. Draw B E, E D, D 2, forming the required pentagon.

**Notes.**—1 The accuracy of this construction depends upon the correct division of the semicircle and the care with which the centre is obtained.

2 A semicircle may be divided into three equal parts by marking off its radius three times. To get 6 equal parts, bisect each third part, to get 9, trisect each third part. To get 4 equal parts, bisect each half, by bisecting again, 8 will be obtained. If the set squares are perfectly accurate, the semicircle may be divided into 4 by using the  $45^\circ$ , and into 8 by using the  $60^\circ$  set square.

**PROBLEM 78.**—To construct a regular hexagon on a given line, A B. (Special method.)

With centres A and B and radius A B describe arcs intersecting in O. With centre O and the same radius describe a circle, and set off A B round it. Join the points, thus forming the required hexagon.

**Note.**—The side B C may be obtained by using the  $60^\circ$  set square, and the hexagon quickly set up by this means.

**PROBLEM 79.**—To construct a regular octagon on a given line, A B. (Special method.)

Erect perpendiculars at A and B. Produce A B both ways. With centres A and B and radius A B describe quadrants. Bisect each quadrant, obtaining B E and A F, two more sides of the octagon. From E and F draw parallels to A C and B D, and make them equal to A B. Draw E F and G H. Make K D and L C equal to J B. Join the points H, D, C, G.

**Notes.**—1 The lines from A and B through M and N will also give the points H and G.

2 B E, A F, H D, G C may be readily obtained by using the  $45^\circ$  set square.

**PROBLEM 80.**—To construct a regular polygon by using the protractor.

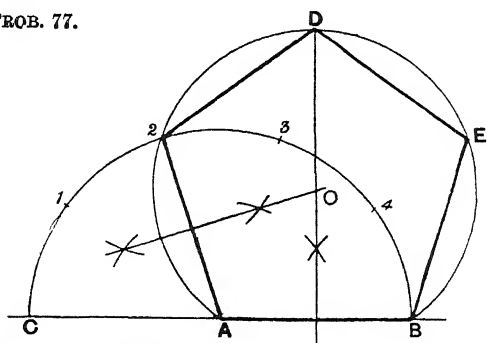
The number of degrees in each angle of a regular polygon may be readily found as follows:—*From twice as many right angles as the figure has sides subtract four right angles, and divide this result by the number of angles the figure contains.* (Euc. I. 32, Cor.)

Suppose the regular polygon to be a pentagon.

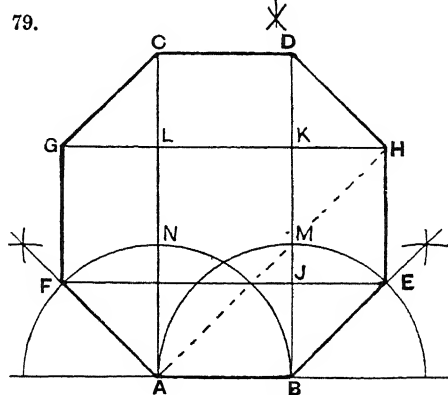
The pentagon has five sides, therefore take ten right angles. Deduct four, leaving six right angles. The solution will then be  $\frac{6 \times 90^\circ}{5} = 108^\circ$ . The angles for other polygons may be found in the same manner. For an octagon the solution would be  $\frac{12 \times 90^\circ}{8} = 135^\circ$ .

Place the protractor on A B, with its centre on B (see page 2). Mark  $108^\circ$ , reading from the left. Place the centre on A, and mark  $108^\circ$ . Make A D and B C equal to A B. With centres D and C and radius A B describe arcs intersecting in E. Draw D E, E C.

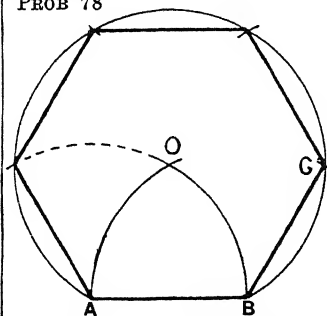
PROB. 77.



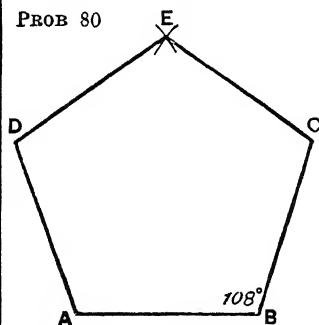
PROB 79.



PROB 78



PROB 80



**PROBLEM 81.**—To inscribe ANY regular polygon in a circle (approximate).

Draw the diameter A B. (If the centre of the circle be not given, draw a chord, and bisect it.) Divide A B into 7 equal parts. With centres A and B describe arcs intersecting at C. From C, through the second part, rule C D, cutting off A D, one of the sides of the polygon. Set off A D round the circle, and join the points.

Notes.—1 The greatest care must be exercised in dividing the line and in drawing the line from C exactly through the point 2

2 The circumference of the circle may be divided into the number of parts required with the dividers, and the points joined to form the figure

3 The hexagon, octagon, and duodecagon are more easily inscribed in a circle by special methods

**PROBLEM 82.**—To inscribe a regular hexagon in a circle. (Special method)

Draw the diameter A B. With centres A and B set off the radius on each side. Join the points.

**PROBLEM 83.**—To inscribe a regular octagon in a circle. (Special method)

Draw the diameter A B. Bisect it at right angles by the diameter C D. Bisect each quadrant thus formed, cutting the circumference into 8 equal portions. Join the points thus obtained.

**PROBLEM 84.**—To inscribe a regular duodecagon in a circle (Special method.)

Draw two diameters, at right angles to each other, as before.

With centres A, B, C, D, describe arcs passing through the centre of the circle and cutting the circumference. Join the points thus obtained, forming the required polygon.

**PROBLEM 85.**—To complete a regular polygon, two adjacent sides and the included angle being given.

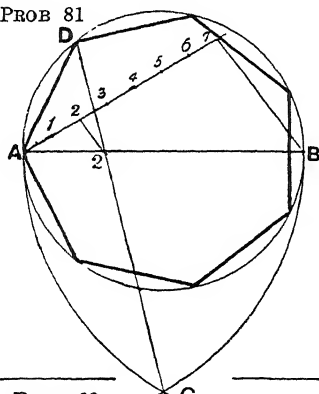
Let A B, B C, be the two sides, and A B C the included angle (in this case  $120^\circ$ ). Bisect the two sides. Describe a circle passing through the points A, B, C. Set off the length of one side round the circumference, and join the points thus obtained.

**PROBLEM 86** —To inscribe ANY regular polygon in a circle by using the protractor.

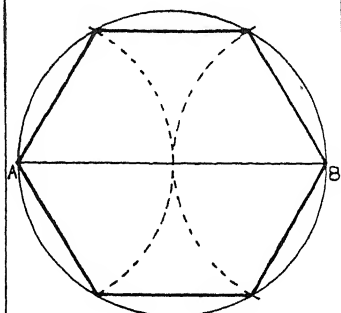
If all the angles of a polygon be joined with the centre, there will be formed as many equal isosceles triangles as the polygon has sides, having their vertical angles at the centre equal. All the angles at the centre equal four right angles. (Euc. I. 15, Cor. 2) Hence, if  $360^\circ$  be divided by the number of sides the regular polygon has, the magnitudes of the central angles will be obtained. For example, it is required to inscribe a regular pentagon in a circle by this method. The central angle in this case will be  $\frac{360^\circ}{5} = 72^\circ$ .

Draw any radius, O A. Make the angle A O B equal to  $72^\circ$ , with the protractor. Set off the distance A B round the circumference, and join the points. A B C D E is the required pentagon.

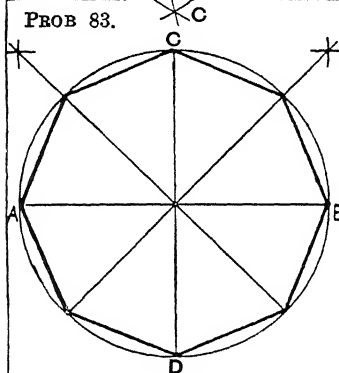
PROB 81



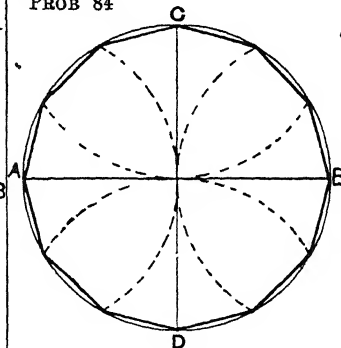
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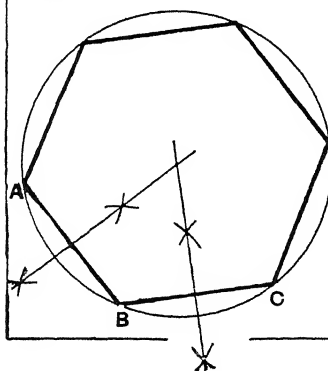
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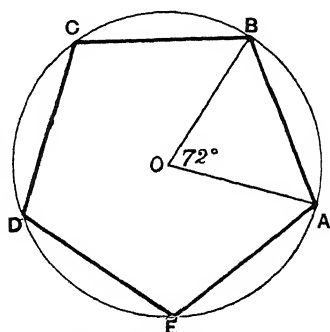
PROB 84



PROB. 85.



PROB 86.



**PROBLEM 87.**—To describe ANY regular polygon about a circle (say a pentagon).

Divide the circumference of the given circle into as many equal parts as the polygon has sides, in the same manner as for the inscribed polygon (*Prob. 81, Note 2*) From the centre, O, draw lines through each point. Draw A B, one of the sides of the inscribed pentagon. Draw the tangent C D parallel to A B. Make O E, O F, O G, equal to O D. Draw D E, E F, F G, G C, tangential to the circle. Then C D E F G is the required pentagon.

**PROBLEM 88.**—To construct ANY regular polygon, the length of any diagonal being given.

Let A B be the length of one of the *longer* diagonals of a regular heptagon.

On any base, C d, construct a regular heptagon. (*Prob. 77.*) From C draw the diagonals, and produce them, if necessary. Mark off C F equal to A B on one of the longer diagonals. Draw F E, E D, F G, G H, H J, parallel to the corresponding sides of the heptagon.

**Notes**—1 If the length of the shorter diagonals be given, proceed in an exactly similar manner.

2 If one of the longer diagonals of a hexagon, octagon, or duodecagon be given, bisect it, describe the circumscribing circle, and proceed as in Problems 82, 88, and 84

**PROBLEM 89.**—To construct ANY regular polygon, having the diameter given (say a pentagon).

**Note.**—The diameter divides the polygon into two equal parts. It is the line passing through the centre of the polygon, and terminated at the middle points of two opposite sides in a polygon having an *even* number of sides. When the number of the sides is *uneven*, the diameter is drawn through the centre, from one angle to the middle of the opposite side.

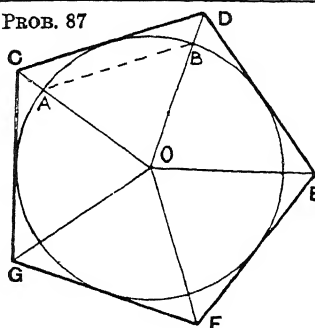
Let A B be the given diameter. Through B draw a line at right angles to A B. Set off B c, B d, any equal distances on each side of B. Describe a pentagon on c d. Through e and f draw lines from B. From A draw A E and A F parallel to a e and a f, and from E and F draw E D and F C parallel to e d and f c. Then C D E A F is the required pentagon.

**PROBLEM 90.**—To construct a regular pentagon, the diagonal being given. (Special method.)

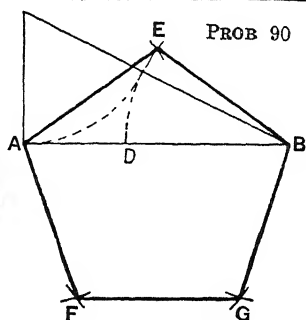
Let A B be the diagonal. An important property of the pentagon is, that if the diagonal be divided into an extreme and mean proportion (*Prob. 30*), the *greater segment* of the diagonal so divided will equal the *side* of the pentagon.

Divide A B into an extreme and mean ratio at D. Then B D, the greater segment, will be the side of the pentagon. With centres A and B and radius B D describe arcs intersecting at E. Draw B E, E A, two sides of the pentagon. With centre B and radius B A describe an arc. With centre A and radius A E intersect the arc in F. With centres F and B and radius F A describe arcs intersecting in G. Draw A F, F G, G B.

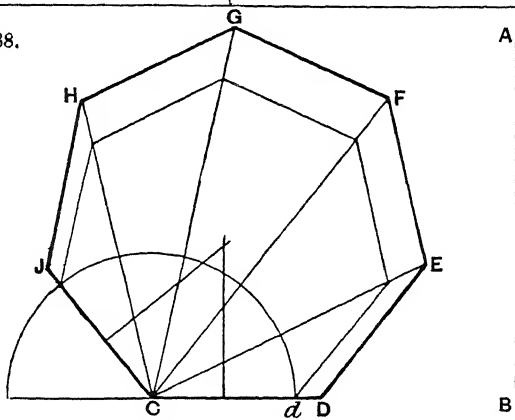
PROB. 87



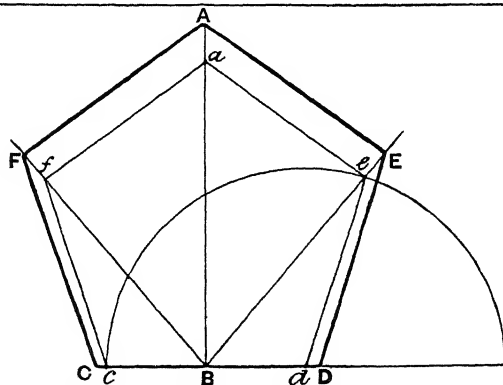
PROB 90



PROB 88.



PROB 89





## CHAPTER IX

## SCALES

Drawings are not usually made of the same size as the objects which they represent. It would be manifestly inconvenient to draw the plan of a building its proper size, as the drawing would be too large for use, but if a drawing were made on which every yard of the object was represented by an inch, then we should have a diagram showing the relative proportion of the parts, but drawn to a different *scale*. As every inch on the drawing represents a true length of one yard, the drawing would be on a *scale of 1 inch to 1 yard*, or  $\frac{1}{36}$ , because each line on the drawing is  $\frac{1}{36}$  part of its true length. This fraction is called the **representative fraction**, and shows the ratio each line on the drawing bears to the object delineated.

The scale of a drawing may be stated in *words*, as *1 in. to 1 yd.*, by its *representative fraction*, as  $\frac{1}{36}$ , or by *drawing a line divided into equal parts*, each representing the unit used. Scales, to be of use, should fulfil the following conditions—1 Divided with great *accuracy*, and carefully numbered 2 Long enough to measure the principal lines of the drawing 3 The zero (0) must always be between the unit and its subdivisions 4 The name of the scale should be written on it, and the representative fraction shown

## PLAIN SCALES

Plain Scales are divided into a suitable number of equal parts or units, the first of which is subdivided into smaller parts

**PROBLEM 91.**—To construct a scale of  $\frac{1}{2}$  in. to 1 ft., or  $\frac{1}{24}$ , to measure 6 ft.

Rule two parallel lines about  $\frac{1}{4}$ th of an inch apart. Set off 6 half inches. Divide the first part into 4, showing spaces of 3 in. Figure the scale as shown, placing the zero between the feet and the subdivision into inches.

**PROBLEM 92.**—To construct a scale of  $\frac{3}{4}$  in. to 1 mile, showing miles and furlongs, and measuring 4 miles.

Draw two lines as before. Set off 4 spaces of  $\frac{3}{4}$  in. each. Divide the first part into 8, showing the furlongs, and number as shown.

**Note.**—The representative fraction in this scale is  $\frac{\frac{3}{4} \text{ in.}}{1 \text{ m.}} = \frac{\frac{3}{4} \text{ in.}}{63360 \text{ in.}} = \frac{1}{84480}$ . Every line of the object drawn to this scale would be 84480 times as long as that of the drawing

**PROBLEM 93.**—Draw a scale of  $\frac{1}{80}$ , to show yards and feet, and measuring 5 yds.

The scale has to be long enough to measure lines 5 yds long; therefore, its total length will be  $\frac{5}{80}$  or  $\frac{1}{16}$  of 1 yard, that is, 3 in. Draw two lines, 3 in. long. Divide, and number as shown

**PROBLEM 94.**—Construct a scale of .875 in. to 10 ft., to measure 40 ft.

If 10 ft. are represented by .875 in., then 40 ft. would be represented by  $.875 \times 4 = 3.5$  in. Take a line  $3\frac{1}{2}$  in. long, divide it into 4 equal parts, giving distances of 10 ft. Divide the first part into 10 equal parts, showing feet.

$$\text{Representative fraction} = \frac{.875 \text{ in.}}{10 \text{ ft.}} = \frac{.875 \text{ in.}}{120 \text{ in.}} = \frac{\frac{7}{8} \text{ in.}}{120 \text{ in.}} = \frac{7}{960}$$

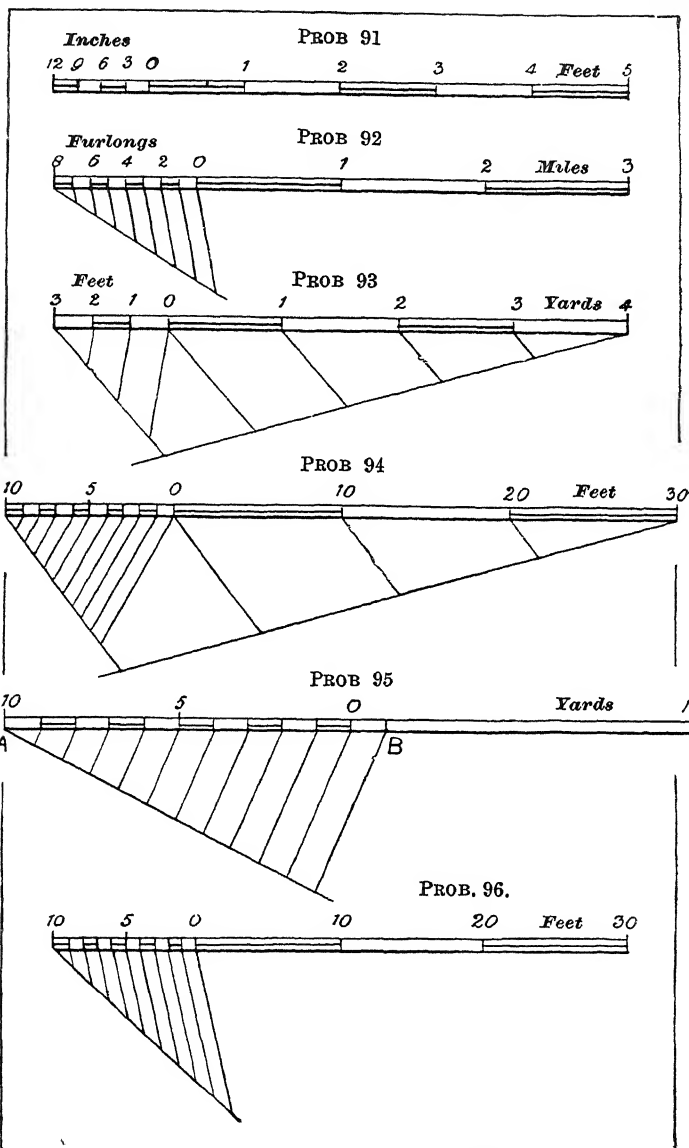
**PROBLEM 95.**—Draw a scale showing  $5\frac{1}{2}$  yds. to 1 in., to measure 20 yds.

If  $5\frac{1}{2}$  yds. be represented by 1 in., then 2 in. will represent 11 yds. Draw a line, and set off A.B., 2 in.

Divide this distance into 11 equal parts. Each of these parts will represent 1 yard. Add 9 more parts, to make up the 20, and complete the scale as shown

**PROBLEM 96.**—One inch represents 13 ft. 4 in. Draw the corresponding scale, divided generally into 10-ft. lengths, with one such length subdivided into single feet

The representative fraction =  $\frac{1 \text{ in.}}{13 \text{ ft. } 4 \text{ in.}} = \frac{1}{160}$ . The *unit* is a distance of 10 ft. If 160 in. be represented by 1 in., then 10 ft., or 120 in., will be represented by  $\frac{120}{160} = \frac{3}{4}$  in. Set off distances of  $\frac{3}{4}$  in., representing 10 ft. Subdivide the left-hand division into 10 equal parts, showing single feet.



**PROBLEM 97.**—The line  $A B$  represents  $3\frac{1}{2}$  yards. Construct a scale showing yards and feet, to measure 10 yards.

Divide  $A B$  into 7 equal parts, showing  $\frac{1}{2}$ -yards. Produce the line to the required length. Subdivide the left-hand division into 3 equal parts, showing feet.

**PROBLEM 98.**—The given line,  $A B$ , is 2 ft. 5 in. long by scale. Produce it so as to make it 6 ft.

Draw  $A C$  at any angle, and set off 2 ft 5 in. to any convenient scale;  $2\frac{5}{12}$  in. will do. Draw  $B C$ , and rule parallels cutting  $A B$  into 2 ft. 5 in. Add on the required number of feet

### DIAGONAL SCALES

By means of the diagonal scale very minute distances may be measured with great accuracy. The principle of its construction is as follows.—If the rectangle  $A B C D$ , on the opposite page, be divided into 8 equal parts by parallels to  $A B$ , and the diagonal  $D B$  be drawn, then a number of similar triangles will be formed (Euc VI 2). If  $D a$  is half  $D A$ , then  $a b$  will be half  $A B$ . In the same manner,  $D 1$  is  $\frac{1}{8}$  of  $D A$ , therefore,  $1 b$  will be  $\frac{1}{8}$  of  $A B$ .

From a plain scale we get two dimensions, such as yards and feet; from a diagonal scale we may obtain three dimensions, such as yards, feet, and inches

**PROBLEM 99.**—Draw a diagonal scale showing inches and tenths.

Draw a line  $A B$ , and set off inches. At  $A$  erect a perpendicular, and set off 10 equal parts to any convenient unit, and from each part draw parallel lines to  $A B$ . Erect verticals at each primary division. Draw the diagonal from 0 to 10.

Note.—We may obtain from this scale inches and tenths, thus.—Suppose  $1\frac{7}{10}$ , or 1.7 in., be required. Place the dividers where vertical 1 meets horizontal 7 (point  $e$ ), and open until point  $f$  is reached.  $a b = 1\frac{7}{10}$ , or 1.1 in.,  $c d = 1\frac{7}{10}$ , or 1.2 in.;  $g h = 2\frac{7}{10}$ , or 2.8 in.

**PROBLEM 100.**—Draw a diagonal scale showing inches and hundredths of an inch.

Draw the 11 parallels, and set up the verticals as before. Now divide  $A O$  into 10 equal parts, and join the first part on the left with 10 at the end of the top line. Rule parallels from each of the parts as shown.

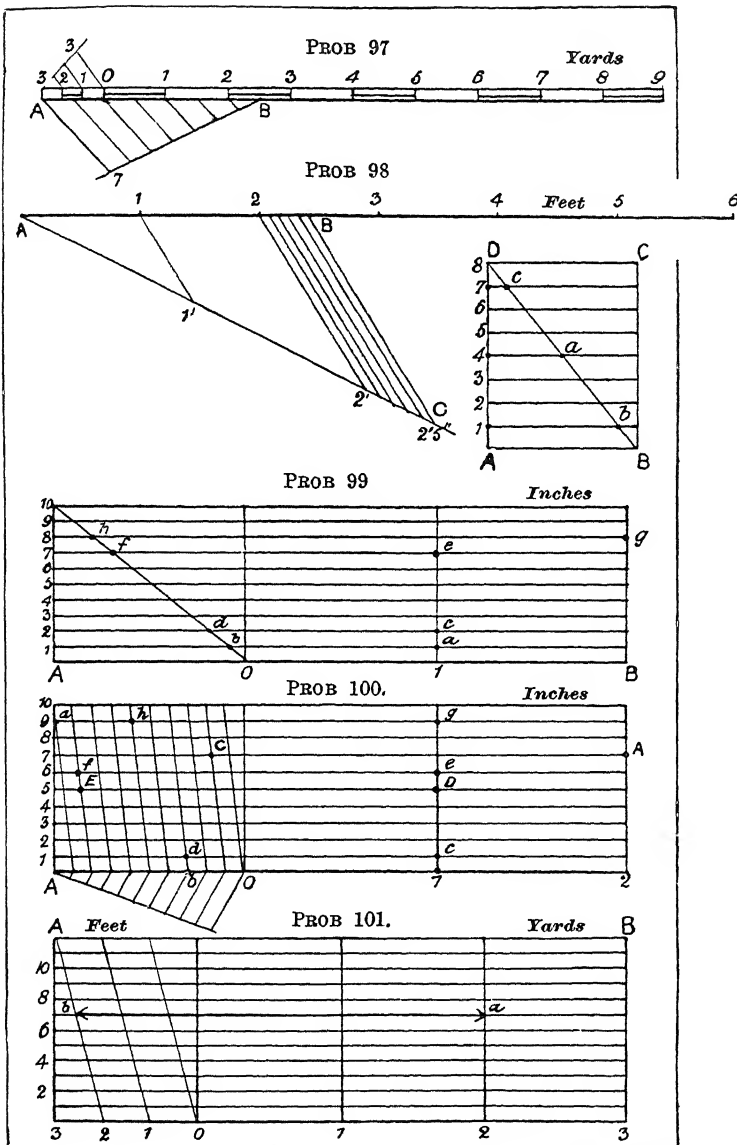
Note.—Each part on the line  $A O$  shows tenths of an inch, and the distance  $9 a$  on the second line from the top will be  $\frac{1}{10}$  of a tenth—that is,  $\frac{1}{100}$  of an inch. The distance  $1 b = 1\frac{3}{10}$ , or 1.3;  $c d = 1\frac{31}{100}$ , or 1.31;  $e f = 1\frac{36}{100}$ , or 1.36

If 1.59 inches were required from the scale, place one point of the dividers on the point  $g$ , where the vertical 1 meets the horizontal 9, open the dividers to the point  $h$ , where the diagonal 5 meets the horizontal 9

**PROBLEM 101.**—Draw a scale of  $\frac{1}{48}$ th to show yards, feet, and inches, and show a distance of 2 yds. 2 ft. 7 in. on the scale.

A scale of  $\frac{1}{48}$  would be  $\frac{3}{2}$  in. to the yard. Draw a line,  $A B$ , and set off distances of  $\frac{3}{2}$  in. Divide the first part into 3, showing feet. Draw the vertical  $A 3$ , and set off 12 equal parts. Rule parallels, and complete the scale.

To obtain the required distance, place one point of the dividers on  $a$ , where vertical 2 meets the horizontal 7, and extend the other point to  $b$ , where the diagonal 2 intersects the horizontal 7.



## THE SCALE OF CHORDS

Angles are sometimes set out and measured by using the scale of chords. It is marked on a scale by the letters 'C' or 'CHO,' and is figured from 0 to 90

**PROBLEM 102**—To construct a scale of chords.

Draw the quadrant **A B C**. Divide the arc **A C** into 9 equal parts. From centre **A** turn down the divisions to **A B**, and complete as shown

**Notes**—1. The divisions are unequal, decreasing gradually from **A** to **D**. 2. The distance from **A** to each division on the scale is the chord of the angle containing that number of degrees. 3. The chord for  $60^\circ$  (**A** to 60) = the radius **A B**. 4. On the scale distances of  $1^\circ$  are shown

**PROBLEM 103**—To construct angles of  $50^\circ$  and  $105^\circ$  by means of the scale of chords

Draw any line, **A B**. With centre **A** and radius 0 to  $60^\circ$  from the scale, describe an arc. Now take the distance from 0 to  $50^\circ$ , and intersect the arc in **C**. Then **B A C** is the angle of  $50^\circ$ . For  $105^\circ$ , take the distance from 0 to  $55^\circ$ , and set off from **C**, giving **B A D**, the angle of  $105^\circ$  ( $50^\circ + 55^\circ = 105^\circ$ ).

**PROBLEM 104**—To measure the size of an angle by means of the scale of chords.

Let **B A C** (*Prob. 103*) be the given angle. With radius 0 to  $60^\circ$  from the scale, describe the arc **B C**. Take the distance **B C**, and apply to the scale, giving  $50^\circ$ .

## THE SECTOR

This instrument is shown on the opposite page. The most important of the scales marked on it are the line of lines, marked **L**, and the line of polygons, marked **POL**. The following problems show some of its uses

**PROBLEM 105**—To bisect a line

Open the sector until the transverse distance from 10 to 10 on **L** equals the given line. Then the distance from 5 to 5 will be half the line

**Notes**—1. Measure with the dividers from the inside line, where the dots are marked. 2. The transverse distances at 8 and 4, or 6 and 3, &c., may also be used for the bisection.

**PROBLEM 106**—To divide a straight line into 7 equal parts

Open the sector until the transverse distance from 7 to 7 on **L** equals the given line. Then the transverse distance from 1 to 1 will be  $\frac{1}{7}$ th of the given line

**PROBLEM 107**—To find a fourth proportional to three given lines, **A**, **B**, **C**

From the centre **O**, on **L**, set off **O D** equal to **A**. Open the sector until the transverse distance at **D** equals **B**. Then, if **O E** be set off equal to **C**, the transverse distance at **E** will be the required fourth proportional

**PROBLEM 108**—To inscribe a regular pentagon in a circle.

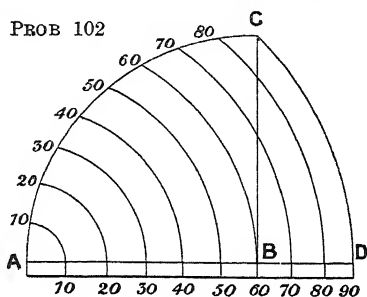
Let **O A** be the radius of the circle. Open the sector until the transverse distance from 6 to 6 on **POL** equals **O A**. Then the transverse distance at 5 will equal the side of the pentagon

**Notes**—1. Always make the radius of the circle equal the distance from 6 to 6. 2. If a heptagon were required, then the distance from 7 to 7 would equal the side

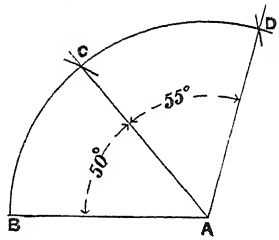
**PROBLEM 109**—To construct a regular heptagon on a given line.

Let **A B** be the given line. Open the sector until the transverse distance from 7 to 7 equals **A B**. With **A** and **B** as centres, and the distance from 6 to 6 as radius, describe arcs intersecting at **O**. With centre **O** and the same radius describe a circle. Set off the distance **A B** round the circumference

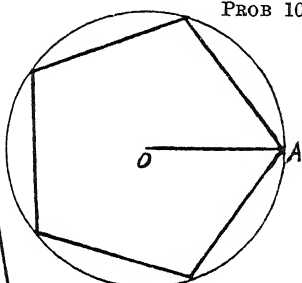
PROB 102



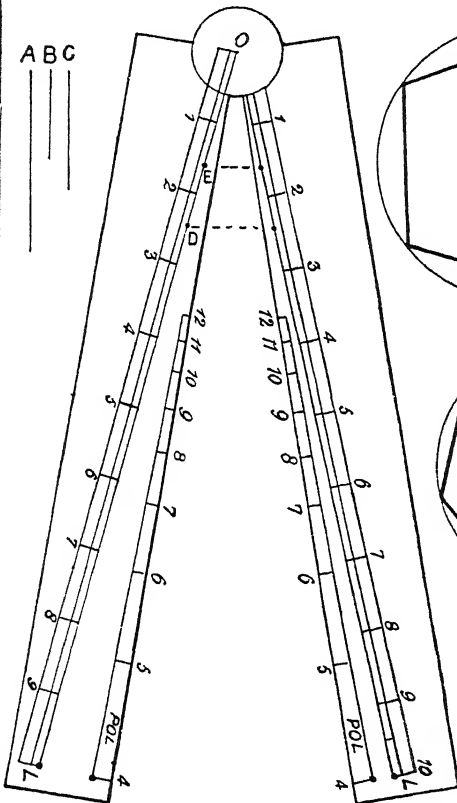
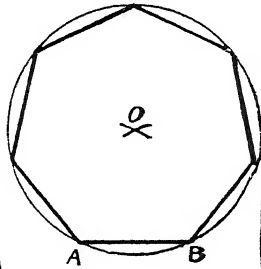
PROB 103



PROB 108



PROB. 109.



## CHAPTER X

## THE USE OF SCALES IN THE CONSTRUCTION OF IRREGULAR POLYGONS, AND IN THE REDUCING, ENLARGING, AND COPYING OF PLANE FIGURES.

**PROBLEM 110**—To construct an irregular polygon having given the lengths of the sides and the magnitude of the angles.

Sides,  $AB = 1\frac{3}{4}"$ ,  $BC = 1\frac{1}{2}"$ ,  $CD = 1\frac{1}{4}"$ ,  $DE = 1\frac{5}{8}"$ ,  $EA = 1\frac{1}{2}"$ .  
Angles,  $\angle ABC = 140^\circ$ ,  $\angle BAE = 100^\circ$ . First make a rough freehand sketch, lettered and figured, as a guide. Draw  $AB$  and make it  $1\frac{3}{4}"$  (1.75) long from the scale. Make the angles  $\angle ABC$  and  $\angle BAE$  equal to  $140^\circ$  and  $100^\circ$  respectively, either with the protractor or scale of chords. Make  $BC = 1\frac{1}{2}"$  and  $AE = 1\frac{1}{2}"$ .

With centre  $C$  and radius  $1\frac{1}{4}"$  describe an arc. With centre  $E$  and radius  $1\frac{5}{8}"$  intersect the arc in  $D$ . Draw  $ED$ ,  $CD$ .

Note.—The angles of figures are usually lettered in order, starting from the first letter.

**PROBLEM 111.**—To construct an irregular polygon, the lengths of the sides and diagonals being given.

Sides,  $AB = 1\cdot9"$ ,  $BC = 1"$ ,  $CD = 1\cdot85"$ ,  $DE = 1\cdot75"$ ,  $EA = 1\frac{1}{2}"$ .  
Diagonals,  $AC = 2\cdot17"$ ,  $BE = 2\cdot4"$ . Make a rough sketch of the figure as before. Draw  $AB = 1\cdot9"$  long. With centre  $B$  and radius  $1"$  describe an arc, and with centre  $A$  and radius  $2\cdot17"$  intersect the arc in  $C$ . (This distance may be taken from the diagonal scale showing hundredths of an inch; it is marked by the letters  $A C$  on the figure for Problem 100.) Draw the triangle  $ABE$  in a similar manner. With centres  $C$  and  $E$  describe arcs of  $1\cdot85"$  and  $1\cdot75"$  radii intersecting in  $D$ . Draw  $BC$ ,  $CD$ ,  $DE$ ,  $EA$ . (The distance  $CD$  is shown on the same scale by the letters  $D E$ .)

Note.—Problems 111 and 112 are drawn half size.

**PROBLEM 112**—To construct an irregular polygon, having given two sides, the lengths of the diagonals drawn from one angle, and the angles between them.

Sides,  $AB = 1\cdot15"$ ,  $BC = 1\cdot3"$ . Diagonals,  $BD = 2\cdot4"$ ,  $BE = 2\cdot8"$ ,  $BF = 2"$ . Angles,  $\angle ABF = 30^\circ$ ,  $\angle FBE = 45^\circ$ ,  $\angle EBD = 35^\circ$ ,  $\angle DBC = 25^\circ$ .

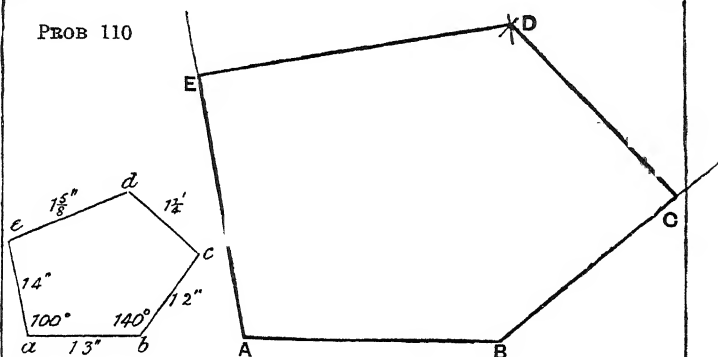
After making a sketch, draw  $AB = 1\cdot15"$ . With the protractor set off angles  $\angle ABF$ ,  $\angle FBE$ ,  $\angle EBD$ ,  $\angle DBC$ . Make  $BC$ ,  $BD$ ,  $BE$ ,  $BF$  equal to the lengths given. Draw  $CD$ ,  $DE$ ,  $EF$ ,  $FA$ .

**PROBLEM 113**—To construct an irregular polygon having given a point  $O$  within the polygon, the distance from the point to each angle, and the angles round the point.

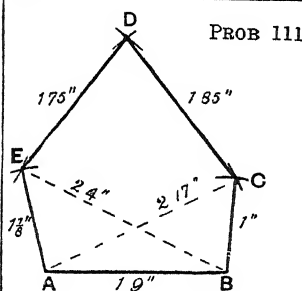
$OA = 50'$ ,  $OB = 55'$ ,  $OC = 60'$ ,  $OD = 25'$ ,  $OE = 75'$ ,  $OF = 40'$ .  
Angles.— $\angle AOB = 65^\circ$ ,  $\angle AOF = 50^\circ$ ,  $\angle FOE = 75^\circ$ ,  $\angle EOD = 30^\circ$ ,  $\angle DOC = 55^\circ$ . Scale,  $45 \text{ ft} = 1 \text{ inch}$ .

First construct the scale, showing distances of 5 ft. After making a sketch, fix a point  $O$ , draw  $OB$ , and set out the angles  $\angle BOA$ ,  $\angle AOF$ ,  $\angle FOE$ ,  $\angle EOD$ ,  $\angle DOC$  with the protractor. Take the lengths of the lines given from the scale and join the points,  $A$ ,  $B$ ,  $C$ ,  $D$ ,  $E$ ,  $F$ , with each other.

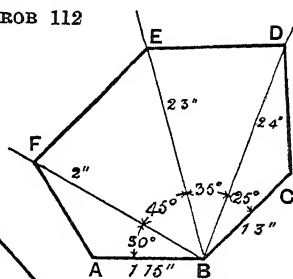
PROB 110



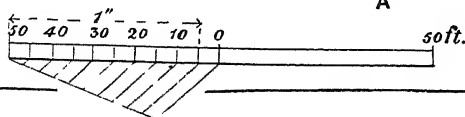
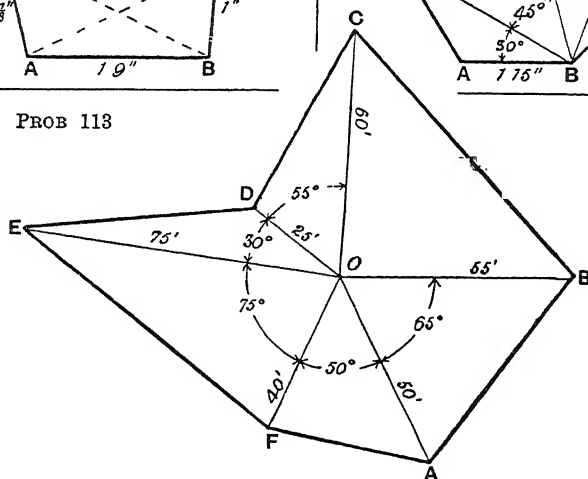
PROB 111



PROB 112



PROB 113





**PROBLEM 114.**—To construct an irregular polygon from a rough diagram, the dimensions on the diagonal and the offsets or ordinates being given

$A E = 3 \text{ ch. } 50 \text{ l.}$ ,  $A b = 75 \text{ l.}$ ,  $A g = 90 \text{ l.}$ ,  $A c = 2 \text{ ch. } 20 \text{ l.}$ ,  $A f = 2 \text{ ch. } 90 \text{ l.}$ ,  $A d = 3 \text{ ch.}$  Offsets,  $b B = 1 \text{ ch. } 10 \text{ l.}$ ,  $g G = 80 \text{ l.}$ ,  $c C = 25 \text{ l.}$ ,  $f F = 1 \text{ ch. } 30 \text{ l.}$ ,  $d D = 90 \text{ l.}$

Scale,  $\frac{3}{4} \text{ in.}$  to represent 1 cham.

$$\text{Representative Fraction} = \frac{\frac{3}{4} \text{ in.}}{66 \text{ feet}} = \frac{\frac{3}{4} \text{ in.}}{792 \text{ in.}} = \frac{1}{1,056}.$$

*First* construct the scale. As no distances in the figure are required smaller than 5 links it will only be necessary to divide the unit into 20 parts. This will be more accurately performed by means of a diagonal scale. Draw 5 parallel lines. Set off 4 spaces of  $\frac{3}{4} \text{ in.}$  each and draw verticals. Divide the first space into 5 equal parts giving distances of 20 links. Draw the diagonals, thus obtaining distances of 15, 10, and 5 links.

Next draw  $A E$ , 3 ch. 50 l. from the scale ( $a b$  on the scale), and set off  $A b$  ( $c d$  on scale),  $A g$ ,  $A c$ ,  $A f$ ,  $A d$ . At the points  $b, g, c, f, d$  draw the offsets, taking the distances from the scale. Join the points  $A, G, F, E, D, C, B$ .

**PROBLEM 115**—To construct an irregular figure from a given figured rough sketch. Scale, 20 ft. = 1 in.

$$\text{Representative Fraction} = \frac{1 \text{ in.}}{20 \text{ ft.}} = \frac{1}{240}.$$

*First* draw the scale. As the longest distance to be measured is under 40 ft. the scale need be only 2 in. long. Divide the first inch into 20 parts.

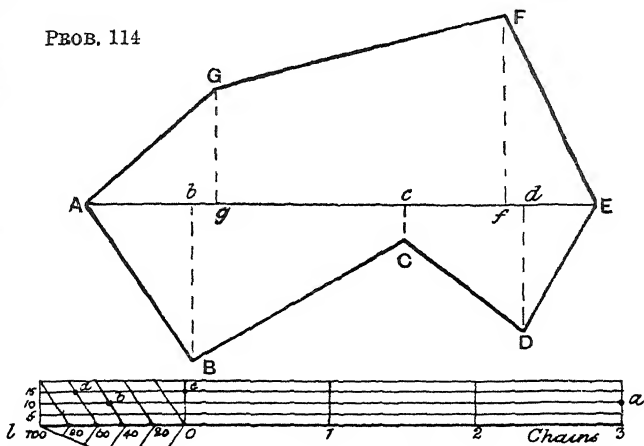
Draw any line  $A B$ . At  $A$  draw  $A C$  at an angle of  $90^\circ$  and make it 27 ft. long by scale. Make an angle of  $30^\circ$  at  $C$  and draw  $C D$  15 ft. by scale. At  $D$  make an angle of  $100^\circ$  and draw  $D E$  35 ft. long. Proceed in a similar manner for the other lines and angles. From  $G$  draw  $G B$  at right angles to  $A B$ .

**PROBLEM 116.**—Construct a six-sided polygon  $a b c d e f$ , such that  $e f = \frac{a e}{2} = \frac{2}{3} a f$ ;  $d e = a d$ ; and  $a b = \frac{2}{3} b c$ . The rest of the data are given on the figure. Scale  $\frac{1}{2}'' = 10' 0''$ .

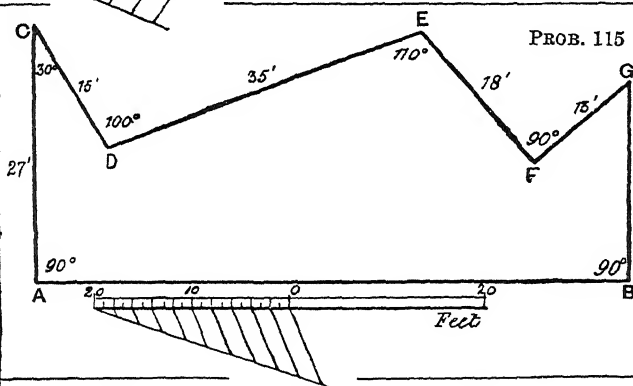
*First* construct a plain scale showing feet, and long enough to measure  $a e$ . Then draw  $a e$  56' by scale. On  $a e$  construct the isosceles triangle  $a d e$ , making the sides  $d a$  and  $d e$  49' long. To obtain point  $c$ , draw a line parallel to  $a e$  and 22' from it, and with centre  $e$  and radius 42' from the scale intersect the parallel. Draw  $d c$ . ( $a b = \frac{2}{3} b c = 18'$ .) With centre  $a$  and radius 18' describe an arc, and with centre  $c$  and radius 27' intersect the arc in  $b$ . Draw  $c b, a b$ . ( $e f = \frac{a e}{2} = 28'$ , and  $\frac{2}{3} a f = e f$ , therefore  $a f = 42'$ .) With centre  $e$  and radius 28' describe an arc, and with centre  $a$  and radius 42' intersect the arc in  $f$ . Draw  $a f, f e$ .

**Note.**—The figure is drawn half its proper size.

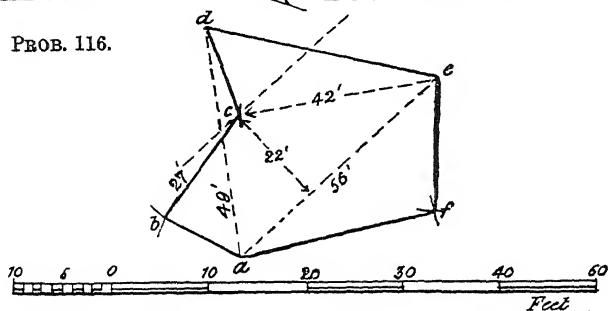
PROB. 114



PROB. 115



PROB. 116.



**PROBLEM 117.**—To make a reduced drawing of a given figure.

Let  $ABCDEF$  be the given figure. It is required to reduce it so that each side shall be  $\frac{2}{3}$  of its length on the given figure.

**Method 1.**—Draw  $BF$ ,  $BE$ ,  $BD$ . Divide  $AB$  into 3 equal parts.  $Ba$  will be one of the required sides. Draw  $af$ ,  $fe$ ,  $ed$ ,  $dc$  parallel to the corresponding sides of the figure.

Note.—The principle is the same as in Prob 88

**Method 2.**—Divide  $AB$  into 3 equal parts as before. Take  $Ba$  and place it in the same straight line or parallel to  $AB$ . Draw  $af$ ,  $bf$  parallel to  $AF$ ,  $BF$ , thus obtaining point  $f$ . From  $f$  draw a parallel to  $FE$  and from  $b$  draw a parallel to  $BE$  giving point  $e$ . Proceed in a similar manner for the other sides.

**Method 3.**—Trisect each side and construct the figure by applying the principle of Prob. 63.

Notes.—1 Two other figures are given showing the application. The sides of both rectangle and triangle are reduced two-thirds.

2 A figure may be reduced to any given size by this method.

**PROBLEM 118.**—To make an enlarged drawing of a given figure

Let  $ABCDEF$  be the given figure. It is required to enlarge it so that  $AB$  may be equal to the given line  $ab$ .

Draw  $AC$ ,  $AE$ ,  $AD$  and produce them. Set off  $ab$  from  $A$ . From  $b$  draw  $bc$  parallel to  $BC$ , and proceed as in the previous problem.

**PROBLEM 119.**—To enlarge or reduce a drawing by means of a **PROPORTIONAL SCALE**.

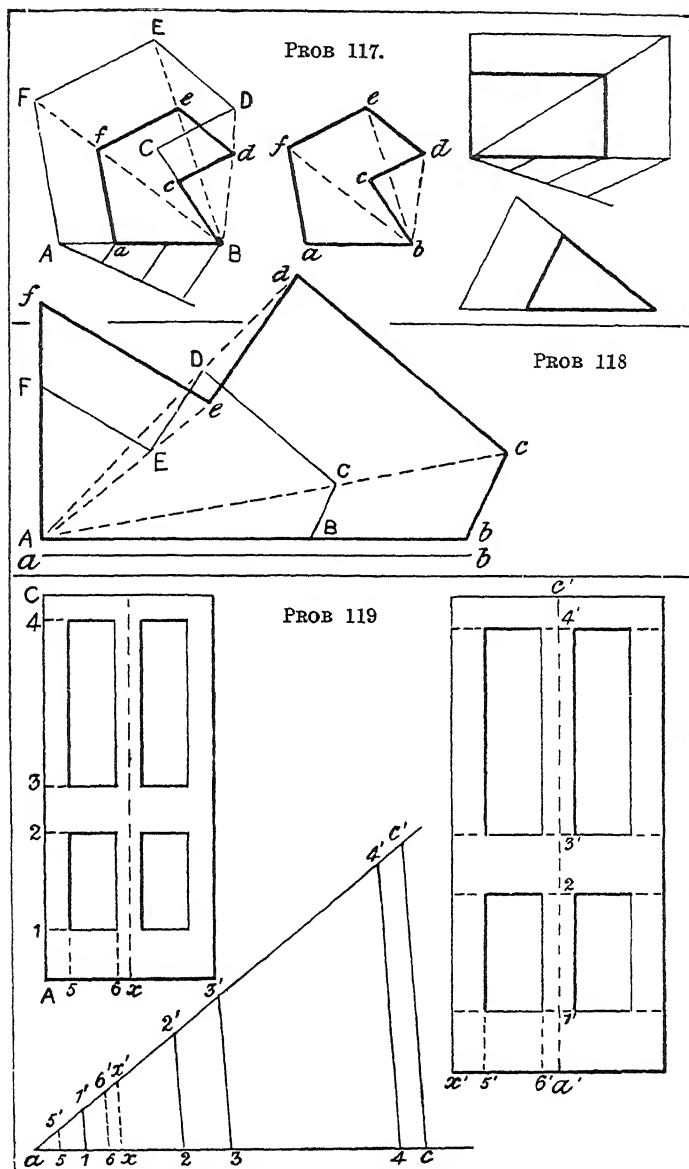
It is required to enlarge the given drawing so that  $AC$  shall equal  $2\frac{1}{2}$  ins.

Construct a proportional scale by drawing two lines at any angle to each other, set off  $ac$  equal to  $AC$  and  $ac'$  equal to the required distance,  $2\frac{1}{2}$  ins. Join  $cc'$ . Set off the distances  $A1$ ,  $A2$ ,  $A3$ ,  $A4$  on  $ac$ . From each point rule parallels to  $cc'$  giving the proportionate distances for the new drawing.

Draw the centre line  $2\frac{1}{2}$  ins long and on it set off the distances  $a1'$ ,  $a2'$ , &c. Through  $a'$  and  $c'$  draw perpendiculars. Set off  $Ax$  on  $ac$  and obtain  $ax'$  the width of half the door. Set this distance off on each side of  $a'$  and erect perpendiculars. Through  $1'$ ,  $2'$ ,  $3'$ ,  $4'$  draw parallels. For the width of the panels set off  $A5$ ,  $A6$  on  $ac$  and proceed as before.

To reduce a drawing proceed in a similar manner. The only difference will be that the *given* dimensions will be set off on the *longer* line of the scale and the *required* dimensions will be obtained from the *shorter* line.

**PROBLEM 119a.**—To reduce or enlarge a drawing by squaring. See Prob. 31.



**PROBLEM 120.—To copy figures.**

The copying of given figures is an exercise of considerable importance in helping to form habits of accuracy and quickness in a draughtsman. The proper construction lines must be first obtained, and always in drawing symmetrical figures remember to set out the centre lines first. Several examples are here given, with the proper method of proceeding.

**Figure 1** represents a Greek fret. It will be found that the pattern is formed upon ten equi-distant horizontal lines intersected by vertical lines forming squares. Draw the pattern in firmer line, keeping the *shade* line darker.

**Note**—The figure is given half size. The same method of construction must be employed when copying any fret.

**Figure 2.**—Rule the parallels first. Draw the verticals, and with the  $60^\circ$  set square obtain the equilateral triangles as shown. Keep the proper relation between thick and thin lines. Make your copy double the size of the given figure.

**Figure 3.**—Draw the figure to the dimensions given.

It will be seen that a square of 3" sides divided into 9 equal squares will contain the inner lines of the four squares of which the figure is composed. Draw the diagonals, and about each square describe another square at the required distance. In thickening, or inking in the pattern, be careful to notice the portions that overlap.

**Figure 4**—Draw the pattern to the dimensions given. As the sides of the small squares and the distances between them are equal, draw a line and set off 5 equal spaces of  $\frac{3}{4}$ " each. On this distance construct a square and divide it into 25 equal squares.

To obtain the other lines divide the alternate squares as shown.

**Figure 5.**—Draw the figure, making the centres of the circles  $1\frac{1}{2}$  inch apart. Radius of the large circle  $\frac{3}{8}$ ", of the smaller one  $\frac{1}{8}$ ". Draw the centre line, set off the centres and describe the circles. For the centres of the connecting arcs describe equilateral triangles as shown.

PROB 120.

FIG 1

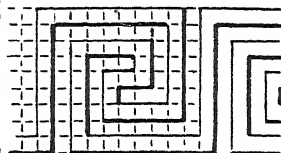


FIG 2.

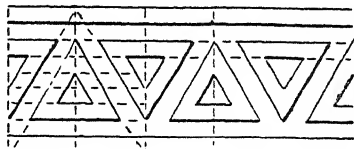


FIG 3

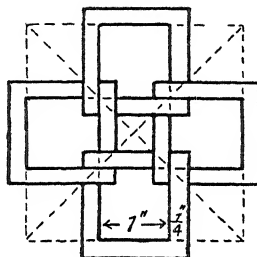


FIG 4

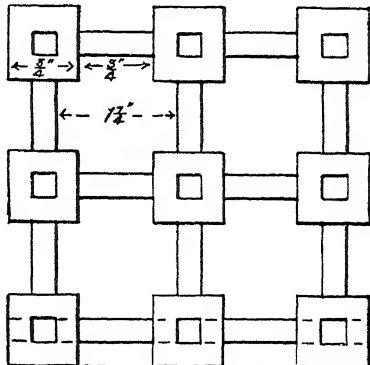
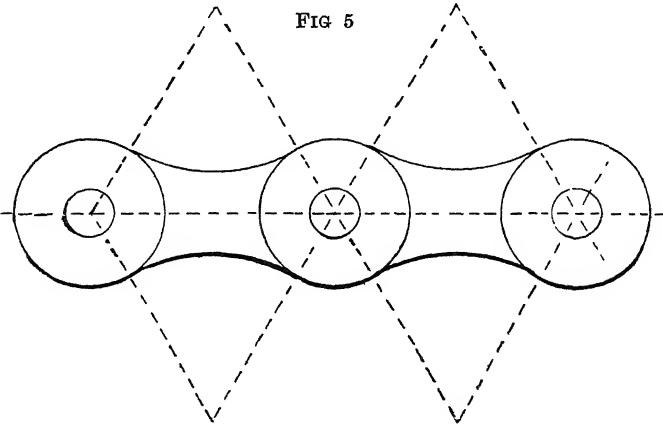


FIG 5



**Figure 6.**—Copy the double spiral, which is composed of semicircles  $\frac{1}{4}$ " apart; the diameter of the smallest semicircle is  $\frac{1}{2}$  in.

Draw a line *AB* and set off 12 equal spaces of  $\frac{1}{4}$ ", and with centres 3 and 9 describe three concentric semicircles above the line from 3, and below the line from 9. The centres for the connecting semicircles will be midway between 2, 3, and 9, 10. Describe the arcs which have equal radii before commencing the next.

**Figure 7.**—Rule the horizontal and vertical lines as shown, and from each centre describe two concentric circles. The finished pattern should be thickened or inked in. Let the centres for each circle be  $\frac{5}{8}$ " apart.

**Figure 8.**—Draw the figure from the given dimensions.

Draw a line *AB* and set off distances of  $\frac{3}{8}$ ",  $1\frac{1}{4}$ ",  $\frac{3}{8}$ ",  $1\frac{1}{4}$ ",  $\frac{3}{8}$ ",  $1\frac{1}{4}$ ",  $\frac{3}{8}$ ". Mark the centre for each of the circles and describe them, omitting the portions which are omitted in the copy.

**Note**—The given figure is drawn to a smaller scale.

**Figure 9.**—Draw the figure from the given dimensions.

Begin with the centre lines *AB* and *CD*.

Set off on *CD* distances of  $\frac{1}{8}$ ",  $\frac{1}{4}$ ",  $\frac{1}{8}$ ",  $\frac{1}{2}$ ",  $\frac{1}{8}$ ",  $\frac{1}{4}$ ",  $\frac{1}{8}$ ". Describe the circles. Set off  $1\frac{1}{8}$ " on each side of the centre of *AB* and draw the vertical lines *E* and *F*. Describe the semicircles and complete the figure.

FIG. 6

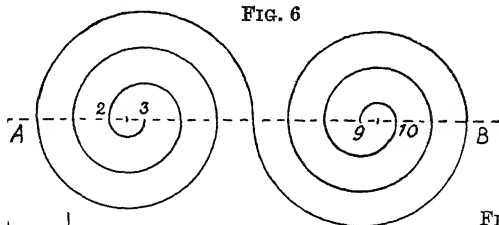


FIG 7

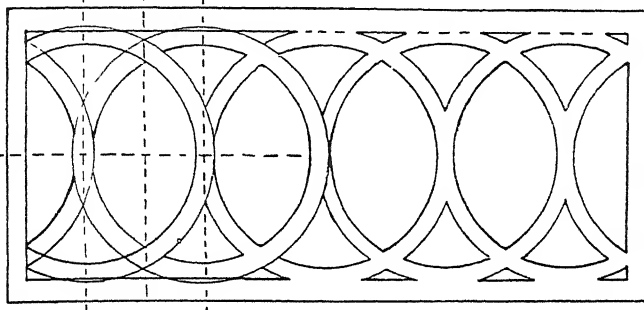


FIG 8

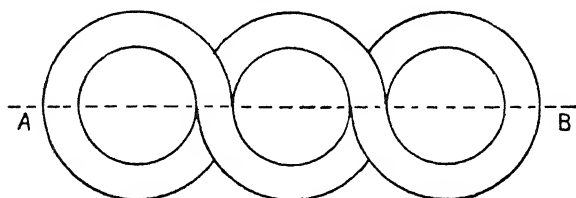
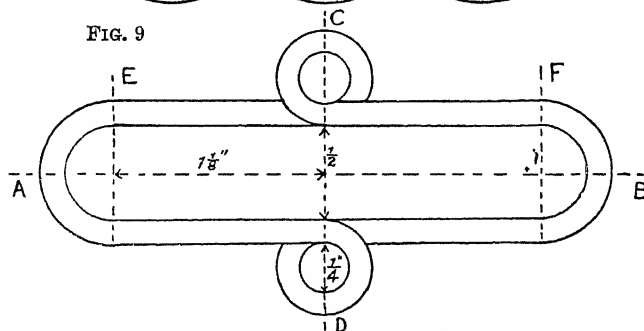


FIG. 9





## EXERCISES

**Note**—Any figures relating to the following exercises will be found on page 63.

## CHAPTER VII

- 1 Describe a circle  $1\frac{1}{4}$ " radius, and from a point A, 3" from its centre, draw a line touching the circle.
- 2 Describe two circles with radii  $\frac{3}{4}$ " and 1", having their centres  $2\frac{1}{2}$ " apart. Draw a right line to touch both these circles
- 3 To a circle of 1.25" radius, draw two tangents which shall contain an angle of  $60^\circ$ . (Sc) (Prob. 72)
- 4 Describe a circle of  $1\frac{1}{8}$ " radius, and draw tangents intersecting each other from two points in the circumference  $100^\circ$  apart
- 5 Describe a circle 1.5" radius. Draw a straight line 1.75" long within the circle. On this line construct an isosceles triangle having its vertical angle in the circumference of the circle
- 6 Draw a circular arc of large radius, and draw a tangent to it without finding the centre of the circle.

## CHAPTER VIII

1. Construct a regular hexagon having its diagonal 3 inches.
- 2 On a base of  $1\frac{1}{4}$ " construct a regular heptagon
3. Describe a circle with a diameter of 2.9". About it describe a regular nonagon.
- 4 On a side of  $\frac{3}{4}$ " set up a regular octagon by means of the set square.
- 5 In a circle of 3.5" diameter inscribe a regular pentagon
6. Construct a regular pentagon whose diagonal shall be 3". (Sc.) (Prob 90)
7. Construct a regular hexagon on a base of 75", and on the same base construct a similar hexagon having its side 1.25" long (Prob 117)
- 8 Copy the hexagons (Fig 1) which are drawn to a scale of  $\frac{3}{4}$ " to 1' 0", and make them  $\frac{1}{2}$ " to 1' 0" (Art)

**Note**—The figure is given half its proper size and the construction lines are indicated.

## CHAPTER IX

1. Construct a scale 5 ft long,  $1\frac{1}{8}$  in to 1 ft. to show inches. What is its representative fraction?
- 2 Construct a scale 2 inches to 1 yard to show  $5\frac{1}{2}$  feet. What is its representative fraction?
- 3 The given line (Fig 2) is 2 ft 6 in long. Produce it to measure 3 ft. 6 in (Art)
- 4 Convert A B 2 in. long into a diagonal scale showing  $\frac{2}{100}$  of the line. (Art.)
- 5 Construct a plain scale  $\frac{3}{4}$  in to 1 ft., and mark upon it a distance of 5 ft 7 in. by the scale. (Art.)

6 Convert the given scale (Fig 3) into a diagonal scale reading inches. State the representative fraction of this scale (Sc)

Note —The first division will require dividing into 10 equal parts, and twelve parallels must be ruled to obtain the inches

7. Construct a scale of chords on a radius of 4 25" to read to 5° By means of this scale plot an angle of 75° (Sc)

8 Given a scale of yards (Fig 4) Deduce from it a scale of feet to read to 1' 0" and show 70' 0"

Note —The given scale is  $\frac{2}{3}$  its proper size and shows 20 yds or 60 feet Divide it into 6 equal parts, showing distances of 10 feet, and add one space on to obtain 70 feet Subdivide the first part into 10 to obtain one foot

9 The line  $ab$  (Fig 5) represents 3' 9" Construct a scale reading inches and showing 10' 0" The scale to be correctly figured (Sc)

Note —Draw a line at any angle from  $a$ , and set off  $3\frac{3}{4}$  to any unit Divide  $ab$  and add the distance required as in Probs 97 and 98

10 Construct a triangle with sides 10' 6", 14' 0", and 16' 3" Scale  $\frac{1}{8}$ " to 1' 0" (Sc)

Note —First construct the scale. As divisions equal to  $\frac{1}{2}$  of  $\frac{1}{8}$ " will be needed, a diagonal scale should be used.

11 Transfer the given scale (Fig 6) to your paper and complete it neatly, with figuring, &c Write down the representative fraction (Sc)

Note —The scale measures 3 75", therefore its representative fraction will be  $\frac{3.75 \text{ in}}{50 \text{ yds} \times 3 \times 12}$  Reduce to lowest terms

12. Make a scale 6" long to read feet and inches Fraction  $\frac{1}{18}$  (Sc)

13. A length of 100 yds is found to measure 3 6" on a drawing What is the fraction of the scale? Construct a scale to read yards, making it not less than 4 ins long (Sc)

14 Give the representative fraction of a scale on which  $3\frac{1}{2}$ " represents 2247' 0" Construct a scale of  $\frac{1}{250}$ , reading inches (Sc)

15 Construct a scale of  $\frac{1}{48}$  showing yards. Scale to be properly figured and not less than 7" long (Sc)

Note —1 in stands for 48 ins, that is 13 yards

16 Construct a scale of  $\frac{1}{18}$  to read inches and show 10' 0" (Sc)

Note —To find the length of the scale If 19 feet be represented by 1 foot, what distance will be represented by 10 feet? 19 ft 10 ft as 12 ins required length, whence required length =  $\frac{12 \times 10}{19} = 6\frac{6}{19}$  ins Draw a line  $6\frac{6}{19}$  ins long and divide it into 10 equal parts showing feet Subdivide the first unit into 12 equal parts showing inches.

17 Draw a scale of 12 5 feet to 1 inch and give the fraction. (Sc)

18 A line 100 ft long is represented on a drawing by a line 4" long Make a scale of feet for the drawing and give the representative fraction

19 A line 50 yds long is measured from A to B at right angles to a line joining A with another point C The angle subtended by the line AC is found to be 47° Find by construction the distance from A to C Scale 20 yds to an inch

20 Describe a circle of  $1\frac{1}{2}$ " radius Within it inscribe a regular nonagon by means of the sector. (Prob. 108)

## CHAPTER X

- 1 On a base 2" long construct a figure similar to the given figure (Fig 7).
- 2 Construct an irregular pentagon, sides  $2\frac{1}{2}"$ , 2",  $1\frac{1}{2}"$ , 1", and  $\frac{1}{2}"$ . The angle between the longest and shortest sides to be  $100^\circ$  and the diagonal joining the extremities of the two longest sides  $2\frac{1}{2}"$ . (First draw a rough sketch)
- 3 Draw any irregular four-sided figure, no side less than  $1\frac{1}{4}"$ . Construct a similar figure whose sides are  $1\frac{1}{2}$  times those of the first figure (Sc.)
- 4 The tops of two vertical poles are 85' 0" apart The poles are 40' 0" apart The height of one pole is 12' 0". Determine the height of the other pole. Scale 1" to 10' 0" (Sc.)
- 5 Draw a circle of 1.25" radius with centre O The corners of the polygon inscribed in this circle are so that the angles at the centre are as follows  $\angle AOB = 60^\circ$ ,  $\angle BOC = 70^\circ$ ,  $\angle COD = 50^\circ$ ,  $\angle DOE = 80^\circ$ ,  $\angle EOF = 50^\circ$  Write down the lengths of AB, BC, CD. (Sc, 1871)
- 6 Construct an irregular polygon from the following dimensions. Sides  $AB = 2"$ ,  $AF = 1.8"$ . Diagonals,  $AD = 3.5"$ ,  $AE = 3"$ . Angles,  $\angle ABC = 83^\circ$ ,  $\angle BAC = 40^\circ$ ,  $\angle BAD = 59^\circ$ ,  $\angle BAE = 118^\circ$ ,  $\angle BAF = 130^\circ$
- 7 Construct an irregular pentagon having its sides 2",  $2\frac{1}{4}"$ ,  $2\frac{3}{8}"$ ,  $2\frac{1}{2}"$ ,  $2\frac{3}{4}"$  respectively and with two of its angles right angles.
- 8 Draw to a scale of 40 yds to an inch an irregular polygon ABCDEF.  $AB = 140$  yds,  $BC = 113$  yds, angle  $\angle ABC = 130^\circ$ ,  $CD = 82$  yds,  $AD = 200$  yds,  $DE = 138$  yds,  $AE = 140$  yds,  $EF = 67$  yds,  $FA = 90$  yds.  
Note—A diagonal scale will be required, showing the inch divided into 40 parts.
- 9 The sides of a quadrilateral figure ABCD are as follows— $AB = 1"$ ,  $BC = 1"$ ,  $CD = 1\frac{1}{4}"$ ,  $AD = 1\frac{1}{2}"$ , and the diagonal  $BD = 1\frac{1}{2}"$ . Construct the figure and obtain a similar figure whose perimeter is 4".  
Note—Find a fourth proportional to the perimeter of the constructed figure ( $4\frac{1}{2}"$ ), the perimeter of the required figure (4"), and the side AB of the constructed figure (1") This will give the side of the required figure Set off this distance on BA and proceed as in Prob. 117
- 10 The given figure (Fig 8) represents a Maltese cross Two dimensions and an angle are given Draw the cross from the figured dimensions to a scale of  $\frac{1}{8}"$  to 1' 0". (Sc.)  
Note—The chief construction lines are indicated on the figure, which is drawn to a smaller scale Commence with the inner square 5' 6" side Draw the diagonals and diameters of the square, and produce them. From the centre set off 7' 3" on each side and draw parallels. On each side of the diagonal produced set off  $20^\circ$  From a draw a line parallel to the vertical diameter to meet the diagonals Complete the figure as shown.
11. Copy the given figure (Fig 9), making the diameter of the circle 4" and the width between the parallel lines  $\frac{1}{4}"$   
Note—Obtain the points as for the inscribed pentagon by using the protractor
- 12 Copy the figure (Fig. 10) to the dimensions given.  
Note.—Commence with the dotted lines

FIG. 1.

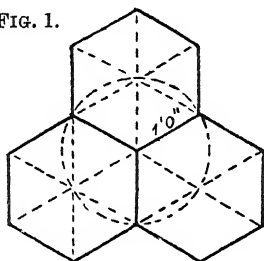


FIG. 7.

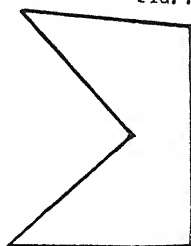


FIG 2



FIG 3

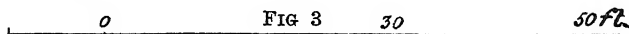


FIG 4

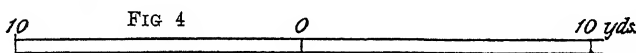


FIG 5

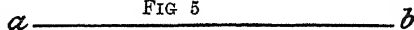


FIG 6

30

40  
yds.

FIG 8

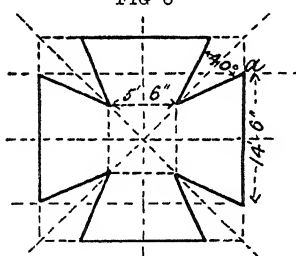


FIG 9.

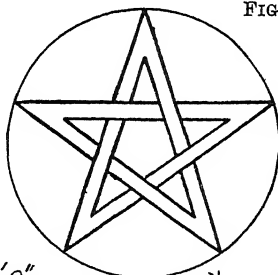
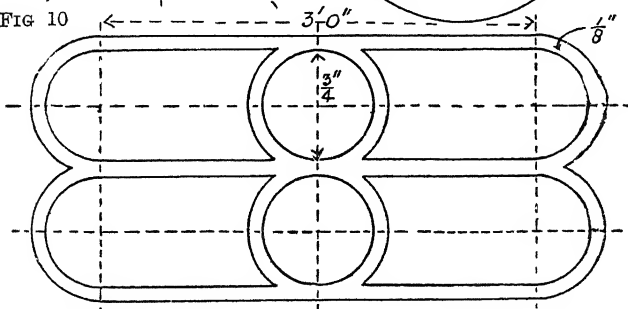


FIG 10



## CHAPTER XI

## CIRCLES TOUCHING LINES AND CIRCLES

**PROBLEM 121.**—To describe a circle passing through a given point, and touching a given straight line in a given point.

Let A be the given point, and B the point in the given straight line.

At B draw B O perpendicular to the given line Join A B.

At A construct the angle O A B equal to the angle O B A. With centre O and radius O B describe the circle.

Notes —1 The centre must lie in the line B O, because the given line will be a tangent to the circle 2 The bisection of A B will also give the centre.

**PROBLEM 122.**—To describe a circle passing through a given point, touching a given straight line, and having a given radius.

Let A be the given point, B the given line, and C the radius

Draw a line parallel to B and at a distance equal to C.

With centre A and radius equal to C intersect this line in O.

With centre O and radius equal to C describe the circle.

**PROBLEM 123.**—To describe a circle of a given radius to touch two converging lines

Let A B and C D be the two lines and E the radius.

At a distance equal to E draw lines parallel to A B and C D.

From O where the two parallels intersect, with radius equal to E, describe the circle.

**PROBLEM 124.**—To describe a circle touching three given straight lines which make angles with each other.

Let A B, A C, C D be the given lines.

Bisect the angles at A and C by lines meeting at O. Draw O E perpendicular to C D. With centre O and radius O E describe the circle.

**PROBLEM 125.**—To describe a circle touching two converging lines and passing through a given point between them.

Let A B, A C be the two converging lines and D the given point.

Bisect the angle B A C by the line A X; the centre of the circle must lie in this line. From any point E draw E F perpendicular to A B and describe a circle touching A B and A C Join A with the given point D. Draw E G, and from D draw D H parallel to G E With centre H and radius H D describe the circle

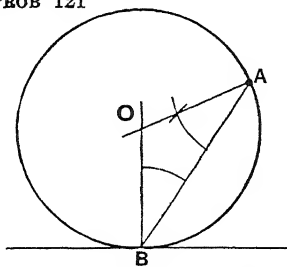
**PROBLEM 126.**—To describe a succession of circles touching each other and two converging lines

Let A B, A C be the two lines. Draw A D bisecting the angle B A C.

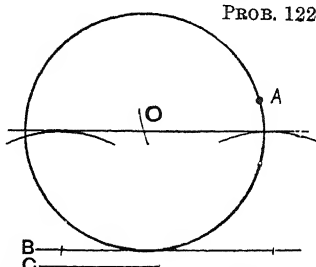
Take any point E, draw E F perpendicular to A C and describe a circle touching the two lines. Draw G H tangential to the circle at J.

Make G K equal to G F. From K draw K L parallel to F E for the centre of the next circle. Proceed in a similar manner for the other circles.

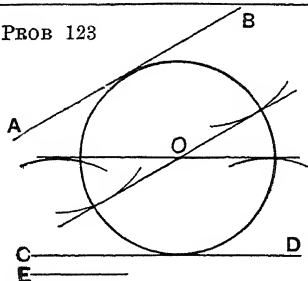
PROB 121



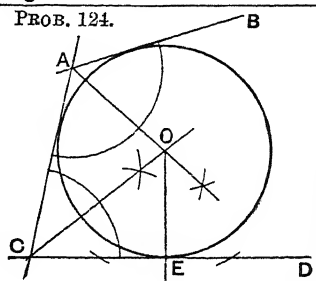
PROB. 122



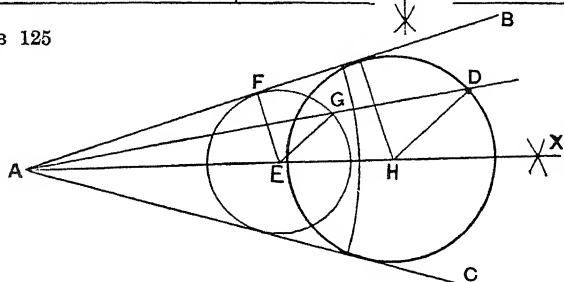
PROB 123



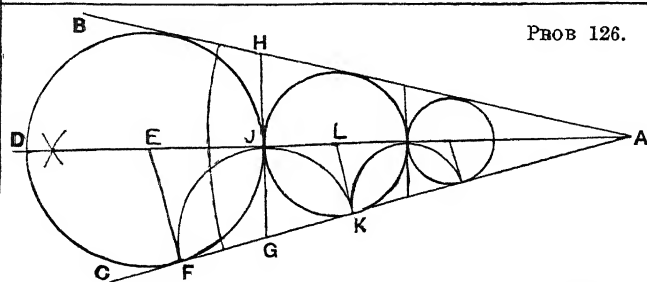
PROB. 124.



PROB 125



PROB 126.



**PROBLEM 127.**—To inscribe a circle in a triangle.

Let  $ABC$  be the triangle.

Bisect any two angles, and proceed as in *Prob. 124*.

**PROBLEM 128.**—To inscribe two equal circles in an isosceles triangle, touching each other and two sides of the triangle.

Let  $ABC$  be the triangle. Draw the perpendicular  $AD$  dividing the triangle into two equal triangles. Find the centre  $O$  of the triangle  $ABD$ . For the centre of the other triangle join  $CE$ , and from  $O$  draw a line parallel to  $BC$  and meeting  $CE$  in  $P$ . From centres  $O$  and  $P$  describe the circles.

**PROBLEM 129.**—To inscribe three equal circles in an equilateral triangle, each touching one side and two circles.

Let  $ABC$  be the triangle. Divide it into three equal triangles by bisecting the sides. Find the centre of the triangle  $OBC$  and inscribe a circle. Set off  $O2, O3$ , each equal to  $O1$ . With centres 2 and 3 describe the circles.

Note.—If parallels to the sides of the triangle be drawn through 1, 2, and 3, the angles 4, 5, and 6 of the triangle thus formed will be the centres for three more equal circles which may be inscribed in the triangle.

**PROBLEM 130.**—To inscribe four equal circles in a square, each touching one side and two circles.

Let  $ABCD$  be the given square.

Draw the diagonals and the diameters of the square.

Find the centre of the triangle  $AOB$  as in the preceding problem, and inscribe a circle. Set off the centres as shown, and describe the circles.

**PROBLEM 131.**—To inscribe in any regular polygon as many equal circles as the figure has sides, each touching one side and two circles.

Let the polygon in this case be a pentagon.

Divide the figure into as many equal triangles as it has sides, and inscribe a circle in each triangle as in the preceding problems.

**PROBLEM 132.**—To inscribe three circles in a spherical triangle, each touching one side and two circles.

Let  $ABC$  be the triangle. Bisect the angles at  $A$  and  $B$  by lines meeting at  $O$ , and draw  $CD$ . At  $D$  draw a tangent to meet  $OA$  produced. Bisect the angle thus formed and inscribe the circles as in *Prob. 129*.

**PROBLEM 133.**—To inscribe three equal circles in a hexagon, each touching one side and two circles.

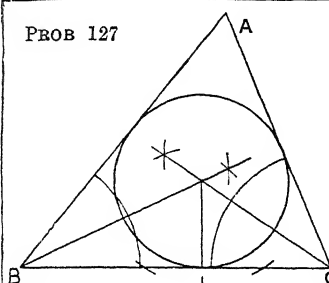
Draw the diameters of the figure. Produce two of these, and form a triangle as shown. Inscribe the circles.

Note.—Problems 128 to 134 all depend on Problem 127.

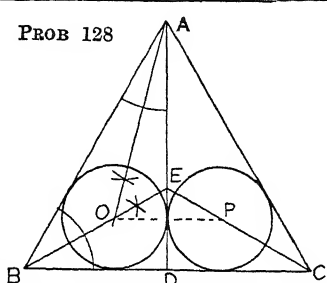
**PROBLEM 134.**—To inscribe a circle in a square.

Draw the diagonals to find the centre  $O$ . For the radius, draw  $OD$  perpendicular to the side. With centre  $O$  and radius  $OD$  describe the circle.

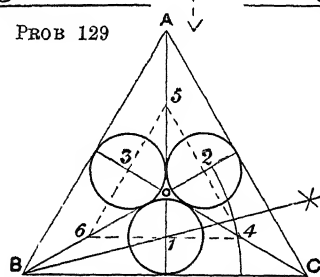
PROB 127



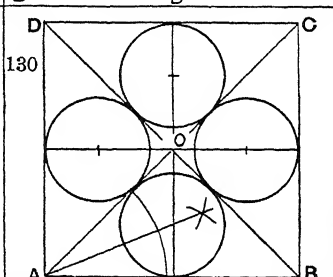
PROB 128



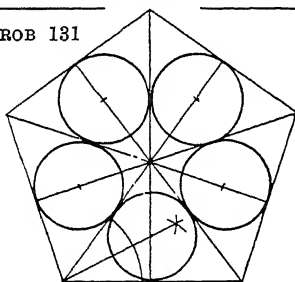
PROB 129



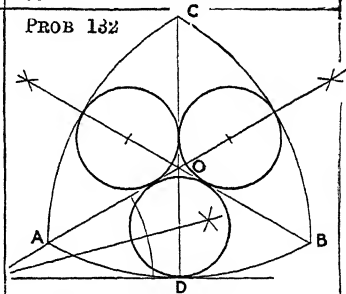
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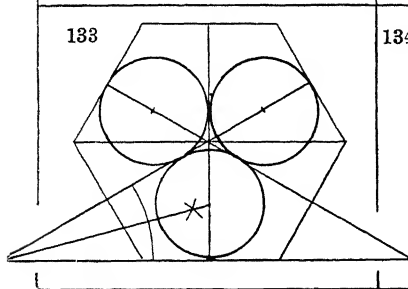
PROB 131



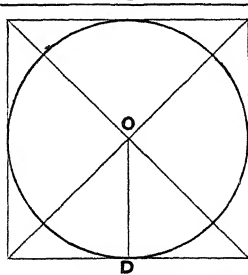
PROB 132



133



134





**PROBLEM 135.**—To inscribe four equal circles in a square, each touching two sides and two circles.

Draw the diagonals and the diameters of the given square. Draw the diagonals  $AB$ ,  $BC$ ,  $CD$ ,  $DA$ , giving the centres of each of the four squares. For the radius of the circles join  $E$  and  $F$ .

**PROBLEM 136.**—To inscribe a circle in any regular polygon.

Bisect two of its angles as in the given pentagon.  $O$  will be the centre, and  $OA$  the radius.

Notes —1. Bisecting two of the sides by perpendiculars will also give the centre

2 If the polygon has an even number of sides, the radius must be found by drawing a line from the centre perpendicular to one of the sides

3 To describe a circle about the figure,  $O$  will be the centre and  $OB$  the radius

**PROBLEM 137.**—To inscribe a circle in a rhombus

Draw the diagonals to obtain the centre. From  $O$  draw  $OA$  perpendicular to one of the sides for the radius.

**PROBLEM 138.**—To inscribe a circle in a trapezion.

Draw the diagonal  $AB$ . Bisect one of the other angles. The point  $O$  is the centre. For the radius draw a perpendicular to one of the sides

**PROBLEM 139.**—To inscribe three equal circles in an equilateral triangle, each touching two sides and two circles.

Bisect each of the angles by the lines  $AE$ ,  $BD$ , and  $CF$ , thus dividing the triangle into three equal trapezions. Bisect the angle  $CFB$ , obtaining centre 1. Set off centres 2 and 3. To obtain the radius join 1, 2. With centres 1, 2, 3 inscribe a circle in each trapezion.

Note—To inscribe in any regular polygon as many circles as the figure has sides, each touching two sides and two circles, divide the polygon into as many equal trapezions as the figure has sides, and inscribe a circle in each as above

**PROBLEM 140.**—In ANY regular polygon having an even number of sides, to inscribe half as many equal circles as the figure has sides, each touching two sides and two circles.

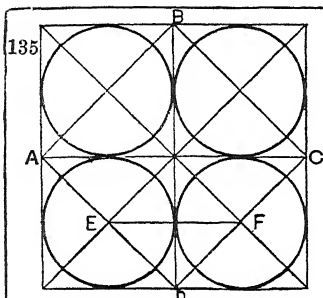
Let the given polygon be an octagon. Draw the diagonals. Inscribe a circle in the trapezion  $ABCO$ . Set off the other centres, and proceed as shown.

**PROBLEM 141.**—To inscribe a circle in a sector.

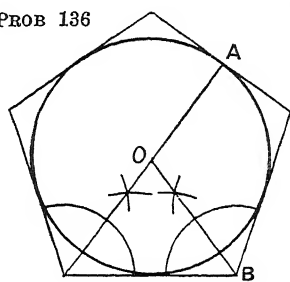
Let  $ABC$  be the sector. Bisect the angle  $BAC$  by the line  $AD$ . Through  $D$  draw a tangent to meet  $AB$ , and  $AC$  produced. Bisect one of the angles of the triangle thus formed. With centre  $O$  and radius  $OD$  inscribe the circle

**PROBLEM 142.**—To describe a circle to touch the arc of a sector externally and the two radii produced.

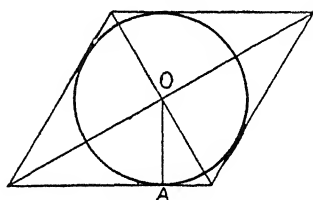
Bisect the angle as before, and produce  $AD$ . Draw a tangent at  $D$ . Bisect the exterior angle at  $E$  by the line  $EO$ . With centre  $O$  and radius  $OD$  describe the circle.



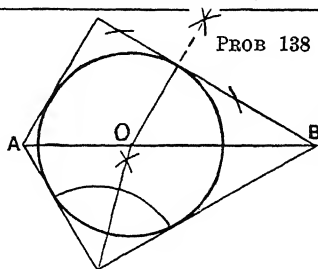
PROB 136



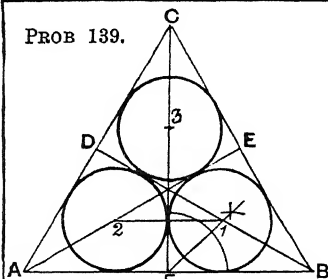
PROB 137



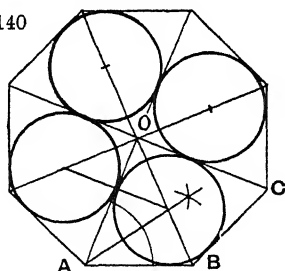
PROB 138



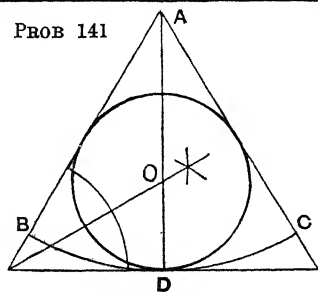
PROB 139.



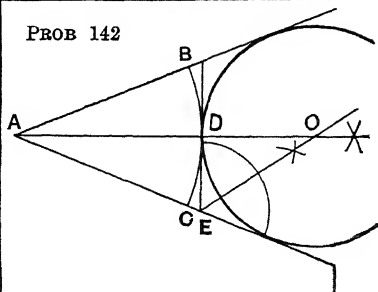
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PROB 141



PROB 142



**PROBLEM 143**—To inscribe ANY number of equal circles in a circle In this case four.

Divide the circle into twice as many sectors as circles required. In this case divide the circle into eight equal parts (Use the  $45^\circ$  set-square) In the sector  $OAB$  inscribe a circle (*Prob 141*) With centre  $O$  and radius  $OC$  mark off the centres for the other circles and describe them.

**PROBLEM 144.**—To describe ANY number of equal circles about a given circle Say six.

Divide the given circle into twice as many sectors as there are circles required, and produce the diameters At  $A$  draw a tangent, and proceed as in *Prob 142*.

Set off the other centres and describe the circles.

**PROBLEM 145.**—To describe about ANY regular polygon as many circles as the polygon has sides, each touching one side and two circles

Let the given polygon be a hexagon Draw the diameters and diagonals of the polygon, and produce them both ways Complete as in the preceding problem

**Note.**—Circles may be described about a triangle or a quadrilateral by applying the same principles.

**PROBLEM 146.**—To describe a circle having a given radius, and touching another circle either externally or internally in a given point.

Let  $A$  be the given radius, and  $B$  the point in the given circle Find the centre  $O$  Draw  $OB$  and produce it Set off  $B1$  and  $B2$  equal to the given radius  $A$  The circle described from  $1$  will touch externally, that from  $2$  internally.

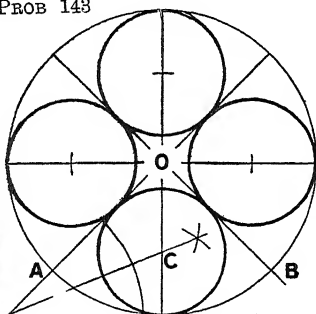
**PROBLEM 147.**—To describe a circle passing through a given point  $A$ , and touching a circle in a given point  $B$ .

The three positions of the point  $A$  should be worked. The same explanation applies to each figure The centre of the required circle must lie in the line drawn through  $B$  and  $O$ . Join  $A$  and  $B$ , and bisect. The centre must also lie in this perpendicular bisector Therefore  $C$  will be the centre in each case. With centre  $C$  and radius  $CB$  describe the circle.

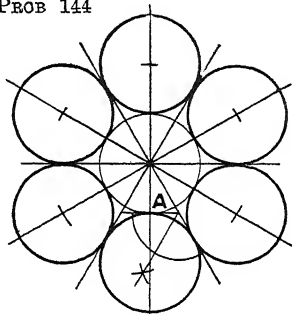
**PROBLEM 148.**—To describe a circle with a given radius, touching a given line  $AB$  and a given circle.

Let the given radius be  $0.75''$ . Draw a line parallel to  $AB$  and  $0.75''$  from it Add  $DE = 0.75''$  to the radius of the given circle, and with radius  $CE$  describe an arc cutting the line in  $O$ . With centre  $O$  and radius  $0.75''$  describe the circle.

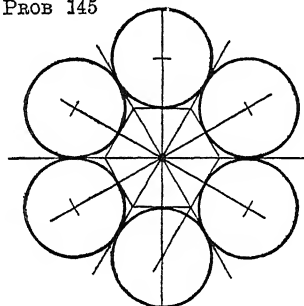
PROB 143



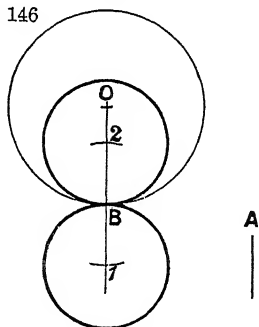
PROB 144



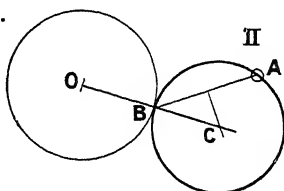
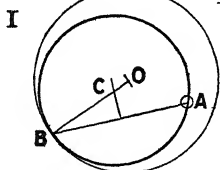
PROB 145



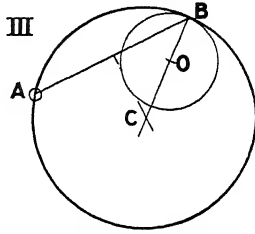
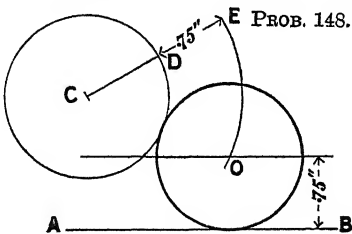
PROB. 146



PROB 147.



PROB. 148.



**PROBLEM 149**—To describe a circle touching a given circle at a given point *A*, and also touching a given straight line *BC*.

**I. Externally.**—Join *O*, the centre of the given circle, and the given point *A*, and produce the line both ways. At *A* draw the tangent *AB*. Make *BC = BA*. From *C* draw *CE* perpendicular to *BC*. With centre *E* and radius *EC* describe the circle.

**II. To include the given circle.**—Repeat the same construction on the other side of *AB*. The required circle is shown in dotted line.

**PROBLEM 150**—To describe a circle touching a given circle, and also a straight line at a given point *A*.

**I Externally**—From *A* draw *AE* perpendicular to *AB*. Through *O*, the centre of the given circle, draw *BC* also perpendicular to *AB*. Draw *AC*, cutting the given circle in *D*. Draw *OD*, and produce to meet *AE*. From centre *E* with radius *EA* describe the circle.

**II To include the given circle.**—The construction is shown in Fig. II. The explanation is the same as in I.

**PROBLEM 151**—To describe a circle touching two given circles, one of them in a given point *A*.

**I. To include both circles.**—From *A* draw a line through centre *O*. The centre of the required circle must be in this line. From *P*, the centre of the other circle, draw *PB* parallel to *AO*. Draw *AB* and produce to *C*. Draw *CP* to meet *AO* produced in *D*. With centre *D* and radius *DA* describe the circle.

**II. To include one circle.**—This may be followed from Fig. II. The explanation is similar to that for I.

**Note**—*CBP* and *CAD* are similar triangles.  $PC = PB$  and  $DA = DC$ .

**PROBLEM 152**—To describe a circle with a given radius to touch two given circles externally.

Let the given radius be  $0.5''$ . Add  $0.5''$  to the radii of both circles. From centre *A* with radius *AC* describe an arc, and from centre *B* with radius *BD* intersect this arc in *O*. With centre *O* and a radius of  $0.5''$  describe the circle.

**PROBLEM 153.**—To describe a circle with a given radius to touch two given circles: **I. Including both circles.** **II. Including one circle.**

**I** Let the given radius be  $2''$ . Draw the diameters of both circles and produce them, making *CD* and *EF* each =  $2''$ . With centre *A* and radius *AD* describe an arc, and with centre *B* and radius *BF* intersect this arc in *O*. With centre *O* and radius =  $2''$  describe the circle.

**II** Let the given radius be  $1.25''$ , and let the smaller circle be included. As the circle is to touch the larger one, add  $1.25''$  to the radius *AC*, and with centre *A* and radius *AD* describe an arc. As it is to include the smaller, make *EF* =  $1.25''$ , and with centre *B* and radius *BF* describe an arc intersecting the first arc in *O*. With centre *O* and radius =  $1.25''$  describe the circle.



**PROBLEM 154**—To describe a circle passing through two given points **A** and **B**, and touching a given line **CD**

**I** When the line joining the two points is parallel to the given line. —Join **A** and **B**, and bisect by a perpendicular meeting **CD** in **C**. The problem is now to describe a circle passing through the points **A**, **B**, and **C**. The perpendicular bisecting **AC** gives the centre.

**II** When the line joining the given points is not parallel to the given line —Join **A** and **B**, and produce to meet the given line in **C**. Find **CE**, the mean proportional to **CA** and **CB** ( $Cb = CB$ ) (*Prob. 29*). Make **CF = CE**. **F** is the point of contact. At **F** draw a perpendicular to **CD** to meet the perpendicular bisecting **AB**. The intersection of these lines gives the centre.

**PROBLEM 155**—To describe a circle passing through two points **A** and **B**, and touching a given circle.

Describe any circle passing through **A** and **B** and cutting the given circle in **C** and **D**. Draw **CD**, and produce to meet **AB** produced in **E**. From **E** draw a tangent **EF** to the given circle (*Prob. 69*). **F** is the point of contact. Describe a circle passing through **A**, **B**, and **F**.

**PROBLEM 156**—To inscribe within a given circle of 1.5" radius two other circles having radii of 0.5" and 0.7" respectively, touching each other and the given circle.

Draw the diameter **AC**. Set off **A1 = 0.5"**, and describe one of the circles. From **B** and **C** set off **B2** and **C3** each = 0.7". From centre 1 with radius 1.2 describe an arc, and from centre **O** with radius 0.3 intersect this arc in 4. From 4 with radius = 0.7" describe the second circle.

**PROBLEM 157**.—To describe three circles touching each other, their radii being 1", 0.7", and 1.2" respectively.

Draw any line and set off **AB = 1"**, and **BC = 0.7"**. Describe two of the circles. Make **B1** and **B2** each = 1.2". From centre **A** with radius 1.2 describe an arc, and from centre **C** with radius 0.7 intersect this arc in **O**. With **O** as centre and 1.2" as radius describe the third circle. The lines **OA** and **OC** show the points of contact.

**PROBLEM 158**.—To describe three circles touching each other, the position of their centres being the points **A**, **B**, and **C**.

Join the points, and find the centre **O** of the triangle thus formed. From **O** draw a perpendicular **OE** to one of the sides. From centres **A** and **B** describe two of the circles touching at **E**. From centre **C** with radius **CF** describe the third circle.





**PROBLEM 159**—To describe a circle with a given radius touching two curved lines  $AB$  and  $CD$ .

Let the given radius be 1.5 cms. Find the centres  $E$  and  $F$  from which the arcs  $AB$  and  $CD$  are described ( $P_1 ob$  67). Join  $E$  and  $F$  and produce each way to cut the arcs in  $G$  and  $K$ . Set off  $GH$  and  $KL$  each = 1.5 cms. From centre  $E$  with radius  $EH$  describe an arc, and from centre  $F$  with radius  $FL$  intersect this arc in  $O$ . From  $O$  with radius = 1.5 cms describe the circle.

**PROBLEM 160**—Describe a circle of 1.25" diameter touching each pair of adjacent lines  $oa, ob, oc, od$ , produced if necessary. Describe two circles touching the three circles.

Describe a circle touching each pair of lines, having a radius of  $\frac{5}{8}$ " by Problem 123. The only two circles that can touch the three are the circles touching them externally and internally.

From  $o$  draw a line passing through the centre of one of the circles. With centre  $o$  and radii  $oe, of$  describe the circles.

Notes—1 If the angles between each pair of adjacent lines are unequal, then find the centre for the touching circles by bisecting the lines joining the centres of the circles. The point where the two bisecting lines meet will be the centre.

2 In working problems of a similar character to Problems 146–160, the following facts should be constantly borne in mind.

$a$  'If two circles touch one another either internally or externally, the straight line, or the straight line produced, which joins their centres passes through the point of contact' (Euc III 11 and 12).

$b$  'Equal straight lines in a circle are equally distant from the centre, and those which are equally distant from the centre are equal' (Euc III. 14).

$c$  'If a straight line touch a circle, the straight line drawn from the centre to the point of contact is perpendicular to the line touching the circle'. Also the converse of this 'If a straight line touch a circle, and from the point of contact a straight line be drawn at right angles to the touching line, the centre of the circle shall be in that line' (Euc III 18 and 19).

3 There is a large variety of problems of this kind, and in cases of difficulty a sketch of the required figure should be made, assuming it to be completed. Then by working backwards step by step, endeavour to find out what principles must be used to secure this result.

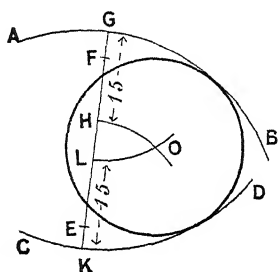
**PROBLEM 161.**—The given figure is made up of circular arcs, all of  $\frac{3}{4}$ " radius. Draw it full size.

First draw the equilateral triangle  $ABC$  of  $1\frac{3}{4}$ " sides. With centres  $A, B, C$  and radius  $\frac{3}{4}$ " describe arcs intersecting at  $D, E, F$ , giving three of the centres. The points  $a, b, c$  are obtained in each case by describing two circles tangential to those already obtained. Join the points  $D, E, F$ , and produce the lines each way. Set off on each line  $\frac{3}{4}$ " as shown, and from the centres thus obtained describe arcs completing the figure.

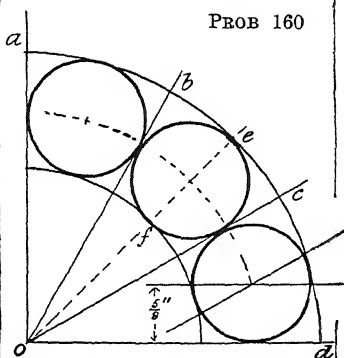
**PROBLEM 162.**—Draw the given geometrical pattern.

Describe a circle of  $3\frac{1}{2}$ " diameter. From the extremity of the diameter set off  $\frac{3}{4}$ " and describe the inner circle. Divide the circumference into six equal parts, and draw the diameters. From  $a$  set off  $1\frac{3}{4}$ " and describe one of the smaller circles. Set off the centres for the other two inner circles and describe them. From the same centres describe the remaining circles. Omit the parts where the lines are not continuous.

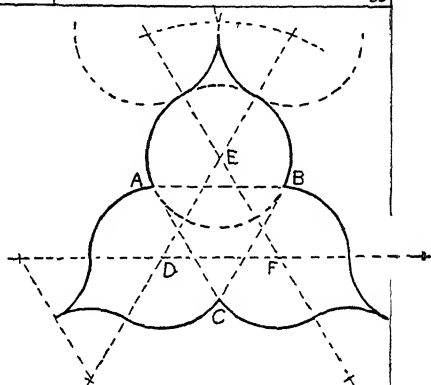
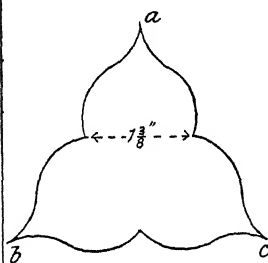
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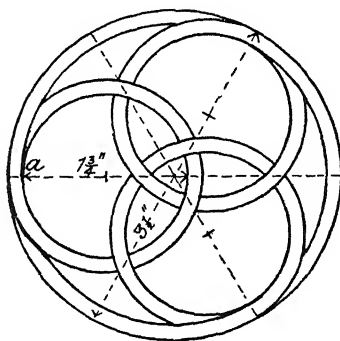
PROB 160



PROB 161.



PROB 162



**PROBLEM 163**—Draw the geometrical pattern shown, adhering strictly to the figured dimensions, and showing the construction lines.

It will be seen that the figure is formed by six equal circles, each touching two other circles. But when six equal circles are inscribed in a circle, a seventh equal circle may always be inscribed touching the six circles internally. The problem then resolves itself into describing a circle of  $\frac{3}{4}$ " radius, placing six equal circles about it (*Prob 144*), and omitting those portions of the circles which are not needed. Describe a circle of  $\frac{3}{4}$ " radius, divide its circumference into 12 equal parts. Find the centres for the circles, and complete as shown.

Note—Problems 161–163 indicate a few of the numerous ways in which the preceding problems on circles may be applied

**PROBLEM 164.**—To inscribe a semicircle in an isosceles triangle.

Bisect the angle  $A\ C\ B$  by the line  $C\ D$ . Bisect the angle  $C\ D\ B$  by the line  $D\ E$

Draw  $E\ F$  parallel to  $A\ B$ , and on it describe a semicircle.

**PROBLEM 165.**—To inscribe three equal semicircles in an equilateral triangle having their diameters adjacent, and each touching one side of the triangle.

Bisect each angle of the triangle by the lines  $A\ D$ ,  $B\ E$ , and  $C\ F$ .

Bisect the angle  $C\ F\ B$  by the line  $F\ G$ . From  $G$  draw  $G\ J$ ,  $G\ H$  parallel to  $A\ B$ ,  $B\ C$ . Join  $J$  and  $H$ . On the lines  $H\ G$ ,  $G\ J$ ,  $J\ H$  describe semicircles forming a *trefoil* of semicircular arcs.

**PROBLEM 166.**—To inscribe four equal semicircles in a square, having their diameters adjacent, and each touching one side of the square.

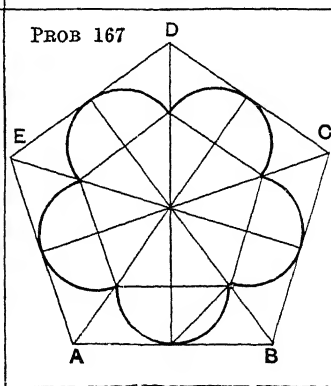
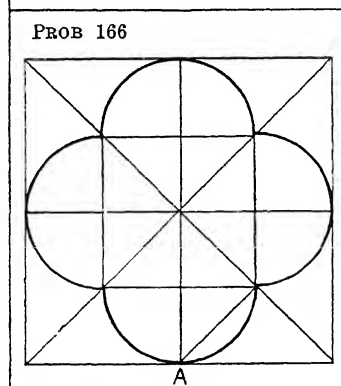
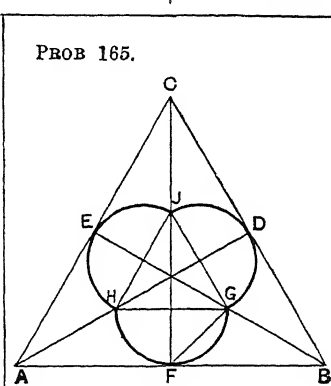
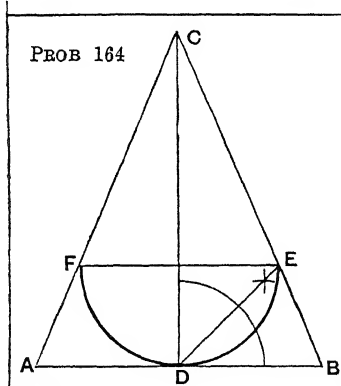
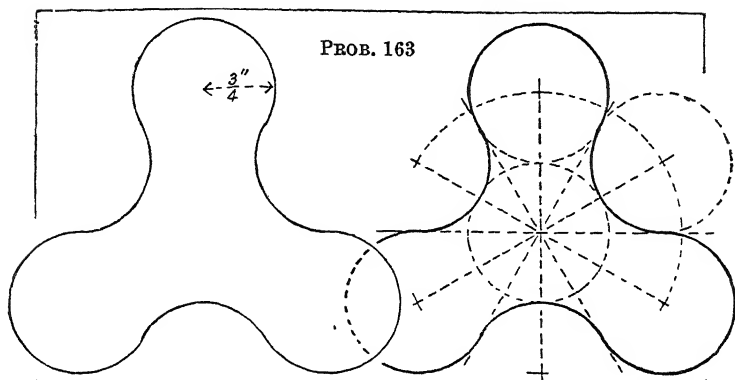
Draw the diagonals and diameters of the given square. Bisect one of the angles at  $A$ , and obtain the inner square. Describe a semicircle on each side of this square, forming a *quatrefoil* of semicircular arcs.

**PROBLEM 167.**—To inscribe within any regular polygon as many semicircles as the figure has sides, each touching one side and having their diameters adjacent.

Let  $A\ B\ C\ D\ E$  be the given polygon, in this case a pentagon. Divide the polygon into equal isosceles triangles. Inscribe a semicircle in each as before, forming a *cinquefoil* of semicircular arcs.

Notes—1 In Problems 165–173 notice that the foiled figures are made by *semicircular arcs*. In Problems 174–177 they are formed by *tangential arcs*. The problems relating to the inscription and circumscription of circles and foiled figures are exceedingly useful in geometrical design, as they furnish the leading lines for window tracery and ornamental forms of various kinds

2 Problems 165–168 depend upon the same principles of construction as those used in Problem 164.



**PROBLEM 168**—To inscribe any number of equal semicircles in a circle, having their diameters adjacent, and each touching the circumference

Divide the circumference of the given circle into twice as many parts as there are semicircles required. (In this case four.) Draw the diameters. Draw a tangent at A and bisect the angle. Set off the distance O B on the alternate diameters, and draw the square. On each side of the square describe a semicircle.

**PROBLEM 169.**—To inscribe a semicircle in a square.

Draw the diagonals of the square, and on one of them A B describe a semicircle. From O the centre of the square draw O C at right angles to the side of the square. Draw C F, and from D where the line C F cuts the side of the square draw D E parallel to C O. Through E draw a line parallel to A B, and on it describe the required semicircle.

**PROBLEM 170.**—To inscribe four equal semicircles in a square, having their diameters adjacent, and each touching two sides of the square

Draw the diagonals and diameters of the given square, and inscribe a semicircle in one of the four squares thus obtained by the preceding problem. Complete the inner square, and describe a semicircle on the other three sides.

**PROBLEM 171.**—To inscribe a semicircle in a trapezoid, or kite.

Draw the diagonals of the given figure. On A B, the shorter diagonal, describe a semicircle. From O draw O C perpendicular to the side of the trapezoid. Draw C F, and from D draw D E parallel to C O. Through E draw a line parallel to A B, and upon it describe the required semicircle.

**PROBLEM 172.**—To inscribe three equal semicircles in an equilateral triangle, having their diameters adjacent, and each touching two sides of the triangle.

Bisect each angle of the triangle by lines dividing the triangle into three equal trapezoids. Inscribe a semicircle in the trapezoid A F B G by the preceding problem. Complete the inner equilateral triangle, and upon the other two sides describe semicircles.

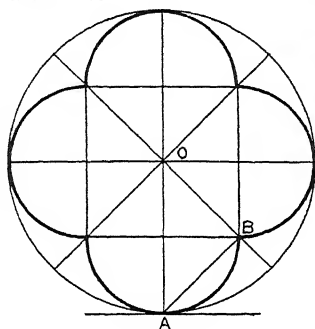
**PROBLEM 173.**—To inscribe in any regular polygon a number of equal semicircles, having their diameters adjacent, and each touching two sides of the polygon.

Let the given polygon be a regular hexagon. Draw the diameters and diagonals of the figure, cutting it into six equal trapezoids, and inscribe a semicircle in each, as shown.

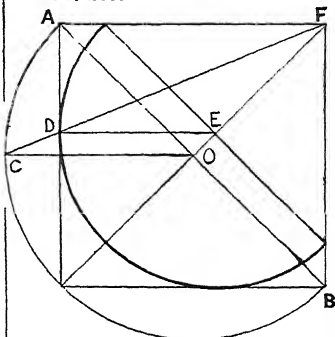
Notes.—1 Problems 170–173 depend upon the same principles of construction as those used in Problem 169

2 Notice that in all cases the line joining the points C and F is drawn from C to the angle of the quadrilateral which is opposite to the semicircle described upon A B.

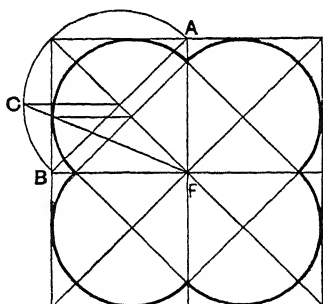
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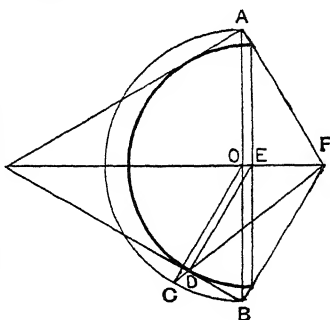
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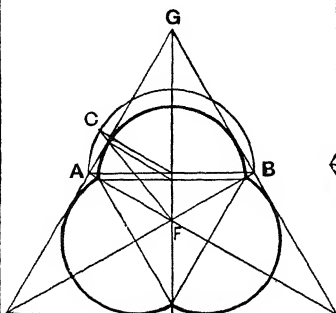
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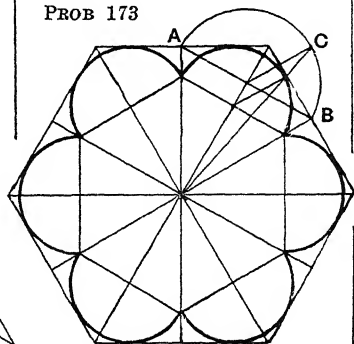
PROB 171



PROB 172



PROB 173



**PROBLEM 174**—To describe a trefoil of tangential arcs, the radius of the arc being given.

Construct an equilateral triangle having each of its sides double the given radius. From each angle with the given radius describe the arcs.

**PROBLEM 175.**—To describe a quatrefoil of tangential arcs, the radius being given

Construct a square having each of its sides double the given radius and describe the arcs as shown

Note—The same principle may be employed in the construction of all foiled figures formed by tangential arcs.

**PROBLEM 176.**—About any regular polygon, to construct a foiled figure of tangential arcs.

Let the given polygon be a regular hexagon Bisect one side to obtain the radius, and proceed as in the preceding problems.

Note—These problems are identical with the problems connected with the describing of a number of equal circles each touching two others For example, in Problem 140, if the centres of the circles be joined, and the portions of the circles enclosed by the square thus formed be removed, we have the figure of Problem 175 inscribed in an octagon.

**PROBLEM 177.**—To draw the given geometrical pattern to the figured dimensions

The inner arcs are identical with *Prob. 176*. Construct a hexagon having each side double the given radius ( $\frac{5}{8}''$ ), and from each angle describe the tangential arcs. From the same centres with a radius of  $\frac{3}{4}''$  describe the outer arcs.

## CHAPTER XII

### THE INSCRIPTION AND CIRCUMSCRIPTION OF RECTILINEAL FIGURES

**PROBLEM 178.**—To inscribe or describe an equilateral triangle in or about a circle

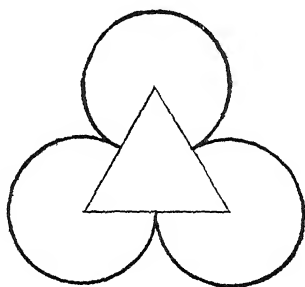
Draw a diameter A B, and find its centre. Set off A C, A D equal to the radius. Draw B C, B D, C D

For the triangle about the circle find the points of the inscribed triangle, and through each draw a parallel to the opposite side.

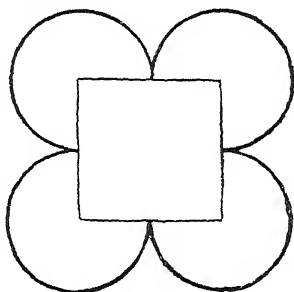
**PROBLEM 179.**—To inscribe or describe a square in or about a circle.

Draw two diameters at right angles to each other, and join the ends. For the square about the circle, with centres A, B, C, D, and radius A O, describe arcs intersecting without the circle and join the points; or through the points A, B, C, D draw parallels to the diagonals.

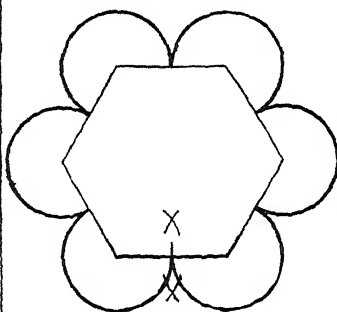
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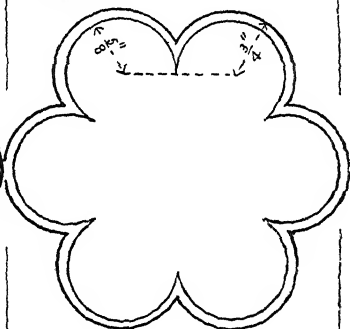
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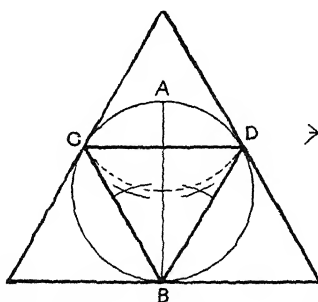
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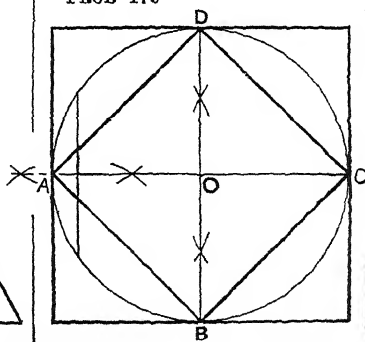
PROB 177



PROB 178



PROB 179





**PROBLEM 180.**—To inscribe an equilateral triangle in a square.

Draw the diagonal  $AB$ . On  $AB$  construct an equilateral triangle. From  $C$  draw  $CE$ ,  $CD$ , parallel to the sides of the triangle. Join  $D$  and  $E$ .

**PROBLEM 181.**—To inscribe an equilateral triangle in a pentagon.

Find  $B$ , the middle point of one side, and join it with  $A$ , the opposite angle. On each side of  $AB$  construct an angle of  $30^\circ$  (*Prob. 33*), and join  $CD$ .

Notes—1 This method may be applied to the square by making an angle of  $30^\circ$  on each side of the diagonal

2 The triangle may also be obtained, as in Problem 180, by describing an equilateral triangle on  $EF$ , and drawing parallels from  $A$ .

**PROBLEM 182.**—To describe an equilateral triangle about a square.

On one side  $AB$  of the given square describe an equilateral triangle  $ABE$ . Produce  $EA$ ,  $EB$  to meet the side  $CD$  produced.

**PROBLEM 183.**—About a given square to describe a triangle similar to a given triangle.

Let  $ABCD$  be the given square and  $EFG$  the given triangle. This problem depends upon exactly the same principle as the preceding. On  $AB$  construct a triangle similar to the triangle  $EFG$ . Produce the sides to meet  $CD$  produced.

**PROBLEM 184.**—About a given triangle to describe another triangle similar to a given triangle.

On  $AB$  construct a triangle similar to the triangle  $DEF$ .

Through  $C$  draw a line parallel to  $AB$ , and produce  $GA$ ,  $GB$  to meet it.

**PROBLEM 185.**—Within a given triangle to inscribe another triangle similar to a given triangle.

On  $AC$ , a side of the given triangle, construct a triangle similar to the triangle  $DEF$ . Draw  $GB$ . From  $H$  draw  $HJ$  and  $HK$  parallel to  $GC$  and  $GA$ . Join  $J$  and  $K$ . Then  $HJK$  is the required triangle

**PROBLEM 186.**—Within a given circle to inscribe a triangle similar to another triangle.

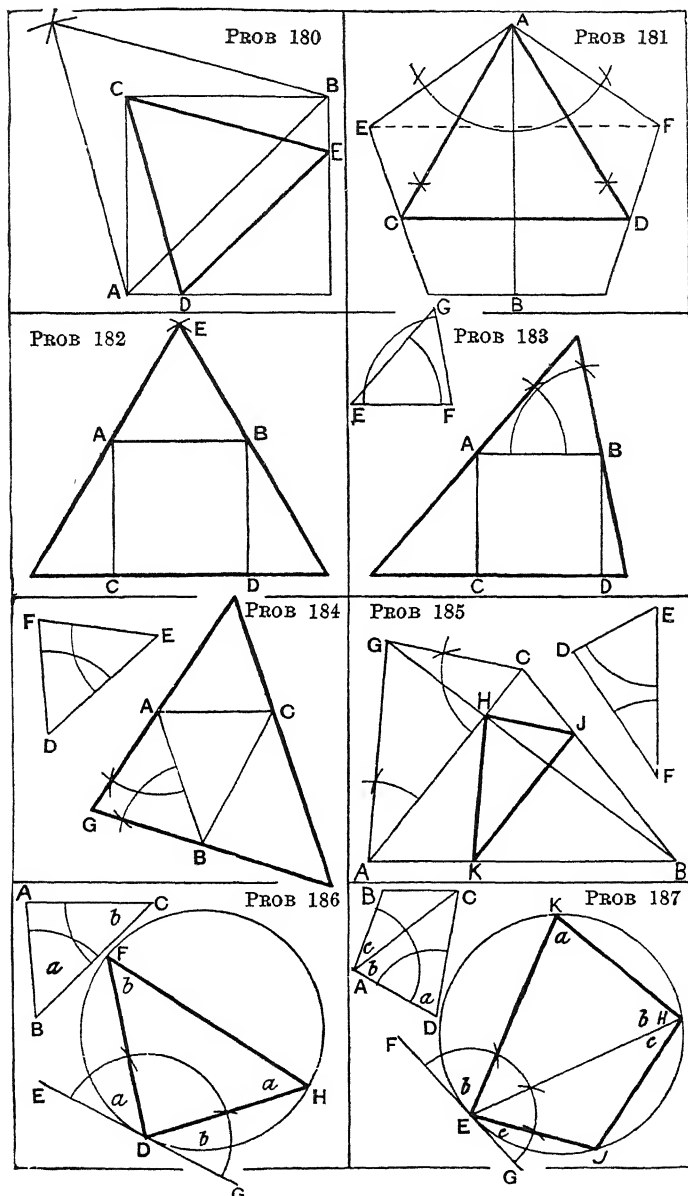
At any point  $D$  in the circumference of the given circle draw a tangent. Make the angle  $EDF$  equal to the angle  $ABC$ , and the angle  $GDH$  equal to the angle  $BCA$ . Join  $FH$ . Then  $DFH$  is the required triangle, the angle at  $F$  being equal to the angle at  $C$ , and the angle at  $H$  to the angle at  $B$  (*Euc. iv. 2*).

**PROBLEM 187.**—Within a given circle to inscribe a quadrilateral figure similar to a given one.

Note—A quadrilateral figure can only be inscribed in a circle when the sum of the opposite angles equals two right angles

Let the given quadrilateral have angles of  $100^\circ$ ,  $70^\circ$ ,  $80^\circ$ , and  $110^\circ$ , as shown.

Through any point  $E$  draw a tangent  $FG$ . Make the angle  $HEG$  equal to the angle  $ADC$ ,  $GEJ$  equal to  $CAB$ , and  $FEK$  equal to  $CAD$ . Draw  $JH$ ,  $HK$ .



**PROBLEM 188.**—About a given circle to describe a triangle similar to a given triangle.

Produce the base  $AB$  of the given triangle. Find the centre  $O$  of the given circle, draw any radius  $OG$ , and produce it. Construct the angle  $FOG$  equal to the exterior angle  $CBD$ , and the angle  $HOG$  equal to the exterior angle  $CAE$ . Produce  $OH$ ,  $OF$ , and draw tangents as shown. (Euc iv. 3.)

Note—Angle  $FOG$  + angle  $FJG$  = angle  $CBD$  + angle  $CBA$  = two right angles, because the angles at  $F$  and  $G$  are right angles

**PROBLEM 189.**—In a given square to inscribe an isosceles triangle, the base being given.

Draw the diagonals  $AC$  and  $BD$  of the given square, and on  $AC$  set off  $AF$  equal to the given base  $E$ . Draw  $FG$  parallel to  $AB$  and  $GH$  parallel to  $AF$ . Draw  $DH$  and  $DG$ .

Note— $GH = AF$  (Euc i. 34)

**PROBLEM 190.**—To describe a square about an isosceles triangle,

Bisect the base  $BC$  of the given triangle by the line  $AD$ . On  $BC$  describe a semicircle. Then  $AD$  will be a diagonal of the required square. Draw  $DC$  and  $DB$  of indefinite length, and from  $A$  draw parallels to meet them.

**PROBLEM 191.**—In a given hexagon to inscribe an isosceles triangle, the base being given.

Draw the diagonal  $AB$  of the given hexagon. Draw  $CD$ , one of the diameters, at right angles to  $AB$ . Set off  $DF$  equal to  $E$ . Draw  $FG$  parallel to  $BD$ , and  $GH$  parallel to  $CD$ . Draw  $AG$  and  $AH$ .

**PROBLEM 192.**—Within a given circle to inscribe an isosceles triangle, the base being given.

Draw two diameters of the given circle,  $AB$  and  $CD$ , at right angles to each other. Make  $OF$  equal to half the given base  $E$ . Draw  $FG$  parallel to  $AB$ , and  $GH$  parallel to  $CD$ . Draw  $AG$  and  $AH$ .

Note—A similar method may be used for inscribing an isosceles triangle in a square, rhombus, or polygon.

**PROBLEM 193.**—To inscribe a square in a triangle

Draw  $CD$  perpendicular to  $AB$ . From  $C$  draw  $CE$  parallel to  $AB$  and equal to  $CD$ . Draw  $AE$ . From  $F$  draw  $FG$  parallel to  $AB$ , and  $FH$  parallel to  $CD$ . From  $G$  draw  $GJ$  parallel to  $CD$ . Then  $GFHJ$  is the required square.

**PROBLEM 194.**—To inscribe a square in a trapezion.

Draw the diagonals  $AB$ ,  $CD$ . Draw  $CE$  parallel to  $AB$ , and equal to  $CD$ . Draw  $AE$ . From  $F$  draw  $FG$  parallel to  $AB$ , and  $FH$  parallel to  $CD$ . From  $G$  draw  $GJ$  parallel to  $CD$ , and join  $J$  and  $H$ .

**PROBLEM 195.**—To inscribe a square in a pentagon.

If two sides of the pentagon be produced until they meet, a trapezion will be formed. The same construction as in *Prob. 194* will then apply.



**PROBLEM 196.**—To inscribe a square in a sector.

Join B and C. Draw CD perpendicular to CB and equal to it. Draw AD, and from E draw EG parallel to BC, and EF parallel to CD.

Draw GH and FH parallel to EF and EG.

**PROBLEM 197.**—To inscribe a square in a segment

Bisect the chord AB of the segment. Draw BD equal to AB and perpendicular to it. Draw DC. From E draw EF parallel to BD and EG parallel to AB. Draw GH parallel to EF.

**PROBLEM 198**—Within a given square to inscribe another square, one angle to touch a side at a given point.

Let A be the position of one angle. Draw the diagonals of the given square.

With centre O and radius OA describe a circle. Join the points A, B, C, and D.

Note—If the length of the diagonal be given, proceed in a similar manner, taking half the given diagonal as radius.

**PROBLEM 199.**—To inscribe a square in a rhombus.

Draw the diagonals, and bisect the angles thus formed.

Join A, B, C, and D.

**PROBLEM 200.**—To inscribe a square in a hexagon.

Draw the diagonal AB, and bisect it at right angles by the diameter CD.

Complete as in the preceding problem.

Note—To inscribe a square in an octagon, join the alternate corners

**PROBLEM 201.**—To inscribe a rhombus in a parallelogram, having one of its angles at a given point.

Let A be the given point. Draw the diagonals of the parallelogram.

From A draw AB passing through the centre O of the parallelogram.

Bisect AB by a line CD at right angles to it. Draw AC, CB, BD, and DA.

**PROBLEM 202.**—To inscribe a regular hexagon in an equilateral triangle.

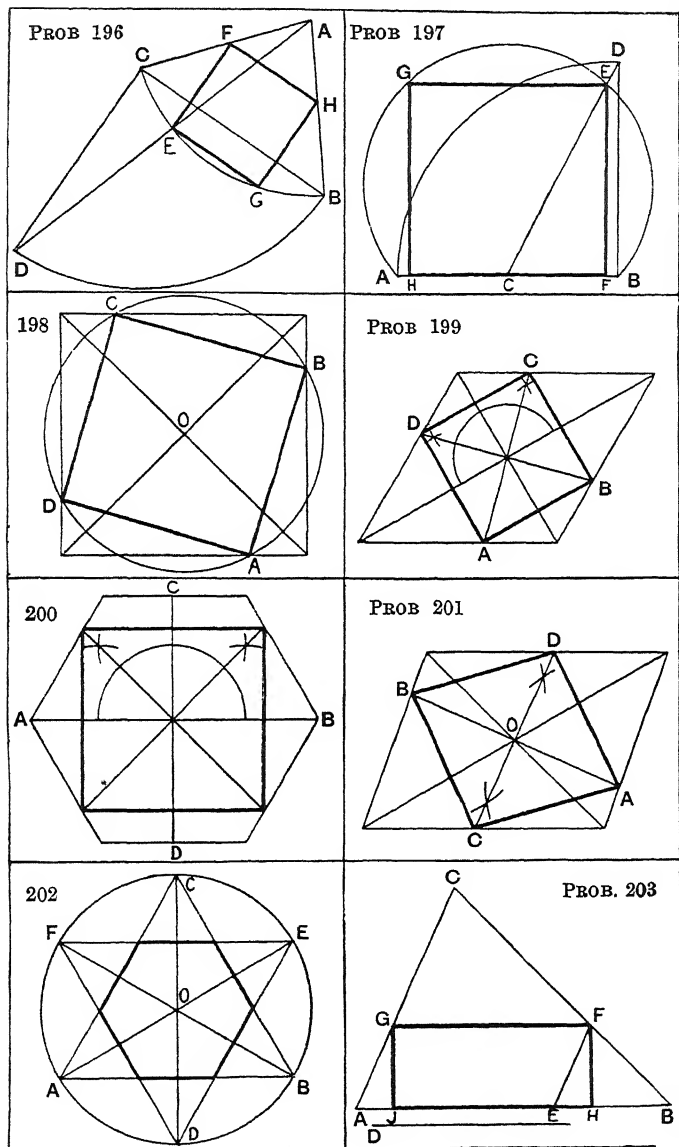
Bisect each of the angles of the given triangle by the lines AE, BF, and CD. With centre O and radius OA describe a circle. Draw DE, EF, and FD.

**PROBLEM 203.**—Within a given triangle to inscribe a rectangle, the length of one side being given.

Let ABC be the given triangle and D the given side of the rectangle

Set off AE equal to D. Draw EF parallel to AC.

From F draw FG parallel to AB. Draw FH and GJ perpendicular to AB.



**PROBLEM 204.**—Within any given quadrilateral to inscribe a parallelogram, having given the position of one angle.

Let  $E$  be the position of one angle. Draw the diagonals  $AC$ ,  $BD$ . Draw  $EF$  parallel to  $AC$ ,  $EH$  and  $FG$  parallel to  $BD$ . Join  $G$  and  $H$ .  $EFGH$  will be the required parallelogram.

**PROBLEM 205.**—Within any given quadrilateral to inscribe a parallelogram, having given the length of one side

Let  $E$  be the length of one side. Draw the diagonals  $AC$ ,  $BD$ . On one of them set off  $AF$  equal to  $E$ .

From  $F$  draw  $FH$  parallel to  $AB$ . Draw  $HG$  parallel to  $AC$ ,  $HK$  and  $GJ$  parallel to  $BD$ . Join  $JK$ .

Notes—1  $GH = AF$  (Euc 1 34)

2 The same construction will apply for inscribing a rectangle in a square, rhombus, or trapezoid

**PROBLEM 206** —Within a given triangle or any regular polygon to inscribe another similar figure, having its sides parallel to and equidistant from those of the given figure, the length of one side being given.

Let  $ABC$  be the given triangle, and  $D$  the length of one of the sides of the required triangle. Bisect the angles and obtain the centre  $E$ . Set off  $AF$  equal to  $D$ . Draw  $FG$  parallel to  $AE$ ,  $GH$  parallel to  $AB$ ,  $HJ$  parallel to  $AC$ , and  $GJ$  parallel to  $BC$ .

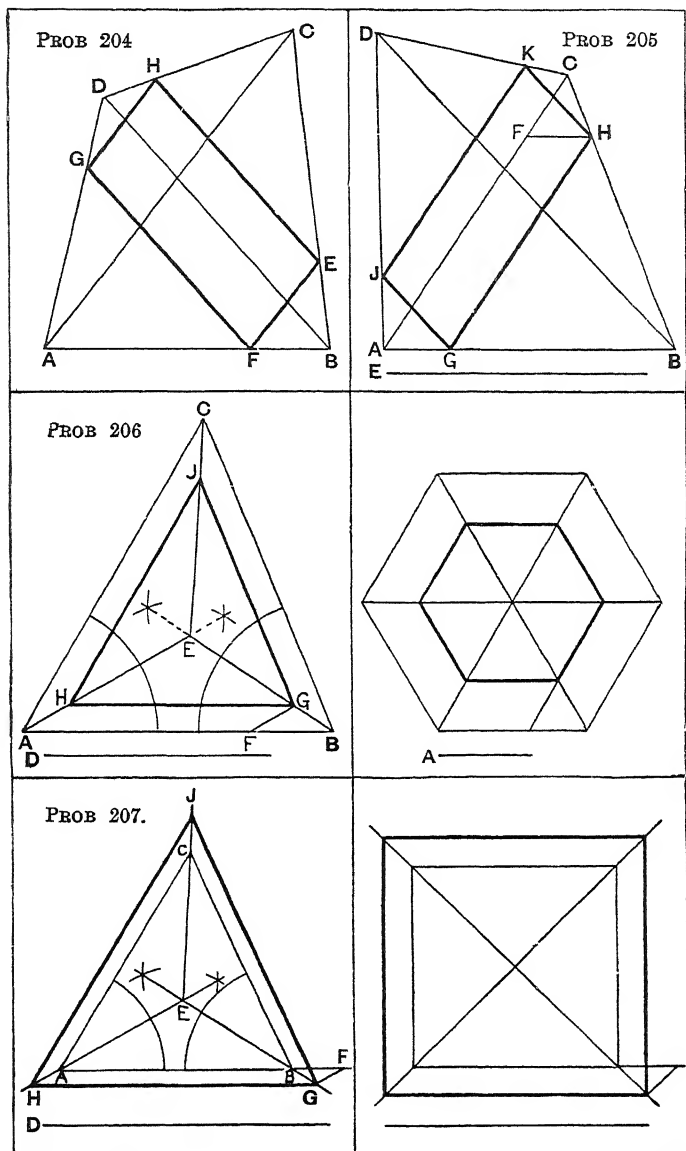
Note —The construction for the inscription of a hexagon within a hexagon is also shown,  $A$  being the length of one side of the inscribed figure.

**PROBLEM 207.**—About a given triangle or any regular polygon to describe another similar figure, having its sides parallel to and equidistant from those of the given figure, the length of one side being given.

Let  $ABC$  be the given triangle, and  $D$  the length of one of the sides of the required triangle.

Find the centre as before, and produce the lines bisecting the angles. Produce  $AB$ , and from  $A$  set off  $AF$  equal to  $D$ . Draw  $FG$  parallel to  $AE$ ,  $GH$  parallel to  $AB$ ,  $HJ$  parallel to  $AC$ , and  $GJ$  parallel to  $BC$ .

Note —The construction for the description of a square about a square is also shown.





## EXERCISES

## CHAPTER XI

1. To describe a circle touching two lines and passing through a point between them (*Art*) (*Prob* 125)

2. Describe a circle of 3" diameter, and within it inscribe six equal semi-circles (*Art*)

3. Describe two circles touching each other and having radii of 1" and  $\frac{3}{4}$ " respectively. Draw a third circle having a radius of  $\frac{1}{2}$ " to touch the other two circles (*Art*) (*Prob* 152)

4. Within a given equilateral triangle of 3" sides inscribe three equal semi-circles, having their diameters adjacent and each touching one side of the triangle. (*Art*)

5. Within a given circle of  $1\frac{1}{2}$ " radius inscribe four equal circles (*Art*)

6. Within a square of  $2\frac{3}{8}$ " sides inscribe four equal semicircles each touching two sides of the square (*Art*)

7. Within a square of  $2\frac{1}{2}$ " sides inscribe four equal circles, each touching two sides and two circles (*Art*)

8. Describe a circle of  $\frac{3}{4}$ " radius which shall touch a given circle and a given straight line (*Art*) (*Prob* 148)

9. Construct a quatrefoil of tangential arcs of  $\frac{1}{2}$ " radius. (*Art*)

10. Describe a circle to enclose two other given circles (*Art*)

11. Describe a triangle with sides of 3", 2", and  $2\frac{1}{2}$ " respectively, and within it inscribe three circles each touching one side and two circles (*Prob* 132)

12. Within a given isosceles triangle inscribe two equal circles touching each other and two sides of the triangle (*Art*)

13. Two parallel lines A B, C D are 2" apart and  $1\frac{1}{2}$ " long. Describe a circle to pass through points A and B and touch C D (*Art*)

14. Within a given pentagon inscribe five tangential arcs each touching two sides of the pentagon (*Art*) (*Note, Prob* 139)

15. About a square of 1" sides describe four equal circles each touching a side of the square and two circles (*Art*) (*Prob* 145)

16. Describe a circle  $\frac{3}{4}$ " radius to pass through a point A and touch a given straight line (*Art*)

17. Construct a rhombus of  $2\frac{3}{4}$ " sides and within it inscribe four equal circles each touching one side and two circles

18. Describe a triangle about a circle of 1" diameter, having angles of  $30^\circ$  and  $105^\circ$

19. About a circle of  $\frac{1}{2}$ " radius place five equal circles each one touching two others and the given circle

20. Draw two right lines meeting at an angle of  $38^\circ$ . Describe a circle of  $\frac{3}{4}$ " radius to touch these lines (*Sc*)

21. Draw three equal circles of  $\frac{7}{8}$ " radius, each touching the other two. (*Sc*, 1870)

22. Two circles of 1" and 5" radius respectively have their centres 2.5" apart. Draw a circle of 1.5" radius to touch both, but to contain the smaller one (*Sc*) (*Prob*. 153)

23. Draw three circles, each touching the other two, their radii being .5", .75", and 1", respectively. (*Sc*.) (*Prob*. 154)

24. Construct an equilateral triangle of  $2\frac{1}{4}$ " sides. On each side as diameter describe a circle. Circumscribe the three circles by a circle. (*Sc*)

25. Describe a circle of  $2\frac{1}{2}$ " radius touching two given circles of 1" and  $\frac{3}{4}$ " radius, and having their centres  $2\frac{1}{4}$ " apart (*Sc*.)

- 26 Inscribe a circle in a rhombus of 2" side and  $2\frac{1}{4}$ " diagonal. (Sc)
- 27 Draw three circles of 1", 1.25", and 1.5" diameter, each touching the other two externally. Draw a circle which shall be touched internally by the largest and smallest of these three circles (Sc.) (Prob 152)
- 28 Draw two circles touching the same straight line at points 2.5" apart and touching one another, the radius of the smaller circles to be 1" (Sc) (Prob 150)
- 29 Construct a square of  $4\frac{1}{2}$ " side, and place in it four equal circles, each touching one side and two diagonals (Sc)
- 30 Draw two lines cutting each other at  $57^\circ$ , and describe four circles of  $2\frac{1}{8}$ " diameter, each touching both lines (Sc)
- 31 Describe a circle passing through two given points  $a$  and  $b$  and touching a given line  $cd$ . The line joining  $a$  and  $b$  is not parallel to  $cd$ . (Sc) (Prob 158)
- 32 Describe a circle passing through a point  $p$  and touching a line  $ab$  in a given point  $c$ . (Sc.) (Prob 121.)

## CHAPTER XII

1. Draw a triangle and within it inscribe a square
- 2 Within a square of 2" sides inscribe the largest possible isosceles triangle having its base  $\frac{1}{2}$ " long (Art) (Prob 189)
- 3 Draw a trapezoid with sides of 1" and 2" respectively, and within it inscribe a square (Art)
- 4 Within a square of 2" sides inscribe the largest possible equilateral triangle (Art)
5. About a regular pentagon of 1" sides describe a similar figure, having its sides parallel and equidistant to those of the given figure, and  $1\frac{1}{2}$ " in length. (Art)
6. Within an equilateral triangle of 3" sides inscribe a similar figure, base  $1\frac{1}{4}$ " (Art)
7. About a circle of  $1\frac{1}{2}$ " diameter describe an equilateral triangle (Art)
- 8 Within a circle of  $1\frac{1}{4}$ " radius inscribe a triangle having angles of  $30^\circ$  and  $60^\circ$  (Art)
9. Construct a parallelogram, sides  $2\frac{1}{2}$ " and  $1\frac{1}{2}$ ", included angle  $50^\circ$ , and within it inscribe a rhombus having one angle touching one side of the parallelogram at a point  $\frac{1}{2}$ " from one of the corners (Art)
10. About an isosceles triangle describe a square. (Prob 190)
- 11 Within a given circle of 3" diameter inscribe an isosceles triangle having its equal sides  $2\frac{1}{2}$ " long (Art)
12. Construct an isosceles triangle, the two equal sides to touch the circumference of a given circle at two given points  $A$  and  $B$ ; the angle made by the radii from  $A$  and  $B$  to be  $100^\circ$
- 13 Draw a triangle two of whose angles are  $50^\circ$  and  $65^\circ$ , and the radius of the inscribed circle 1" (Sc)
- 14 Within a square of 3" sides inscribe an octagon, so that the alternate sides of the octagon shall coincide with the sides of the square (Sc)
- 15 Construct a quadrilateral base 3", base angles  $90^\circ$  and  $75^\circ$ , sides 2" and 2" Within it inscribe a parallelogram having a side of 2"

## CHAPTER XIII

## AREAS

Before beginning the problems on areas, the following principles should be thoroughly understood; and it will be of considerable service to the student to go through Euclid's demonstrations of these principles —

1 The area of a plane figure is the amount of surface enclosed by its boundary, or perimeter. It depends upon both the shape and the perimeter of the figure.

2. Parallelograms upon the same base, and between the same parallels, are equal. (Euc. I. 35)

$ABCD = ABDE$  (Fig. 1)  $F G H J = F G K L$ . (Fig. 2)

3 Parallelograms upon equal bases, and between the same parallels, are equal. (Euc. I. 36.)

$M N O P = Q R S T$ , because each of them is equal to  $M N S T$  (Fig. 3.)

4. Triangles upon the same base, and between the same parallels, are equal. (Euc. I. 37.)

$A B C = A B D$ . (Fig. 4)

5. Triangles upon equal bases, and between the same parallels, are equal (Euc. I. 38.)

$A B C = D E F$ . (Fig. 5)

6. If a parallelogram and a triangle be upon the same base and between the same parallels, the parallelogram shall be double of the triangle. (Euc. I. 41)

$A B C D = \text{twice } A B C$ . (Fig. 6.)  $E F G H = \text{twice } E F J$ . (Fig. 7)

7. The square on the hypotenuse of a right-angled triangle is equal to the sum of the squares on the other two sides. (Euc. I. 47.)

The sq.  $C B D E = \text{the sq. } A B F G + \text{the sq. } A H J C$ . (Fig. 8.)

Note—The same principle applies to other figures constructed upon the sides of a right-angled triangle as long as they are similar (Euc. VI. 31)

8. The area of a triangle is equal to the area of a rectangle upon the same base, but having half the altitude

Triangle  $A B C = \text{rectangle } A B E F$ .  $A E = \frac{1}{2} C D$ . (Fig. 9.)

9. Parallelograms and triangles upon the same base have their areas in the same ratio as their altitudes

$A B E F = \text{twice } A B C D$ , because the altitude  $B E = \text{twice the altitude } B G$ . (Fig. 10.)

$A B D = \text{three times } A B C$ , because the altitude  $D E = \text{three times the altitude } C E$ . (Fig. 11)

10. Parallelograms and triangles of the same altitude are to one another as their bases. (Euc. VI. 1)

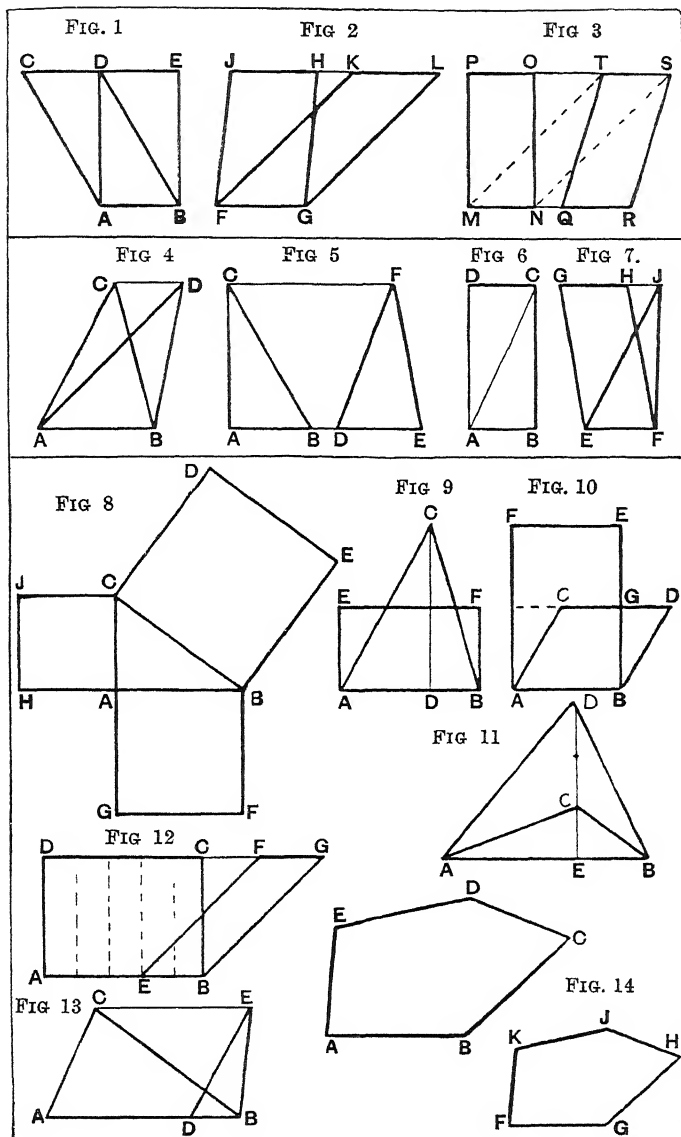
$E B F G = \frac{2}{3} A B C D$ , because  $E B = \frac{2}{3} A B$ . (Fig. 12)

$D B E = \frac{1}{4} A B C$ , because  $D B = \frac{1}{4} A B$ . (Fig. 13)

11. The areas of similar figures are proportional to the squares on their homologous, or corresponding, sides. (Euc. VI. 19, 20.)

$A B C D E : F G H J K \text{ as } A B^2 : F G^2$ . (Fig. 14.)

12 The areas of circles are proportional to the squares on their diameters.



**PROBLEM 208.**—To construct a triangle equal in area to any parallelogram.

Set up twice the altitude of the given figure and join with the extremities of the base as shown in the first two figures, or double the base and keep the same altitude as in the third figure. The triangle  $ABC$  in each case is equal to the given parallelogram. The second figure shows the construction if an isosceles triangle be required.

**PROBLEM 209** —To construct a parallelogram equal in area to a triangle

Draw the altitude of the given triangle, and bisect it by the line  $DE$  parallel to  $AB$ . At  $A$  and  $B$  draw perpendiculars. Then  $ABDE$  is the rectangle equal to the given triangle.  $ABFG$  is a rhombus equal to  $ABC$ .

**PROBLEM 210.**—To construct a triangle equal in area to a given trapezium.

Let  $ABCD$  be the given trapezium. Draw the diagonal  $DB$ , and from  $C$  draw  $CE$  parallel to  $DB$  to meet  $AB$  produced in  $E$ . Draw  $DE$ . Then  $ADE$  is the required triangle.

Note — $DBE = DBC$  (Euc 1 37). Add  $ABD$  to each. Then  $ADE = ABCD$

**PROBLEM 211.**—To construct a triangle equal in area to an irregular pentagon.

Draw  $DA, DB$ . From  $E$  draw  $EF$  parallel to  $DA$  and meeting  $AB$  produced in  $F$ . Draw  $DF$ . From  $C$  draw  $CG$  parallel to  $DB$  and meeting  $AB$  produced in  $G$ . Draw  $DG$ . Then  $FDG$  is the required triangle.

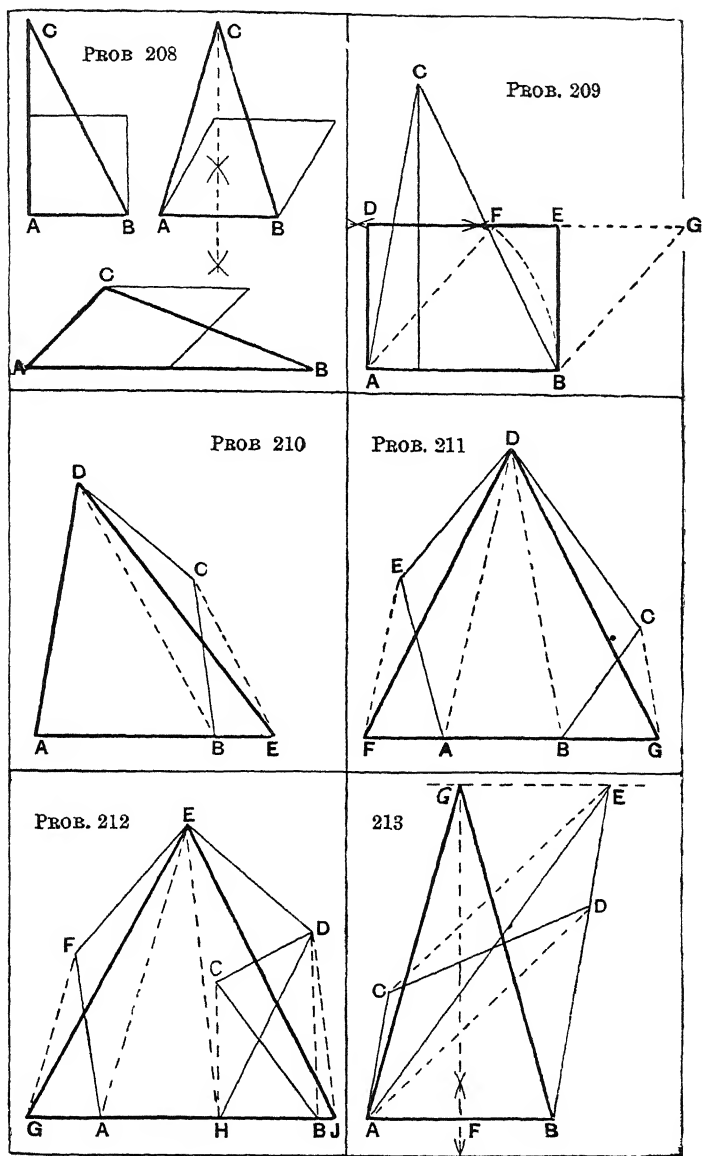
**PROBLEM 212.**—To construct a triangle equal to any irregular polygon.

Let  $ABCDEF$  be the given polygon. Draw  $EA$ . From  $F$  draw  $FG$  parallel to  $AE$ . Draw  $EG$  (The figure  $GBCDE = ABCDEF$ .) Draw  $DB$ , and from  $C$  draw  $CH$  parallel to  $DB$ . Join  $D$  and  $H$ . ( $GHDE = GBCDE$ .) Draw  $EH$ , and from  $D$  draw  $DJ$  parallel to  $EH$ . Join  $E$  and  $J$ . Then  $GEJ$  is the required triangle.

**PROBLEM 213.**—To construct an isosceles triangle equal to a trapezium, one side to be common to both figures.

Let  $ABCD$  be the given trapezium and  $AB$  the side common to both figures. Draw  $AD$ , and from  $C$  draw  $CE$  parallel to  $AD$  and meeting  $BD$  produced. Join  $A$  and  $E$ . Then the triangle  $ABE$  is equal to the trapezium. To get an isosceles triangle equal to it, bisect the base by the perpendicular  $FG$ , and through  $E$  draw  $EG$  parallel to  $AB$ . Draw  $GA$  and  $GB$ . Then  $ABG$  is the required triangle.

Note —In the case of a pentagon proceed in a similar manner, first converting the pentagon into a trapezium on  $AB$ .



**PROBLEM 214**—To construct a triangle equal in area to the sum of two given triangles.

Let  $ABC$  and  $DEF$  be the given triangles. Make the triangle  $CEF$  equal to the given triangle  $DEF$ , forming an irregular pentagon,  $ABEFC$ . Draw  $AF$ , and from  $C$  draw  $CH$  parallel to  $AF$ , and meeting  $AB$  produced. Join  $F$  and  $H$ . Draw  $FB$ , and from  $E$  draw  $EG$  parallel to  $FB$ . Join  $F$  and  $G$ . Then  $F'GH$  is the required triangle.

**PROBLEM 215**.—To construct a triangle equal to any regular polygon.

Let  $ABCDE$  be a regular pentagon. Find the centre, and divide the polygon into five equal triangles. Make the base  $GH$  equal to five times the base  $AB$ . Draw  $FG$ ,  $FH$ . Then  $F'GH$  is the required triangle.

Notes—1 Another method is to proceed as in Problem 211

2 Where the polygon has a larger number of sides, the altitude of the triangle may be doubled, and the base made equal to half the perimeter of the given polygon.

**PROBLEM 216**.—To construct a triangle equal in area to a circle. (Approximate.)

Draw  $AB$ , the diameter of the circle. Divide the radius  $AC$  into 7 equal parts. Draw  $AD$  perpendicular to  $AB$ , and make it  $3\frac{1}{7}$  times  $AC$ . Draw  $BD$ .  $ABD$  is the required triangle.

**PROBLEM 217**.—On a given base to draw a triangle equal in area to another given triangle.

Let  $ABC$  be the given triangle, and  $D$  the given base. On  $AB$  or  $AB$  produced set off  $AE$  equal to  $D$ . Draw  $CE$ , and from  $B$  draw  $BF$  parallel to  $CE$ . Join  $F$  and  $E$ . Then  $A'FE$  is the required triangle. ( $B'FC = FBE$ )

**PROBLEM 218**—To construct a triangle equal in area to a given triangle, having its vertex in a given point, and its base in the same straight line as that of the given triangle.

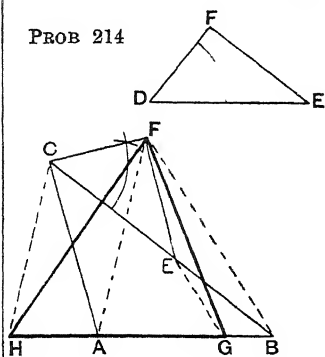
Let  $ABC$  be the given triangle, and  $D$  the given point. Draw  $AD$ . From  $C$  draw  $CE$  parallel to  $AB$ , and join  $B$  and  $E$ . Then the triangle  $ABE =$  the triangle  $ABC$ . The Problem now resolves itself into the same as the previous Problem—that is, to construct a triangle on  $AD$  equal to the triangle  $ABE$ . Draw  $DB$ , and from  $E$  draw  $EF$  parallel to  $DB$ . Join  $D$  and  $F$ . Then  $A'DF$  is the required triangle.

**PROBLEM 219**.—To construct a triangle of a given altitude equal in area to another given triangle.

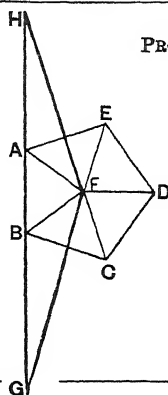
Let  $ABC$  be the given triangle, and  $D$  the given altitude. Draw  $CE$ , the altitude of the given triangle, and on it mark off  $EF$  equal to  $D$ . Draw  $FA$ , and from  $C$  draw  $CG$  parallel to  $FA$ . Join  $F$  and  $G$ . Draw  $FB$ , and from  $C$  draw  $CH$  parallel to  $FB$ . Join  $F$  and  $H$ . Then  $G'FH$  is the required triangle.

Note.— $A'FG = A'FC$ , and  $B'FH = B'FC$ .

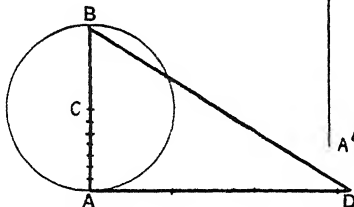
PROB 214



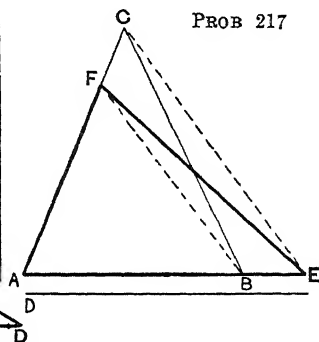
PROB 215



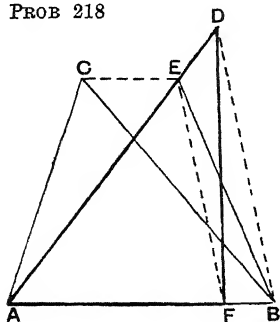
PROB 216



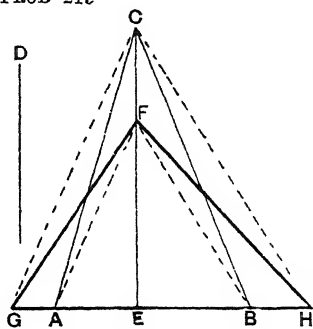
PROB 217



PROB 218



PROB 219





**PROBLEM 220**—To construct a square equal in area to the sum of three squares.

Let  $A$ ,  $B$ , and  $C$  be the lengths of the sides of the given squares. Draw  $EF$  equal to  $A$ , and  $ED$  at right angles to  $EF$ , and equal to  $B$ . Join  $F$  and  $D$ .

Then the square on  $DF$  is equal to the sum of the squares on  $DE$  and  $EF$ . (Euc I 47.) Draw  $DG$  at right angles to  $DF$ , and equal to  $C$ . Join  $F$  and  $G$ . On  $GF$  describe a square.

**PROBLEM 221**—To construct a square equal in area to the difference between two given squares.

Let  $A$  and  $B$  be the sides of the given squares. Draw two lines at right angles to each other. Make  $CD$  equal to  $B$ . With  $C$  as centre and  $A$  as radius mark off  $E$ . On  $DE$  construct the square.

Notes—1 If the square on  $CE$  = the sum of the squares on  $CD$  and  $DE$ , then the square on  $DE$  must be equal to the difference of the squares on  $CE$  and  $CD$ .

2 The same principles may be applied to the circle, or any rectilinear figure. An equilateral triangle, polygon, &c, may be constructed equal in area to the sum or the difference of two similar figures.

**PROBLEM 222.**—To describe a circle equal in area to the sum of two given circles.

Draw  $CD$  and  $DE$  perpendicular to each other, and equal to  $A$  and  $B$ , the diameters of the given circles. Draw  $CE$ , the diameter of the required circle.

Note—To describe a circle equal in area to the difference between two circles, proceed as in Problem 221.  $CD$  will be the smaller diameter,  $CE$  the larger, and  $DE$  the diameter of the required circle.

**PROBLEM 223.**—To construct a triangle similar to a given triangle, but having twice its area.

Let  $ABC$  be the given triangle. Draw  $AD$  perpendicular to  $AB$ , and equal to it. Make  $BE$  equal to  $BD$ . From  $E$  draw  $EF$  parallel to  $AC$ , and meeting  $BC$  produced. Then  $EBF$  is the required triangle.

Note—The same principle may be applied to any other rectilinear figure. A trapezium is shown similar to, and double the area of, a given trapezium.

**PROBLEM 224.**—To construct a trapezium similar to a given trapezium, having half its area.

Let  $ABCD$  be the given trapezium. Bisect  $AB$ , and describe a semicircle. Draw  $BH$ , and make  $BE$  equal to it. Draw  $BD$ , and from  $E$  draw  $EF$  parallel to  $AD$ , and from  $F$  draw  $FG$  parallel to  $DC$ . Then  $BEFG$  is the required figure.

Note—The same principle applies to other rectilinear figures. The triangle  $BEF$  is half the triangle  $BAD$ .

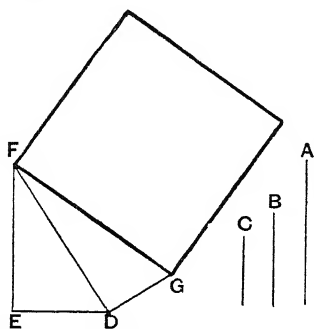
**PROBLEM 225.**—To construct a square having twice the area of a given square.

Draw the diagonal, and describe a square on it.

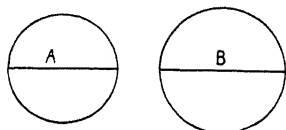
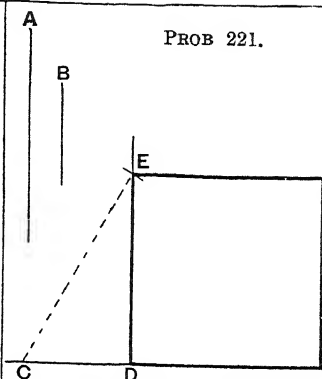
**PROBLEM 226.**—To construct a square having half the area of a given square.

On half the diagonal describe a square.

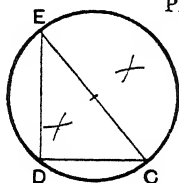
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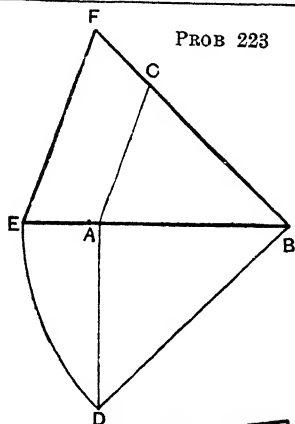
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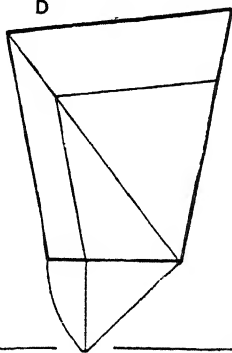
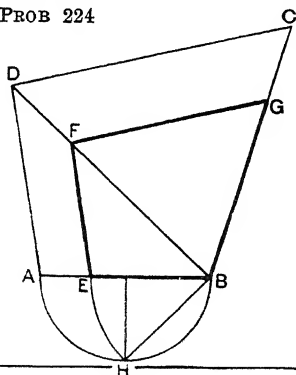
PROB 222



PROB 223



PROB 224



**PROBLEM 227** —To construct a square equal in area to any given parallelogram

Let  $ABCD$  be the given parallelogram. Produce  $AB$ , and make  $BF$  equal to the altitude of the parallelogram. Find  $BG$ , a mean proportional between  $AB$  and  $BF$  (*Prob. 29*). On  $BG$  describe the square.

**PROBLEM 228**.—To construct a square equal in area to a triangle.

Make a rectangle equal to the triangle. (*Prob. 209*) Find a mean proportional between the base and the height of the rectangle, and on it describe the square

**PROBLEM 229** —To construct a square equal in area to a trapezium.

Draw the diagonal. On one side of it make a rectangle equal to the triangle  $ADC$ , and on the other side a rectangle equal to the triangle  $ABC$ . Proceed as in the preceding problem.

**PROBLEM 230**.—To construct a square equal in area to any polygon.

First obtain a triangle equal in area to the polygon, as shown in Problems 211 and 212, then construct a rectangle equal in area to the triangle, and proceed as above.

**PROBLEM 231** —To construct a rectangle of a given perimeter, and equal in area to a given square.

Let the perimeter equal  $4\frac{1}{2}$  inches, and the side of the given square 1 inch. Divide half the perimeter into 2 parts, whose mean proportional shall equal 1 in., as follows —Draw  $AB$ ,  $2\frac{1}{4}$  inches long, and at  $B$  draw  $BC$  perpendicular to  $AB$ , and 1 inch long. Describe a semicircle on  $AB$ . Through  $C$  draw  $CD$  parallel to  $AB$ , and from  $D$  draw  $DE$  parallel to  $CB$ . Then  $AE$  and  $EB$  will be two of the sides of the rectangle. Complete the figure as shown.

**PROBLEM 232**.—To construct a rectangle equal in area to a square, and having its sides in a given ratio.

Let the ratio of the sides be as 2 : 3. Produce the base of the given square. From  $A$  set off to any convenient unit  $AB$  and  $AC$ , equal to 2 and 3 units respectively. Find  $AD$ , the side of a square equal to the rectangle contained by  $AB$  and  $AC$ . From  $E$  draw  $EF$  and  $EG$  parallel to  $DC$  and  $DB$ .  $AF$  and  $AG$  will be the sides of the rectangle. Complete the figure as shown

**PROBLEM 233** —To construct a rectangle equal in area to a square, having the difference between two adjacent sides given.

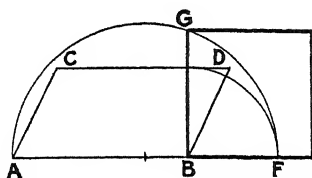
Let  $AB$  be the difference between the two adjacent sides. Bisect  $AB$  in  $C$ . With centre  $C$  and radius  $CD$  describe a semicircle. Then  $AF$  and  $AE$  will be the sides of the rectangle.

**PROBLEM 234**.—On a given base to construct a rectangle equal in area to a given rectangle

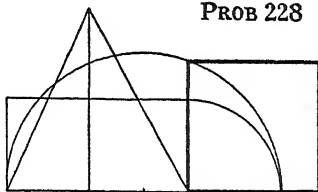
Let  $ABCD$  be the given rectangle, and  $E$  the given base. Produce  $AB$ , and make  $AF$  equal to  $E$ . Join  $C$  and  $F$ , and from  $B$  draw  $BG$  parallel to  $CF$ .  $AG$  is the other side of the rectangle.

Note — $AC \times AG$  as  $AF \times AB$  (*Euc. VI. 2*) But when quantities are in proportion the product of the *extremes* equals the product of the *means* (*p. 18*); therefore  $AC \times AB = AG \times AF$ .

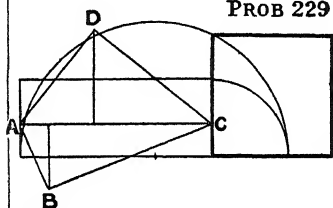
PROB 227



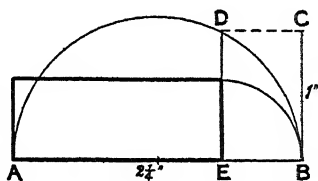
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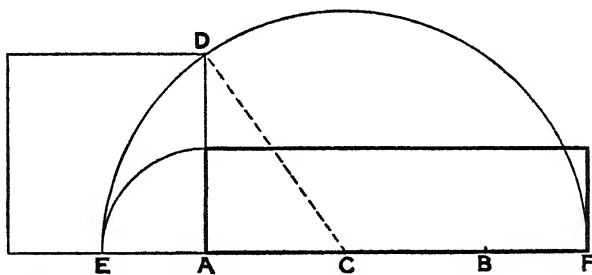
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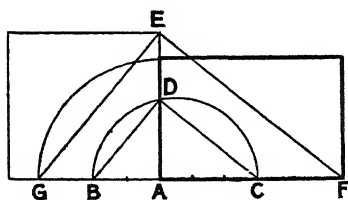
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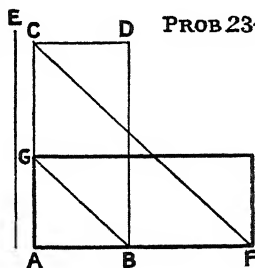
PROB 233



PROB 232.



PROB 234



**PROBLEM 235.**—To divide a triangle into any number of equal parts by lines drawn from one of its angles.

Divide  $AB$  into the required number of equal parts (say 3). Draw  $C1$  and  $C2$ . The three triangles thus formed are equal to each other. (Euc I. 38)

**PROBLEM 236** —To divide a parallelogram into any number of equal parts by lines drawn from one of its angles.

Divide two adjacent sides into the same number of equal parts as the figure has to be divided into (say 3). Join  $D1$ ,  $D1$ .

**PROBLEM 237** —To bisect a triangle by a line drawn from a point in one of the sides

Let  $D$  be the given point. Bisect  $AB$  in  $E$ , and draw  $CE$ . Join  $D$  and  $E$ , and from  $C$  draw  $CF$  parallel to  $DE$ . Join  $D$  and  $F$ . Then  $DF$  bisects the triangle.

**PROBLEM 238** —To bisect a parallelogram by a line drawn from a point in one of the sides.

Let  $E$  be the given point. Find the centre, and draw  $EF$  through it. Then  $EF$  bisects the parallelogram.

Note —This construction will apply for any position of the point

**PROBLEM 239.**—To bisect a trapezium by a line drawn from one of its angles.

Let  $ABCD$  be the given trapezium. Draw the diagonals. Bisect  $AC$  in  $E$ . Draw  $DE$ ,  $EB$ , dividing the trapezium into two equal areas. Through  $E$  draw  $FG$  parallel to  $BD$ . Join  $D$  and  $F$ . Then  $DF$  bisects the trapezium.

Note.—The triangle  $DBF$  = the triangle  $DBE$ .

**PROBLEM 240.**—To divide a triangle into any number of equal parts by lines drawn from a point in one of the sides.

Let  $D$  be the given point. Divide the side  $CB$ , in which the given point is situated, into as many equal parts as the triangle has to be divided into (say 3). Draw  $AD$ . From 1 and 2 draw  $1E$ ,  $2F$ , parallel to  $AD$ . Draw  $DE$ ,  $DF$ , dividing the triangle as required.

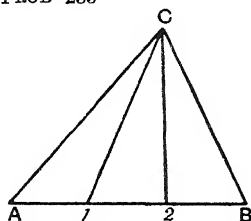
Note —The triangle  $DE1$  = the triangle  $AE1$

**PROBLEM 241.**—To divide a parallelogram into any number of equal parts by a line drawn from a point in one of the sides.

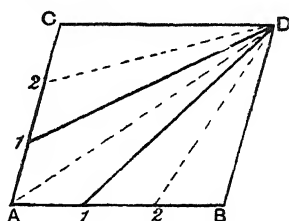
Let  $E$  be the given point. Divide  $AB$  into the required number of equal parts (say 4) in 1, 2, 3. Draw  $11$ ,  $22$  parallel to  $AD$  and bisect them in  $F$  and  $G$ . Through  $F$  and  $G$  draw  $EH$  and  $EJ$ . Bisect the trapezium  $EJCB$  by the line  $EK$ . (Prob. 239.) Then the lines  $EH$ ,  $EJ$ ,  $EK$  divide the parallelogram into 4 equal parts.

Note —If the point  $E$  were placed so that  $EK$  would fall upon the side  $DC$ , then a line  $33$  might have been bisected, and  $EK$  drawn through the points of bisection, as in 11 and 22.

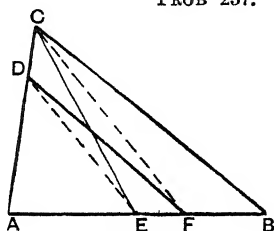
PROB 235



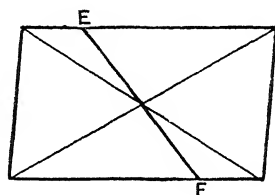
PROB 236



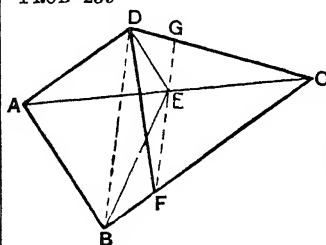
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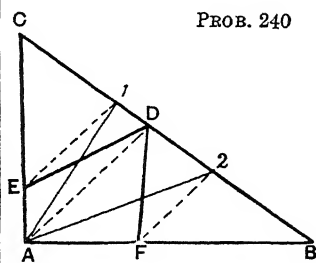
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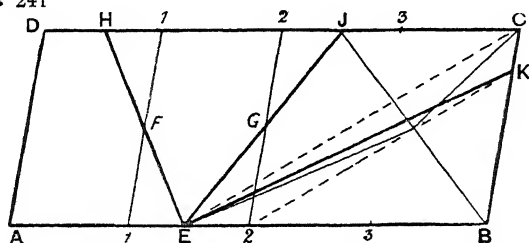
PROB 239



PROB. 240



PROB 241



**PROBLEM 242.**—To divide an irregular polygon into any number of equal parts by lines drawn from one of the angles

Let  $A B C D E$  be an irregular polygon, it is required to divide it into 3 equal parts by lines drawn from the angle at  $D$

Construct the triangle  $D F G$  equal to the polygon (*Prob 211*)

Divide the base,  $F G$ , into 3 equal parts Draw  $D 1$  It is evident that the triangle  $D 1 F$  is  $\frac{1}{3}$ rd of the triangle  $D F G$ , and consequently  $\frac{1}{3}$ rd of the given polygon. As the point 2 does not fall upon the base  $A B$  of the polygon, draw  $2 d$  parallel to the diagonal  $D B$ . Draw  $D d$  The lines  $D 1$  and  $D d$  divide the polygon as required.

Note.—The triangle  $D B 2$  = the triangle  $D B d$  (*Euc 1 37*)

**PROBLEM 243.**—To divide a triangle into any number of equal parts by lines drawn from a point within the triangle.

Let  $D$  be the given point. Divide the base  $A B$  into as many equal parts as required (say 3). Draw  $D 1$ ,  $D 2$ , and  $D C$ . From  $C$  draw  $C E$  parallel to  $D 1$ , and  $C F$  parallel to  $D 2$ . Draw  $D E$  and  $D F$ . The lines  $D C$ ,  $D E$ , and  $D F$  divide the triangle into 3 equal parts.

Notes.—1 The triangle  $E 1 C$  = the triangle  $E D C$  If  $A E C$  be added to each, then  $A 1 C$  =  $A E D C$  But  $A 1 C$  is  $\frac{1}{3}$ rd of the triangle  $A B C$  Therefore  $A E D C$  will equal  $\frac{1}{3}$ rd of the triangle  $A B C$ .

2 If the line  $D F$  does not fall upon  $A B$ , then proceed as shown in **Problem 243 a.** Obtain  $D E$  as in the preceding problem Draw  $2 F$  parallel to  $B C$ . Join  $D$  and  $B$ , and from  $F$  draw  $F G$  parallel to  $D B$ . Draw  $D G$ . Then  $D C$ ,  $D E$ , and  $D G$  divide the triangle into 3 equal parts.

**Proof.**—The triangle  $B F C$  = the triangle  $B 2 C$  (*Euc 1 37*) Triangle  $F G D$  = triangle  $F G B$  Add to each the triangle  $C F G$  Then  $C F G + F G D$  =  $C F G + F G B$ —that is,  $C D G$  =  $C F B$  But  $C F B$  =  $C 2 B$  Therefore,  $C D G$  =  $C 2 B$

**PROBLEM 244.**—To bisect a triangle by a line drawn parallel to one side.

Bisect  $C B$  by the perpendicular  $E F$ , and describe a semicircle. With centre  $C$  and radius  $C F$  describe the arc  $F G$  From  $G$  draw the line  $G H$  parallel to  $A B$ . This line bisects the triangle.

Note.— $C F$ , which equals  $C G$ , is a mean proportional between the side  $C B$  and its half,  $C E$  (*Euc vi 8, Cor*) The triangle  $C G H$  triangle  $C B A$  as  $C G^2$   $C B^2$ —that is, as  $C E$   $C B$ , or as 1 2

**PROBLEM 245.**—To bisect a triangle by a line perpendicular to the base.

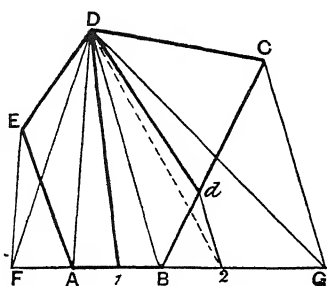
Draw  $A D$  perpendicular to  $B C$ , and bisect  $B C$  in  $E$ . Find a mean proportional,  $C F$ , between the *larger* segment of the base  $C D$ , and the half  $C E$ . Make  $C G$  equal to  $C F$ , and draw  $G H$  perpendicular to the base. Then  $G H$  bisects the triangle.

**PROBLEM 246.**—To divide a triangle into any number of equal parts by lines drawn parallel to one of the sides.

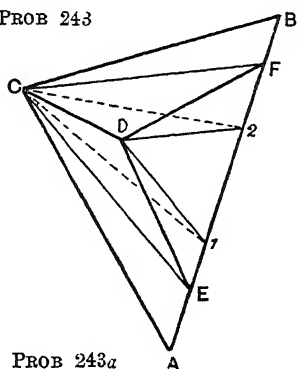
Let the triangle  $A B C$  be divided into 3 equal parts. Divide  $B C$  or  $B A$  into 3 equal parts. Describe a semicircle on  $B C$ , and erect perpendiculars at 1 and 2. Make  $B d$  equal to  $B D$ , and  $B e$  equal to  $B E$  Parallels drawn from  $d$  and  $e$  will divide the triangle as required.

Note.— $B D$  is a mean proportional between  $B C$  and its third,  $B 1$ , and  $B E$  is a mean proportional between  $B C$  and two-thirds,  $B 2$   $B d F B C A$  as  $B d^2$   $B C^2$ —that is, as  $B 1$   $B C$ , or as 1 3.

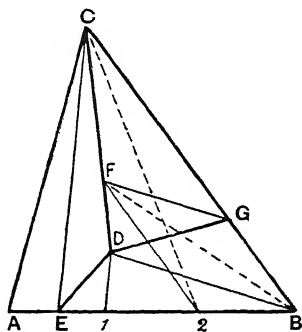
PROB 242



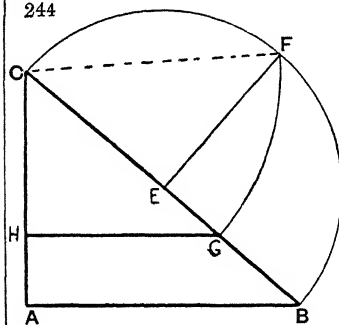
PROB 243



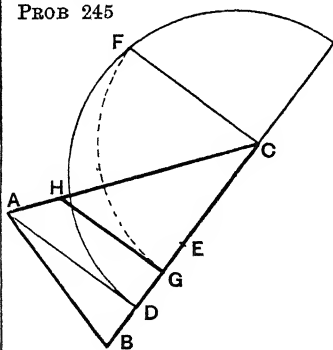
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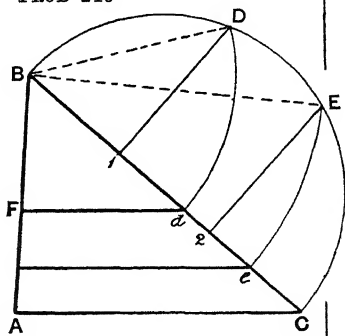
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PROB 245



PROB 246





**PROBLEM 247** —To divide a parallelogram into any number of equal parts by lines parallel to the diagonal.

Let it be required to divide the parallelogram  $ABCD$  into 5 equal parts by lines drawn parallel to the diagonal  $DB$ . Divide  $DC$  into 5 equal parts. Describe a semicircle on  $DC$ , erect perpendiculars at the alternate parts, 2 and 4, and proceed as in Problem 246.

Note —Each of the triangles  $ABD$  and  $DBC$  is divided into 5 equal parts. By drawing the alternate lines only, the whole parallelogram is divided into 5 equal parts.

**PROBLEM 248.**—To divide a circle into any number of equal parts (say 3) by concentric circles.

Divide the radius  $AO$  into 3 equal parts. On it describe a semicircle, and erect perpendiculars at 1 and 2.  $OB$  and  $OC$  will be the radii of the required circles.

Note — $OB$  is the mean proportional between the radius  $OA$  and its third part.

**PROBLEM 249.**—To divide a circle into any number of parts (say 3) equal in area and perimeter.

Divide the diameter of the circle into twice as many parts as there are equal areas required. With centres 1, 2, 4, and 5 describe semicircles.

**PROBLEM 250** —To divide a triangle into 2 parts, having a given ratio to each other, by a straight line drawn through a given point in one of its sides.

Let  $D$  be the given point, and the given ratio as 3 : 2.

Divide  $CB$  into 3 + 2 equal parts. Join  $A2$ , which divides the triangle into 2 parts in the ratio of 3 to 2. Draw  $DA$ , and from 2 draw  $2E$  parallel to  $DA$ . Join  $DE$ . Then  $DE$  divides the triangle as required.

Note —The triangle  $E2A$  = the triangle  $E2D$ . Therefore,  $E2D = A2D$ .

**PROBLEM 251.**—To divide a parallelogram into 2 parts, having a given ratio to each other, by a straight line drawn from a given point in one of the sides.

Let  $E$  be the given point, and the given ratio as 3 : 1.

Divide  $AB$  into 4 equal parts. From 1 draw a line,  $1F$ , parallel to  $AD$ , cutting the parallelogram into 2 parts having the required ratio. Bisect  $1F$ , and from  $E$  draw a line through the point of bisection. This line divides the triangle as required.

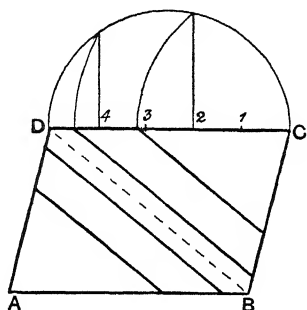
**PROBLEM 252.**—To construct a square, the area being given.

Let the required area be  $2\frac{1}{2}$  square inches. Construct a rectangle, sides 1 inch and  $2\frac{1}{2}$  inches. This rectangle will contain  $2\frac{1}{2}$  square inches. Find a mean proportional,  $BE$ , to the two sides. On  $BE$  construct the square.

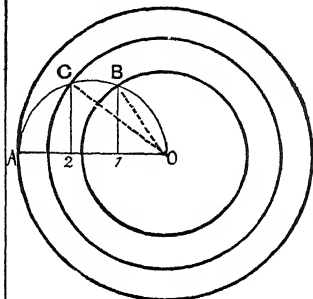
Notes —1 It is not necessary to construct the rectangle, but only to find the mean proportional between the dimensions of the two sides which would contain a rectangle of the given area.

2 The figure is drawn to a smaller scale.

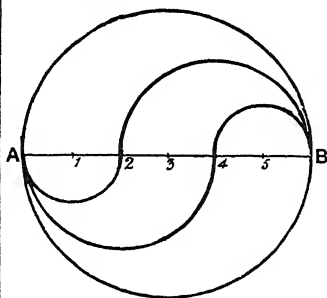
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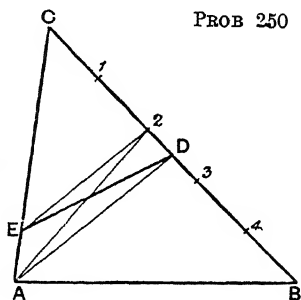
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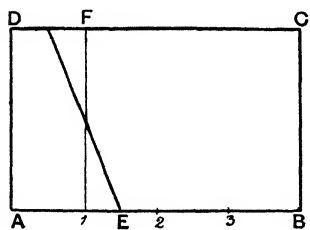
PROB 249



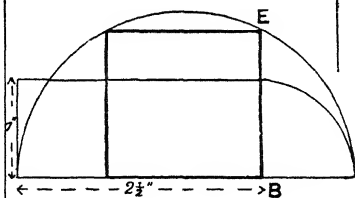
PROB 250



PROB 251.



PROB 252

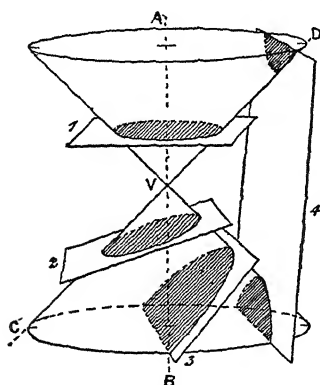


## CHAPTER XIV

## PLANE CURVES

SOME of the curves used in geometrical and mechanical drawings, such as the ellipse, parabola, and hyperbola, cycloids, the involute, various spirals, &c, cannot be described by the ordinary compasses. These curves are obtained by finding a number of points in the required line, and then tracing the curve through these points by hand, or with the help of French curves.

The ellipse, parabola, and hyperbola are known as conic sections, because they are formed when a right circular cone is intersected



by a plane in various positions. In dealing with conic sections it is preferable to consider the cone to be generated in the following manner. Let  $AB$  and  $CD$  be two straight lines intersecting at  $V$ . If  $AB$  remain fixed and  $CD$  revolve around it, always making a constant angle with  $AB$ , then the surface of a right circular cone will be generated.

$AB$  is the axis,  $CD$  the generator, and  $V$  the vertex of the cone. It will be seen that the cone thus formed consists of two symmetrical portions, one on each side of the vertex.

The various positions of the cutting plane to form the conic sections are shown on the figure.

1 Here the plane is perpendicular to the axis, and the curve formed is the circle.

2. The plane is inclined, but still passes through opposite sides of the cone, and the curve formed is the ellipse.

3 The plane is further inclined until it makes an angle with the axis,  $AB$ , equal to that made by the generator. The curve formed is the parabola.

4 The plane is still further inclined so that the angle made by it with the axis is less than the angle made by the generator. The curve formed is the hyperbola. It will be noticed that the plane now cuts both portions of the cone, and the hyperbola consequently has two branches.

**Note.**—As will be seen from the above, the circle and ellipse are closed curves, while the parabola and hyperbola are open and unlimited.

In the construction of these curves it is better to regard them as being traced by a point moving on a plane surface according to some fixed law. In the circle, for instance, the moving point always keeps the same distance from a fixed point, the centre of the circle.

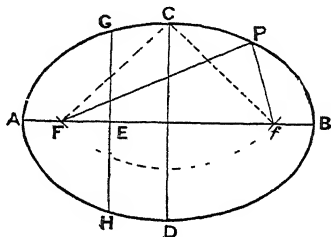
## THE ELLIPSE

This curve is traced by a point moving so that the distance from a fixed point or focus is less than its distance from a fixed line called the directrix

An ellipse has two foci (*Prob 253*,  $F$  and  $f$ ) and two directrices. The line passing through the two foci is called the **transverse** or **major axis** ( $AB$ , *Prob 253*). The line at right angles to the major axis, and bisecting it, is called the **conjugate** or **minor axis** ( $CD$ , *Prob 253*). Any line passing through the centre and terminated by the curve is a **diameter**. Any straight line perpendicular to the major axis, as  $EG$  is an **ordinate**,  $HG$  is called a **double ordinate**. If any point in the curve be joined to the foci by two lines, these two lines are, together, equal to the major axis, for example  $FC + Cf$  and  $FP + Pf$  are each of them equal to  $AB$ . A **tangent** is a line touching the curve in one point. A **normal** is a perpendicular to a tangent at the point of contact.

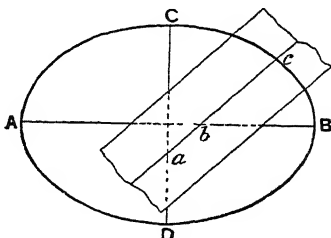
**PROBLEM 253**—To describe an ellipse, the major and minor axes being given. **First Method** (by means of a piece of thread).

Draw the axes  $AB$  and  $CD$  perpendicular to each other. From  $C$  with radius  $= \frac{1}{2} AB$  find the foci  $F$  and  $f$ . At the points  $F, f$ , and  $C$  stick three pins firmly and tie a piece of thread round them, as shown by the lines  $CF$ ,  $Ff$ , and  $fC$ . Remove the pin at  $C$ , and replace it with a well-pointed pencil. If the point of the pencil be now moved round carefully, keeping the thread tightly stretched, the curve traced will be an ellipse, because  $PF + Pf = AB$ , wherever  $P$  may be.



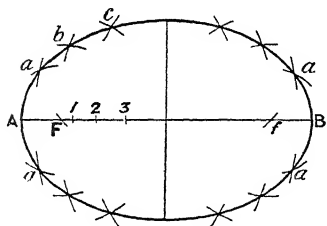
**PROBLEM 254**—The same. **Second Method** (by using a trammel)

Set out the axes as before. On a slip of tracing paper rule a line, and mark off  $ca = \frac{1}{2} AB$ , and  $cb = \frac{1}{2} CD$ . If the paper be now placed so that  $a$  is on the minor and  $b$  on the major axis, then  $c$  will be a point on the curve. Prick through  $c$ . Move the trammel, still keeping  $a$  and  $b$  on the minor and major axes respectively, when  $c$  will give other points on the curve. Through these points draw the ellipse.



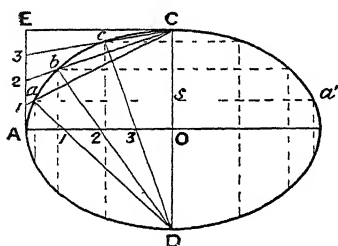
**PROBLEM 255.—The same. Third Method (by intersecting arcs)**

Set out the axes as before and obtain the foci. From  $F$  mark any number of points as 1, 2, 3, making the parts smaller as they near  $F$ . From each focus with radius  $B1$  describe arcs at  $a$ , and with radius  $A1$  (remainder of  $AB$ ) from each focus intersect these arcs, obtaining four points on the curve. Repeat the process with radii  $B2$  and  $A2$ , giving four more points as at  $b$ . Continue with radii  $B3$  and  $A3$  for the points  $c$ . Through the points draw the curve. Here again  $F'a + f'a = AB$

**PROBLEM 256.—The same. Fourth Method (by intersecting lines)**

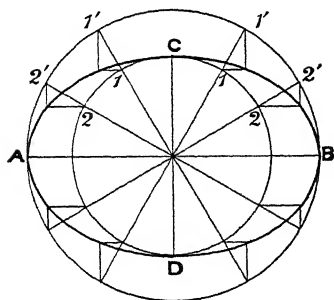
Draw the axes and construct the rectangle  $AOC'E$ . Divide  $AE$  and  $AO$  into the same number of equal parts (say four). Draw  $C1$ ,  $C2$ ,  $C3$ . From  $D$  draw  $D1$  to meet  $C1$ ,  $D2$  to meet  $C2$ ,  $D3$  to meet  $C3$ . Through the points thus obtained draw a quarter of the ellipse. The figure may be completed either by repeating the same process for the other quarters, or by

symmetry, thus — Draw parallels through  $a$ ,  $b$ ,  $c$ , and set off  $s'a' = sa$ , etc. Repeat the process for the lower half as shown

**PROBLEM 257.—The same. Fifth Method.**

Place the axes as before, and describe a circle on each. Divide one of the quadrants into any number of parts, and obtain corresponding points on the others by producing the radii. From the points 1 and 2 on the smaller circle draw parallels to  $AB$ , and from the points  $1'$  and  $2'$  on the larger circle draw parallels to  $CD$  to meet the lines drawn parallel to  $AB$ . Through the intersections of these parallels draw the curve

**Note.**—All the problems on curves should be drawn to a much larger scale than shown.



**PROBLEM 258** —To construct an ellipse, the two conjugate diameters  $A B$  and  $C D$  being given

Draw parallels to the diameters and obtain the parallelogram.

Divide  $A E$  and  $A F$  into the same number of equal parts (four), and draw lines from  $C$  and  $D$  to each point

Divide  $A O$  also into four equal parts From  $D$  draw

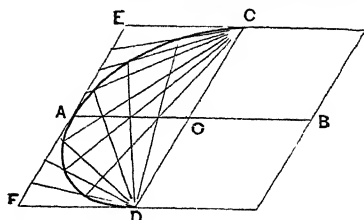
lines through each point on  $A O$  to meet lines from  $C$ ,

and from  $C$  draw lines through the same points to

meet lines from  $D$  Draw the curve. The other half may be obtained in a similar manner,

or by symmetry

**Note.**—By this method an ellipse may be inscribed in any parallelogram, or about a triangle If  $C D$ ,  $D A$ , and  $A C$  be joined, a triangle is formed



**PROBLEM 259** —The curve of an ellipse being given, to find its axes

Draw any two parallel chords

$A B$  and  $C D$  Bisect each

chord, and through the points

of bisection draw  $E F$  This

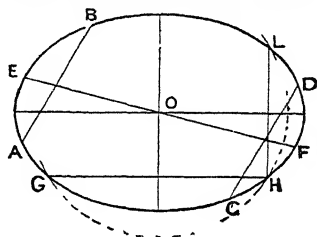
line will be a diameter Bisect

$E F$  in  $O$  From centre  $O$  describe a circle cutting the ellipse

in  $G$ ,  $H$ , and  $L$  Parallels

through  $O$  to the lines  $G H$  and

$H L$  will give the axes.



**PROBLEM 260** —A portion of the curve of an ellipse being given, to complete it.

Let  $C D X$  be the given portion.

Draw two chords,  $A B$

and  $C D$ , and bisect them

in  $E$  and  $F$ . Through  $E$

and  $F$  draw a line If

this line be terminated by

the curve of the ellipse at

each end, obtain the axes

as in Problem 259. Find

the foci, and complete the

ellipse by obtaining points

in the curve as already shown

If the line be not terminated, then

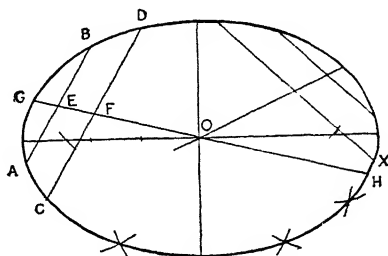
draw two other parallel chords, and bisect them as in the first pair

The line drawn through the points of bisection will intersect the

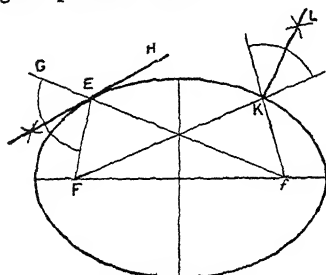
line drawn through  $E$  and  $F$  in  $O$ , the centre of the ellipse Make

$O H$  equal to  $O G$ , and obtain the axes and points in the curve as

before.



**PROBLEM 261.**—To draw a tangent and a normal to an ellipse from given points in the curve.

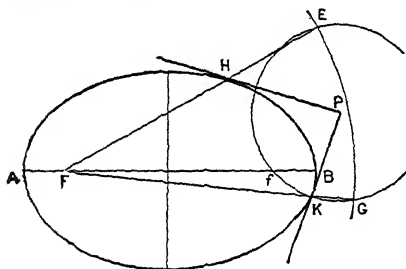


normal or perpendicular

1 The tangent. Find the axes and foci (*Prob 259*) Join the foci with *E*, the given point, and produce *fE*. Bisect the angle *FEG*. The line *EH* is the required tangent.

2 For the normal at *E* draw a perpendicular to the tangent. If another point, *K*, be given, then join the foci with *K* and produce both lines. Bisect the angle thus formed. *KL* is the required

**PROBLEM 262.**—To draw a tangent to an ellipse from a point without the curve.



will be tangents to the ellipse.

Let *P* be the given point. Find the axes and foci of the ellipse (*Prob 259*). With centre *P* and radius *Pf* describe a circle. With centre *F* and radius equal to the major axis *AB* intersect the first circle in *E* and *G*. Draw *FE* and *FG* cutting the ellipse in *H* and *K*. Lines drawn from *P* through *H* and *K*

## THE PARABOLA

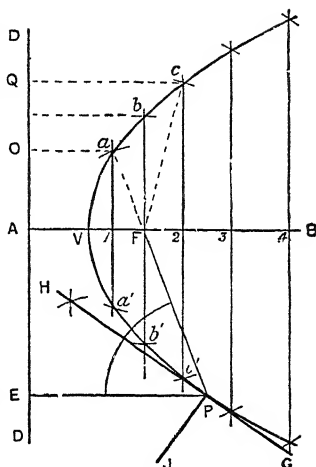
This curve is traced when a point moves so that its distance from the focus (*F*, *Prob. 263*) always equals its distance from the fixed line called the directrix (*DD*, *Prob. 263*). Thus  $Fa = aO$ ,  $Fc = cQ$ , &c.

The path of a projectile forms a parabolic curve. The curve is also largely used in graphic methods for determining the stress upon beams, girders, &c.

The axis, *AB*, is a line drawn through the focus, perpendicular to the directrix, and consequently divides the curve into two symmetrical parts. The point where the curve meets the axis is called the vertex (*V*, *Prob 263*). A perpendicular from any point on the curve to the axis is an ordinate, as *a1*, *c2*, &c. *aa'*, *bb'*, &c, are double ordinates. The double ordinate *bb'*, through the focus, is called the latus rectum. The part of the axis between the ordinate of a point and the vertex of the curve is called the abscissa; thus *2V* is the abscissa of the point *c*.

**PROBLEM 263**—To draw a parabola, the focus and directrix being given

Let  $F$  be the focus, and  $DD$  the directrix. Through  $F$  draw the axis  $AB$  perpendicular to  $DD$ . Bisect  $FA$  in  $V$ . Then  $V$  is a point on the curve, because it is equally distant from the focus and directrix. Take any points, 1, 2, 3, 4, &c., and through them draw perpendiculars to the axis (ordinates). From centre  $F$  with radius  $AV$  cut the ordinate through 1 in  $a, a'$ . With radius  $AF$  and centre  $F$  cut the ordinate in  $b, b'$ . With radius  $A2$  and centre  $F$  obtain the points  $c, c'$ . Any number of points may be obtained similarly. Through the points draw the curve.



**PROBLEM 264**—To draw a tangent and a normal from a point in the curve.

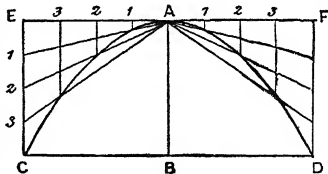
In Problem 263 let  $P$  be the point. Join  $P$  with the focus  $F$ . Draw  $PE$  parallel to  $AB$ . Bisect the angle  $EPF$ . Then  $GH$  is the required tangent.

For the normal draw  $PJ$  perpendicular to the tangent.

Note—In a parabolic arch, the joints of the stones are obtained by drawing normals to the curve.

**PROBLEM 265**—To draw a parabola, when the axis and an ordinate are given

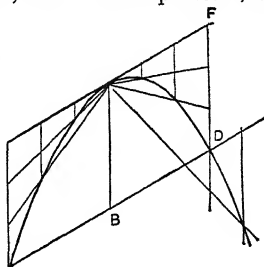
Let  $AB$  be the axis and  $BC$  the ordinate. Make  $BD = BC$ . The problem is now to draw a parabola through the points  $C, A$ , and  $D$ . Construct the rectangle  $CEFD$ . Divide  $AE$  and  $EC$  each into the same number of equal parts (say 4). Set off these parts on  $AF$  and  $FD$ . Draw 1A, 2A, 3A. From points 1, 2, 3 in line  $EF$  draw parallels to  $AB$ , meeting the other lines as shown. Through the points of intersection draw the curve.



**PROBLEM 266**—To inscribe a parabola in any parallelogram.

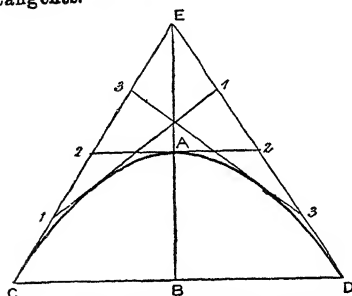
Proceed as in the previous problem. The figure shows a parabola inscribed in a rhomboid.

Note—The curve may be extended by continuing the divisions on lines  $BD$  and  $FD$ .



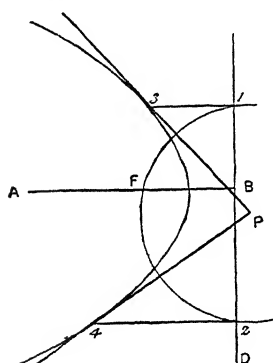


**PROBLEM 267.**—To draw a parabola by means of intersecting tangents.



Given the height,  $AB$ , and a double ordinate,  $CD$ . Produce  $BA$  and make  $AE = AB$ . Join  $E$  with  $C$  and  $D$ . Divide  $EC$  and  $ED$  into any number of equal parts (say 4 or 6). Draw  $11, 22, 33$ . These lines are tangents to the parabola, which touches them midway between the points of intersection.

**PROBLEM 268.**—To draw a pair of tangents to a parabola from a point outside the curve, the focus and directrix being given



Let  $P$  be the given point. Draw the axis  $AB$ . With centre  $P$  and radius  $PF$  describe a circle cutting the directrix  $DD$  in 1 and 2. From these points draw  $13$  and  $24$  parallel to  $AB$ . From  $P$  through 3 and 4 draw the tangents.

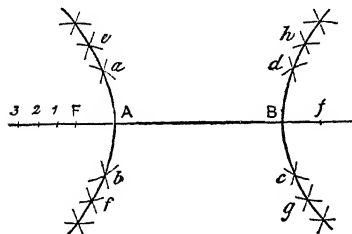
### THE HYPERBOLA

This curve is traced when a point moves so that its distance from the focus is always greater, in a constant ratio, than its distance from the directrix.

The variation in pressure and volume of steam, when it expands in the cylinder of an engine, and the relative changes in pressure and volume of a gas when the temperature remains constant, may be graphically represented by this curve.

**PROBLEM 269**—To describe the hyperbola, the major axis and the foci being given.

The hyperbola, like the ellipse, has two foci. The major axis is the distance between the two branches of the curve.



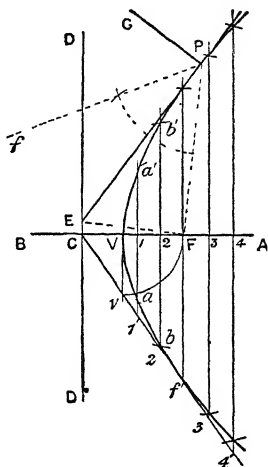
Let  $AB$  be the major axis, and  $F, f$  the foci. In  $AB$  produced and beyond  $F$  take any number of points, 1, 2, 3, &c. From the foci  $F, f$ , with radius  $A1$ , describe four arcs at  $a, b, c$ , and  $d$ , and from the same centres, with radius  $B1$ , intersect these arcs. With radius  $A2$  and the same centres describe arcs at  $e, f, g$ , and  $h$ , and intersect them with

radius  $B2$ . Proceed similarly with radii  $A3$  and  $B3$ .

Note—Compare this construction with that of the ellipse in Prob. 255.

**PROBLEM 270**—To describe the curve of a hyperbola, the focus, directrix, and vertex being given. Also at a point in the curve to draw a tangent and a normal.

Let  $F$  be the focus,  $DD$  the directrix, and  $V$  the vertex. Join  $FV$ , and produce both ways. Draw  $Vv$  perpendicular to  $AB$  and equal to  $VF$ . Draw  $Cv$  and produce. Take any points, 1, 2,  $F$ , 3, 4, &c, on  $AB$ , and through them draw perpendiculars on each side, meeting  $Cv$  in  $1', 2', f', 3', 4',$  &c. From centre  $F$  with radius  $11'$  cut the double ordinate through 1 in  $a, a'$ . With radius  $22'$  and centre  $F$  obtain points  $b, b'$ . Proceed in a similar manner for the other points. Through the points thus obtained draw the curve.



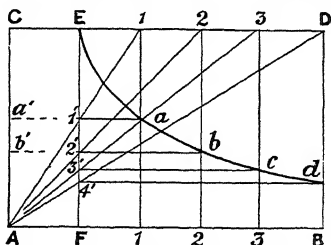
For the tangent, join  $F$  with the given point  $P$ . Draw  $FE$  perpendicular to  $PF$ , meeting the directrix in  $E$ . Then  $EP$  will be the required tangent.

For the normal, draw  $PG$  perpendicular to the tangent.

For the other branch of the curve make the angle  $EPf = EPF$ . To obtain the other focus draw  $Pf$  until it meets  $AB$  produced. Set off distances equal to  $FV$  and  $FC$ , and proceed as for the first branch.

**PROBLEM 271**—To draw a rectangular hyperbola, the axes  $AB$  and  $AC$  and the vertex of the curve,  $E$ , being given.

Complete the rectangle  $ABDC$ . In  $ED$  take any number of points, as 1, 2, 3. From  $E$ , 1, 2, 3 draw perpendiculars to  $AB$ . Draw  $A1$ ,  $A2$ ,  $A3$ , and  $AD$ , cutting  $EF$  in  $1', 2', 3', 4'$ . From each of these points draw parallels to  $AB$  to meet  $11$ ,  $22$ ,  $33$ , and  $DB$ , giving  $a, b, c, d$  points on the curve. Through these points draw the curve.



**Note.**—The rectangular hyperbola is a very useful curve, and is of importance in connection with indicator diagrams. The products of the perpendicular distances from any points on the curve to the axes are equal. Thus  $CE \times EF = a'a \times a1 = b'b \times b2$ , &c.

## CYCLOIDAL CURVES

When a circle rolls along a straight line, and always remains in the same plane, a point on the circumference describes the curve known as the **cycloid**.

If the circle rolls along the outside of another circle, both circles keeping in the same plane, the curve traced by a point is the **epicycloid**. If the circle rolls along the inside of a circle, both circles keeping in the same plane, the curve traced by a point is the **hypocycloid**. The moving circle is the **generating circle**, the line upon which it rolls is the **director** or **base**, and the point tracing the curve is the **generator**.

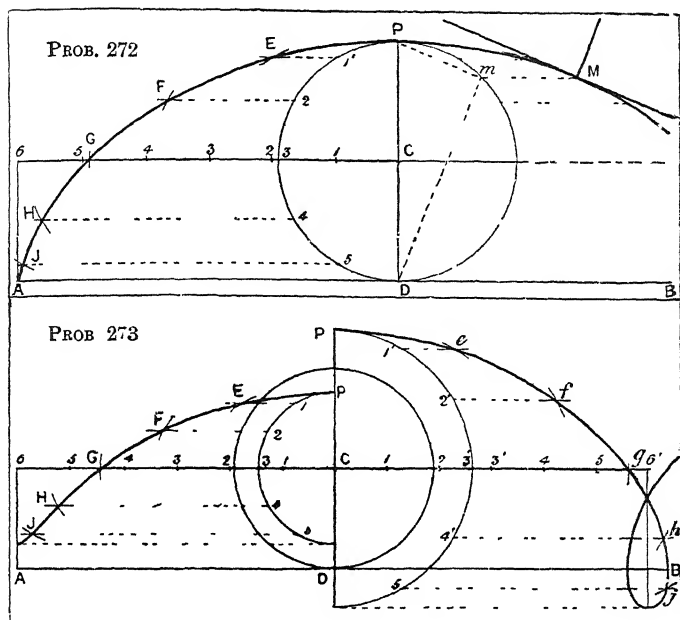
**PROBLEM 272**—To draw a cycloid, the generating circle and the director being given. Also to draw a tangent and a normal from a point in the curve

From **D**, the point where the generating circle touches the director **AB**, draw the diameter **DP**. Through **C**, the centre of the circle, draw a line, **C6**, parallel to **AB**, this line will be the path of the centre of the circle while rolling. Make **DA** = half the circumference of the circle by setting off  $3\frac{1}{2}$  times the radius **CD**. While the circle rolls from **D** to **A**, point **P** will trace half the cycloid. From **A** draw a perpendicular **A6**. Divide both the semi-circumference and **C6** into the same number of equal parts (say 6). Through 1', 2', 4', 5' draw parallels to **AB**. From centre 1, with radius **CP**, cut the parallel from 1' in **E**, and from centres 2, 3, 4, &c., with the same radius, cut the parallels from 2', 3', &c., in points **F**, **G**, **H**, and **J**. Through these points draw the semi-cycloid. It is evident that, as the centre of the circle has moved from **C** to 1, 2, &c., the point **P** will have fallen to the level of the parallels through 1', 2', &c. To obtain the other half, produce the parallels 1' **E**, &c., and set off equal distances to the right of **DP**.

To draw a tangent at **M**. Draw **Mm** parallel to **AB**. Join **m** with **P** and **D**. The tangent will be parallel to **Pm**.

The normal will be parallel to **Dm**.

If the generator be not in the circumference of the circle, the curve is called a **trochoid**, and is obtained in a similar manner.



**PROBLEM 273** —To draw a trochoid, the director, generating circle, and generator being given.

1 When the generator  $P$  is within the given circle. Draw the diameter of the generating circle, and set off the semi-circumference as in the previous problem. Describe a circle passing through  $P$ . Divide this circle and  $C6$  into the same number of equal parts. Describe arcs with radius  $CP$  from points 1, 2, 3, &c., in the line  $C6$  to meet parallels from 1, 2, 3, &c., in the circle. Through the points thus obtained draw the curve. This is called the *inferior trochoid*.

2. When the generator  $P'$  is without the circle. Proceed as before. Divide the circle described through  $P'$  and  $C6$  into the same number of equal parts. Describe arcs from  $1', 2', 3', \&c.$ , to meet parallels from  $1', 2', 3', \&c.$ , in the circle. Through  $e, f, g, \&c.$ , draw half the curve. This is called the *superior trochoid*, and forms a loop as shown, the points of which may be obtained by continuing the centres along  $C6'$ .

**PROBLEM 274.**—To draw the epicycloid, the director and the generating circle being given

Let the arc **AB** be the director, and **P3'D** the generating circle. Join **O**, the centre of the director, with **D**, the point where the generating circle touches the director, and produce to **P**. Then **DP** will be the diameter of the generating circle. Cut off the arc **DA** = the semi-circumference of the generating circle **P3'D**. This is best done as follows.—The angle **DOA** bears the same ratio to  $180^\circ$  that the radius of the generating circle does to the radius of the director. Hence the following proportion for all cases. As **OD : DC ::  $180^\circ$  : DOA**. Thus if **OD** = 3 and **DC** = 1, the angle **DOA** =  $\frac{180}{3}^\circ = 60^\circ$ . Draw **OA**, making  $60^\circ$  with **DO**, and produce. From centre **O** describe an arc through **C**. This will evidently be the path of the centre of the circle when rolling, and **6** will be the position of the centre when **P**, the generating point, has reached **A**. Divide **C6** into any number of equal parts (say 6), and divide the semicircle **PD** into the same number of equal parts, **1', 2', 3', &c**. From centre **O** describe arcs from **1', 2', 3', &c**. From centre **1**, with radius **CD**, cut the arc from **1'** in **E**. From centre **2**, with the same radius, cut the arc from **2'** in **F**. In the same manner cut the arcs from **3', 4', &c**. Through the points thus obtained draw the curve from **P** to **A**. The other half can be obtained similarly.

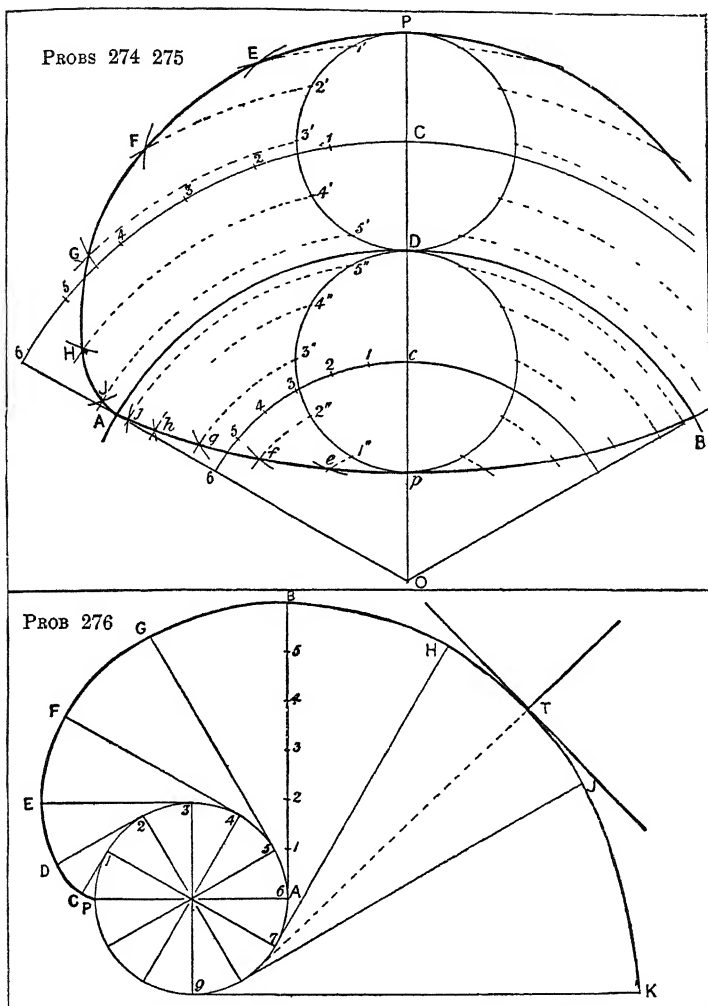
**PROBLEM 275.**—To draw the hypocycloid, the director and the generating circle being given.

Let **AB** be the director, as in the last figure, and **Dp** the generating circle. Proceed exactly as in the previous problem. Number the points **1'', 2'', &c.**, from **p**. From **1, 2, 3, &c.**, in **c 6** cut the arcs from **1'', 2'', &c**. Draw the half-curve **Ap** through the points **e, f, g, h, j**. The other side can be similarly obtained.

**PROBLEM 276**—To draw the involute of a given circle. Also to draw a tangent to the curve at any given point.

If a perfectly flexible thread be unwound from a circle and kept constantly stretched, the extremity of the thread describes a curve known as the *involute* of the circle.

Let **AP** be the circle, and **P** the generating point. Draw the diameter **AP**. At **A** draw the tangent **AB**. Make **AB** = the semi-circumference of the circle. (Radius  $\times 3.1416$ , taken from a scale.) Divide **AB** and the semi-circumference into the same number of equal parts (say 6). Draw tangents to the circle at points **1, 2, 3, 4, 5, 6, &c**. Make **1C = A1**, **2D = A2**, **3E = 3 parts, &c**. To obtain points beyond **B**, proceed in the same manner, **7H = 7 divisions**, **9K = 9 divisions**. Through the points **C, D, E, F, &c**, draw the curve.



To draw a tangent at T. Draw a line from T tangential to the circle. This line is the normal, the tangent is perpendicular to it.

Note—The involute of the circle, and the cycloid, epicycloid, and hypocycloid, are employed in shaping the teeth of wheels.

## SPIRALS

The spiral is a curve which gradually recedes from, or approaches to, a fixed point or centre called its pole, by some definite law. Thus in the Archimedean spiral the radii increase in succession from the pole by equal distances, in the Logarithmic spiral the lengths of the radii are in geometrical progression.

A line drawn from the pole to any point of the curve is a radius. The curve may consist of any number of turns or convolutions.

**PROBLEM 277.**—To construct an Archimedean spiral, the longest radius and the number of convolutions being given.

Let  $OA$  be the longest radius, the number of convolutions being two. Describe a circle with radius  $OA$ . Divide  $OA$  into as many equal parts as convolutions required—in this case two. Divide the circle into any number of equal parts (say 8), and draw the radii  $OB, OC, \&c$ . Divide  $Aa$  into the same number of equal parts (8). Make  $Ob = O7, Oc = O6, \&c$ , each radius of the spiral diminishing by one part. Draw the first convolution through the points  $b, c, d, \&c$ . The second turn, being parallel to the first, is most easily obtained by setting off the distance  $Aa$  from each of the points  $b, c, d, \&c$ .

**Note.**—The spiral is of great service in designing cams—contrivances much used in machinery involving complicated and irregular movements.

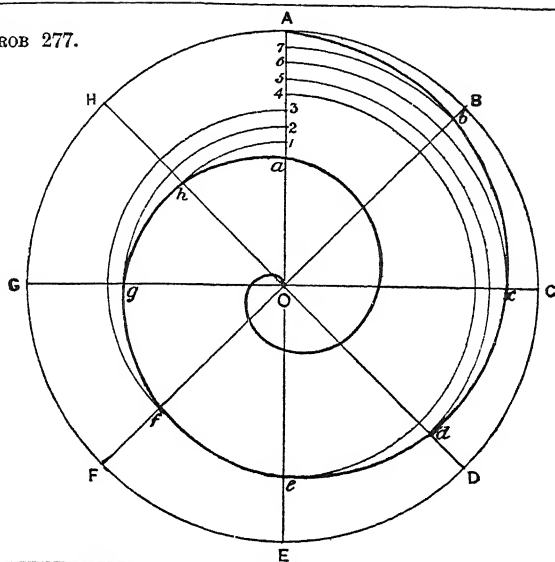
**PROBLEM 278**—To construct a Logarithmic spiral, the greatest radius, the angle between consecutive radii, and the ratio of succeeding radii, being given.

The greatest radius is  $OA$ , the angle between the consecutive radii is  $30^\circ$ , and the ratio of one radius to that which follows it is  $\frac{7}{8}$ . Describe a circle with radius  $OA$ , and set off the radii at angles of  $30^\circ$ . For the lengths of successive radii construct a supplementary figure. Draw  $Oa$  and  $Ob$  at any angle. Make  $Oa = OA$ . Set off  $Ob$  equal to  $\frac{7}{8}$  of  $Oa$ , and join  $ab$ . Make  $O1 = Ob$ , and from 1 draw  $12'$  parallel to  $ab$ , then  $O2 = \frac{7}{8}$  of  $O1$ . In the same manner  $O3 = \frac{7}{8}$  of  $O2, \&c$ . Set off successively  $O1', O2', \&c$ , on the radii  $= O1, O2, \&c$ . Through these points draw the curve.

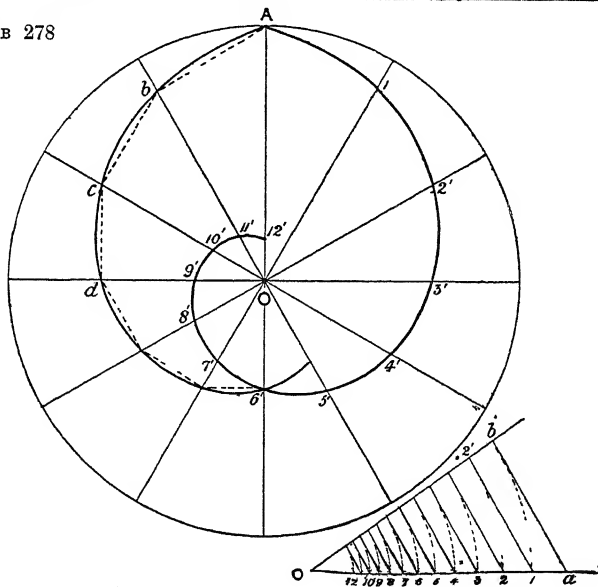
This spiral cuts all the radii at the same angle; hence it is also known as the Equiangular spiral. The angle  $A1'O =$  the angle  $1'2'O$ . This gives a ready way of constructing the spiral as shown on the left of the figure. Let the angle at which the curve meets the radii be  $90^\circ$ . From  $A$  draw  $Ab$  making  $90^\circ$  with radius  $Ob$ . From  $b$  draw  $bc$  making  $90^\circ$  with  $Oc, \&c$ . Through the points  $b, c, d, \&c$ , draw the curve.

**Note.**—This spiral may also be used to determine graphically the powers and roots of numbers.

PROB 277.



PROB 278





**PROBLEM 279.**—To draw the Ionic volute, the cathetus, or greatest radius, being given.

This forms part of the capital of the Ionic column. There are various methods of obtaining the curve, of which that known as *Goldmann's* method is perhaps the most satisfactory. The greatest radius,  $AB$ , is called the *cathetus*. The circle  $DC$  is the *eye* of the volute. The portion between the inner and outer curves,  $BE$ , is the *fillet*. The proportions of these parts are as follows  $AB = 9$ ,  $CD = 2$ ,  $BE = 1$ .

For Examination purposes only draw the outer curve, unless the inner curve also is asked for. Always draw the problem to a large scale.

Divide the cathetus  $AB$  into *nine* equal parts. With centre  $A$  and radius equal to one of the parts describe the eye  $CD$ . Bisect  $AC$  and  $AD$  in 1 and 4, and obtain the square 1234 (A larger drawing is given above). Draw  $A2$  and  $A3$ , and trisect them (Use dividers). From the points of trisection, 6, 7, 10, 11, draw lines obtaining the smaller squares, 5678 and 9101112. With centre 1 and radius  $1B$  describe the first quadrant, to meet 12 produced in  $F$ . With centre 2 and radius  $2F$  describe the second quadrant. Proceed similarly for the other quadrants, taking the centres successively as numbered. The last arc should finish at  $C$ , and be described from centre 12.

To obtain the inner curve, draw  $cb = CB$ , and perpendicular  $b1 = A1$ . Join 1 and  $c$ . Make  $be = BE$ , and draw perpendicular  $ef$ . On each side of  $A$  set off  $ef$ , and obtain the dotted square. Trisect as before for the smaller squares. The corners of the dotted squares will be the centre for the inner curve.

Notes—1. If the height of the volute be given, divide it into 8 equal parts, and on the fourth part from the bottom describe the eye.

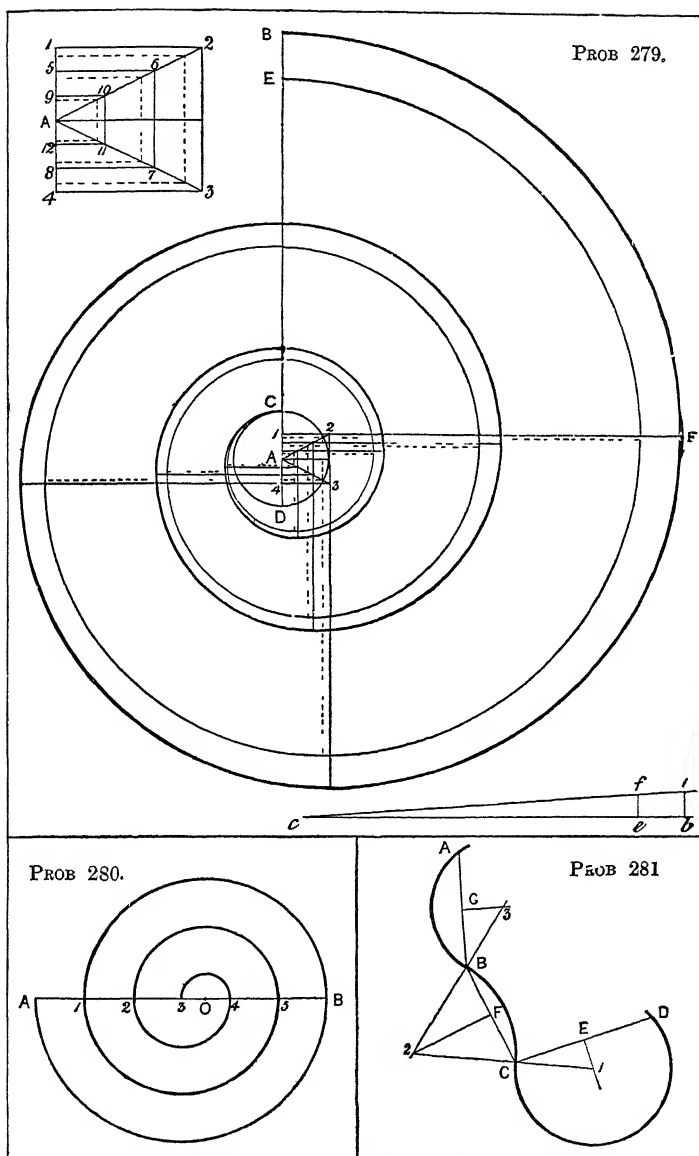
2. The greatest possible care must be taken in the construction of the squares for the centres, as the slightest error will prevent the volute from ending properly.

**PROBLEM 280.**—To draw a spiral scroll by means of semicircular arcs.

Let the spiral be described upon  $AB$ , and consist of three convolutions. Divide  $AB$  into six equal parts. Bisect 34 in  $O$ . With centre  $O$  and radius  $O3$  describe a semicircle. With centre 3 and radius 34 describe another semicircle. With alternate centres,  $O$  and 3, describe the remaining semicircles.

**PROBLEM 281.**—To describe a continuous curve of tangential arcs passing through a number of given points.

Let  $A, B, C, D$  be the given points. Join the points, and bisect the lines by perpendiculars at  $E, F$ , and  $G$ . Take any convenient point, 1, in  $E1$ , and describe an arc on  $CD$ . From 1, through  $C$ , draw 12, meeting the perpendicular from  $F$ . With centre 2 and radius  $2C$  describe the arc upon  $CB$ . Draw 2B to meet the perpendicular from  $G$ . With centre 3 describe the remaining arc.

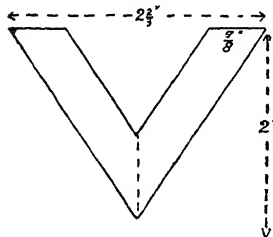


## EXERCISES

## CHAPTER XIII

1. Construct a square equal in area to a rectangle  $2''$  by  $1\frac{1}{2}''$ .
2. Draw a semicircle  $3''$  in diameter, and divide it into 3 equal parts by means of concentric semicircles.
3. Construct a square equal in area to the sum of three squares of  $1''$ ,  $1\frac{1}{2}''$ , and  $2\frac{25}{32}''$  respectively (Sc.)
4. Construct a square of  $2\frac{1}{2}''$  sides, and through one corner draw a line cutting off  $\frac{1}{3}$  of its area (Sc.) (Prob 236)
5. Draw a triangle, sides  $1\frac{1}{2}''$ ,  $2''$ , and  $1\frac{1}{2}''$  respectively. On a base of  $1\frac{3}{4}''$  construct an isosceles triangle of equal area (Sc.) (Prob 217)
6. Draw any irregular hexagon having one re-entering angle, and construct a similar figure whose sides are to those of the given figure as 7 to 3. (Sc.)
- Note.—Divide one side into 3 equal parts, and on 7 of these parts construct the similar figure
7. In a square of  $3''$  sides inscribe another square having  $\frac{2}{3}$  the area of the given square. The corners of the required square must lie in the sides of the given one (Sc.)
- Note.—A mean proportional between the side of the given square and  $\frac{2}{3}$  of the side will be the side of the required square. From this side obtain the diagonal, and from the centre of the given square, with half the diagonal as radius, describe a circle. The points where the circle cuts the sides will be the corners of the required square
8. Construct a parallelogram having sides  $4''$  and  $1\frac{1}{2}''$ , and the included angle  $50^\circ$ . Determine a rhombus of equal area, and having the same included angle.
- Note.—The side of the rhombus will equal the mean proportional between the two sides of the parallelogram
9. Reduce the given figure (Fig. 1) to a square of equal area. (Sc.)

FIG. 1.



- Note.—Divide the figure into 2 equal parts. Obtain a rectangle equal to 1 part, double it, and get a square equal to the rectangle
10. Divide an equilateral triangle of  $2\frac{1}{2}''$  sides into 4 equal parts by perpendiculars to one side.
- Note.—Bisect the triangle, and apply Problem 244 to each half
11. Draw an irregular pentagon, and bisect it by a line drawn from one angle
- Note.—First convert the pentagon into a triangle
12. The side of a rhombus is  $3''$  long, and one angle is  $75^\circ$ . Construct the figure, and divide it into 3 equal parts by lines drawn from one angle (Prob 236)
13. Construct an equilateral triangle of  $1\frac{1}{2}''$  sides, and a rectangle of equal height and area
14. Construct a square equal in area to an equilateral triangle of  $1''$  sides
15. Divide a triangle whose sides are  $2\frac{1}{2}''$ ,  $3''$ , and  $4''$  respectively, into 3 equal parts, by a line drawn from the middle of the longest side.
16. Construct a triangle, base  $3''$ , and base angles  $45^\circ$  and  $75^\circ$ . On the same base construct an isosceles triangle equal to it in area.
17. Draw a rectangle equal in area to a square of  $1\frac{7}{8}''$  side, making the shorter side  $1\frac{1}{8}''$  long. (Sc.)
18. Having given a circle of  $1''$  radius, draw another  $\frac{1}{3}$  of its area. (Sc.)
19. On a line,  $AB$ ,  $2''$  long, as base, construct a triangle,  $ACB$ , whose altitude is  $2\frac{1}{2}''$  and angle  $ABC$   $105^\circ$ . On  $AB$  as base describe a second triangle,  $ADB$ , equal to the triangle  $ACB$ , and having the side  $BD$  parallel to  $AC$
20. Draw a quadrilateral figure  $ABCD$ , with the following dimensions —  $AB = 2''$ ,  $BC = 1\frac{1}{2}''$ ,  $AD = 1\frac{1}{2}''$ . The diagonal  $BD = 2''$  the diagonal  $AC = 2\frac{1}{2}''$ . Find the length of the side of a square equal in area to the quadrilateral.

## CHAPTER XIV

- 1 Construct a semi-ellipse, major axis 3", and half the minor axis 1". (*Art.*)
- 2 Find the centre, axes, and foci of a given ellipse. (*Art.*) (*Draw the ellipse by means of thread and pins.*)
- 3 Describe an ellipse by means of intersecting arcs Axes  $3\frac{1}{2}"$  and 2" (*Art.*)
- 4 Construct a rhombus, side  $2\frac{1}{2}"$ , diagonal  $4\frac{1}{4}"$ . In this rhombus inscribe an ellipse. (*Sc.*) (*Prob.* 258)
- 5 Describe an ellipse, the longer diameter of which is half as long again as the shorter, and at any point on the curve draw a normal to it
- 6 Draw a spiral curve composed of five semicircles, whose diameters are, successively, 1,  $1\frac{1}{2}$ , 2,  $2\frac{1}{2}$ , and 3 inches
- 7 Draw an undulating, continuous curve of six arcs of circles, each containing  $90^\circ$ , and described with a radius of 1".
- 8 The major axis of an ellipse is 3" long, and the foci 1" from the centre; describe the curve, and show how to draw a tangent to it
- 9 Construct a triangle, sides  $3\frac{1}{2}"$ , 2", and  $2\frac{3}{4}"$ , and about it describe an ellipse (*Prob.* 258.)
- 10 Construct an ellipse having its conjugate diameters each  $3\frac{1}{2}"$  and intersecting at  $70^\circ$
- 11 Draw a half-ellipse with axes 4" and 2 5" Draw a sufficient number of normals to the curve, and produce them externally Through points on these, 0 5", from the curve, draw a second curve parallel to the first.
- 12 Draw a parabolic arch, height  $2\frac{1}{2}"$ , width 5".
- 13 A point moves on a plane surface so that every point of its path is equidistant from a fixed line and a fixed point. Draw the curve
14. The major axis of a hyperbola is 2", and its focus is  $\frac{1}{2}"$  from one extremity of the major axis. Draw the curve, and show how to obtain a tangent from a point in the curve
15. The focus, directrix, and vertex of a hyperbola are given Draw the curve
- 16 The radius of a circle is 1" Draw the cycloidal curve traced by a point on the circumference during one revolution.
- 17 A circle of 1" radius rolls round a circle of 3" radius. Draw the epicycloid traced by a point on the generating circle.
- 18 Draw the involute of a circle of 1 75" diameter The curve to be shown from its starting-point on the circumference of the circle till it cuts the produced diameter at that point. (*Sc.*)
19. The line *a b* (Fig. 2) represents a piece of thread unwound from the given circle Draw the curve traced by the extremity *a* when the thread is wound back on to the circle. (*Prob.* 276.)
- 20 Construct an Archimedean spiral of three convolutions, longest radius 3"
- 21 Draw the spiral which would cut all its radii at an angle of  $90^\circ$ .
- 22 Draw the outer curve of Goldmann's volute, the cathetus being 4".
23. The focus of a parabola is  $\frac{3}{4}"$  from the directrix Describe the curve making the axis  $2\frac{1}{2}"$
24. Draw a line *A B*, 1" in length. At *A* draw a line *A D*,  $\frac{3}{4}"$  long, making an angle of  $120^\circ$ , and at *B* draw a line *B C*,  $1\frac{1}{4}"$  long, making an angle of  $100^\circ$  Show how to describe a continuous curve of tangential arcs passing through the points *D A B C*.

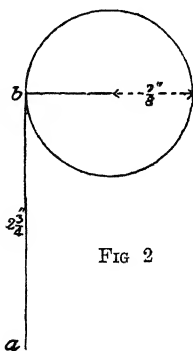


FIG 2

## CHAPTER XV

### SOLID GEOMETRY

#### PLANS, ELEVATIONS, AND SECTIONS OF SOLIDS IN SIMPLE POSITIONS

PLANE Geometry deals with the construction of figures having two dimensions only, *length* and *breadth*, and which can be represented on one surface or plane, but in the case of solids, all the faces do not lie in one plane, and the use of a third dimension, *thickness* or *height*, is necessary. To represent these three dimensions, at least two drawings are required—one, called the **plan**, to show the length and breadth, and the other, called the **elevation**, to show the thickness or height. This representation of three dimensions is known as **Solid or Descriptive Geometry**.

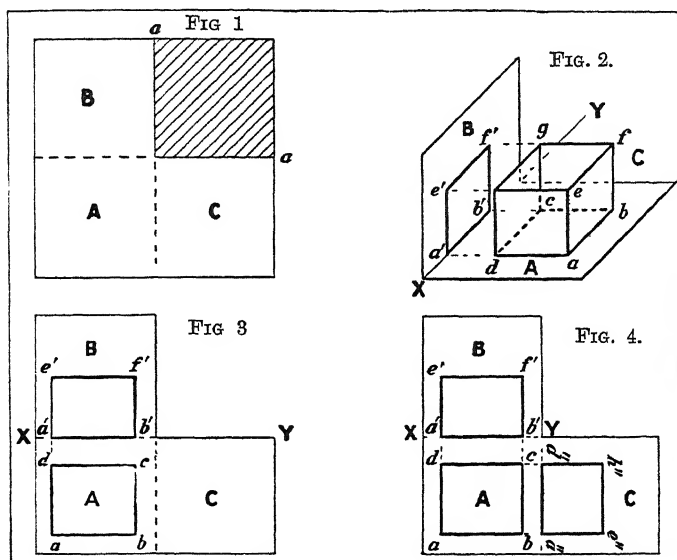
To illustrate this, take a sheet of cardboard and divide it into four parts, as shown in Fig. 1. Cut out the shaded portion, and cut half through the dotted lines. Place **A C** on a table and turn up **B** at right angles, placing it against a wall or any upright surface. There are now two planes—**A C** the **horizontal**, and **B** the **vertical**. The line represented by the crease where these two planes intersect, or cut each other, is called the **intersecting** or **ground-line**, and is usually denoted by the letters **X Y**.

Place a small box or other simple object on the horizontal plane **A C** (Fig. 2), and trace its plan  $a b c d$ . Now look horizontally forwards, bringing the eye level with each point of the object in turn, and trace the elevation  $a' b' f' e'$  on the vertical plane **B**. If we now remove the box and turn **B** back into the horizontal plane, we shall have two drawings on one surface, as in Fig. 3—the plan  $a b c d$  below, and the elevation  $a' b' f' e'$  above the line **X Y**.

But it is obvious that these two drawings do not give sufficient information, as they might also represent the plan and elevation of a cylinder; hence a third drawing is sometimes necessary showing the shape of the end. Replace the cardboard planes and the box as at first, and turn up **C** at right angles to **A**, so that the points  $a$  and  $a'$  (Fig. 1) coincide. Now look horizontally forwards at the end and draw its elevation on **C** in a similar manner to that drawn on **B**. Remove the box and turn down **B** and **C** into the horizontal plane, when three drawings will be represented on one surface, as in Fig. 4—the plan on **A**, the side elevation on **B**, and the end elevation on **C**, thus giving all the information necessary. These drawings are known as **projections**, because each point of the object is projected or thrown upon the portion of the plane exactly opposite to it; thus  $a, a', a''$  are all projections of the actual point **A**. The lines  $a a'$  and  $a a''$  are called the *projectors* of point **A**. The three planes used in Fig. 4 are usually spoken of as the **three co-ordinate planes**.

**Note**—If the portion marked **C** in Fig. 1 be cut out, then the two vertical planes would both fall back above **A**, showing the side and end elevations both

above the intersecting line. It is more convenient, however, in the projection of solids to obtain additional elevations without rotating the plan.



When the projection is effected by *parallel lines at right angles* to the co-ordinate planes, it is termed **orthographic projection**, and every part of the object is represented its correct size by *scale*, no matter how distant. In a **perspective projection** the eye is fixed, and the rays of light converge towards it; hence, the back lines of an object are represented as they appear, that is, shorter than the corresponding front ones.

The system of lettering is as follows — The same point, whatever its position, is denoted by the same letter, thus, if **A** represents an actual point on an object, then *a* shows its plan, *a'* its elevation, etc. The student should now draw roughly the plans and elevations of a number of simple objects, and in cases of difficulty place the object in the required position. Remember — 1. The **plan** is seen by *looking vertically downwards*, the eye being supposed to be exactly opposite each point of the object in turn. 2. The **elevation** is seen by *looking horizontally forwards*. 3. That every point in the plan is directly under the corresponding point in the elevation.

When a solid is bounded entirely by plane surfaces it is called a **polyhedron**, the plane surfaces are the **faces**, and the lines of intersection of the faces are the **edges**. If the faces are equal and regular polygons, the polyhedron is **regular**.

There are **five regular solids**, each of which can be described in a sphere :—

The **tetrahedron** is contained by four equal equilateral triangles (Fig 5)

The **cube** is contained by six equal squares (Fig 6).

The **octahedron** has eight faces, all equal equilateral triangles (Fig 7)

The **dodecahedron** has twelve faces, all regular pentagons (Fig. 8)

The **icosahedron** has twenty faces, all equal equilateral triangles (Fig 9).

A **prism** is a polyhedron having its end faces equal, similar, and parallel, and its side faces parallelograms. The straight line joining the centres of the end faces is the **axis** (A B, Fig 10).

A **pyramid** is a polyhedron having a plane figure for its base, and its side faces triangles meeting at a point called the **vertex**. The straight line joining the centre of the base with the vertex is the **axis** (C D, Fig 11)

When the axis is perpendicular to the ends or bases, the solid is a **right prism** as in Fig 10, or a **right pyramid** as in Fig 11, when it is inclined to the ends, the solid is an **oblique prism** or **pyramid** as in Figs 12 and 13.

**Note** —Prisms and pyramids are named from the shapes of their bases, thus a square prism has two square ends, a hexagonal pyramid has a hexagon for its base, etc

A **sphere** may be defined as a solid generated by the revolution of a semicircle about its diameter; every part of its surface is equally distant from the centre (Fig 14)

A **cylinder** resembles a prism. It may be generated by the revolution of a rectangle about one of its sides (Fig 15).

A **cone** resembles a pyramid. It may be generated by the revolution of a right-angled triangle about one of its sides (Fig. 16)

If the upper part of a pyramid, cone, or other solid be cut away the portion left is called the **frustum**. It is said to be **truncated** (Fig 17).

The **volume** of any solid may be found by immersing it in a vessel full of water, and finding the volume of the water displaced

### THE CUBE AND THE RIGHT SQUARE PRISM

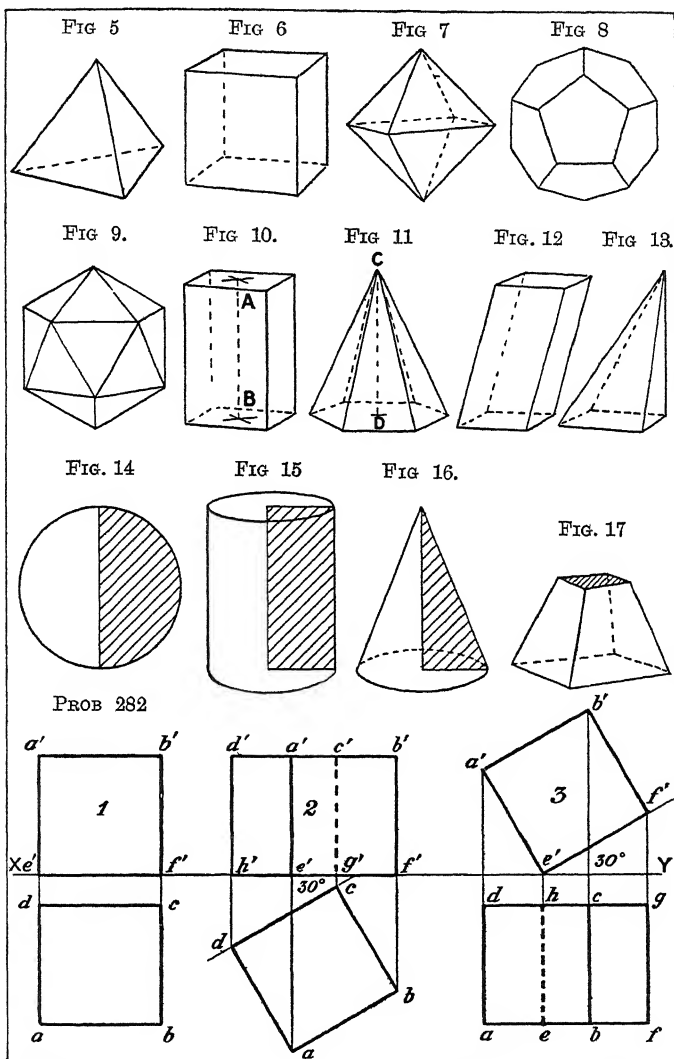
The method of treatment is precisely the same for both solids, hence they are taken together.

**Notes** —1 In all cases the object should be used to illustrate the required position, as shown on page 129.

2 Where dimensions are not given, draw to a large scale.

3 The following contractions will be used —H P. (Horizontal Plane) V.P (Vertical Plane) for the two co-ordinate planes, and X Y for the Intersecting Line

**PROBLEM 282** —Draw the plan and elevation of a cube having an edge of 2" —1 With one face horizontal, and one face parallel to the V.P.  
2 With one face horizontal, and one face inclined at  $30^\circ$  to the V.P.  
3 With one face inclined at  $30^\circ$  to the H.P, and one face parallel to the V.P





1 The plan is the square  $abcd$  with  $ab$  parallel to  $XY$ . The elevation is the square  $e'f'b'a'$ .

2 Draw the plan with  $cd$  inclined at  $30^\circ$  to  $XY$ . Project as shown for the elevation. The edge  $c'g'$  is dotted because, though a necessary part of the construction, it is not visible.

3 The elevation will be the square  $e'f'b'a'$  inclined at  $30^\circ$  to the H.P. Project as shown for the plan.  $eh$  is shown in dotted line because it would not be visible.

**PROBLEM 283** —To draw the plan and elevation of a square prism —

1. When one face is horizontal. 2. When one face is inclined to the H.P. at  $30^\circ$ , and the axis is parallel to the V.P.

1 Five positions are shown, all of which may be readily followed with the help of the model.

Nos 1 and 2 show the plan and elevation when the prism is standing on its end, and are similar to the projections of the cube in Problem 282.

In 3 the prism is above the H.P., with its axis parallel to the V.P.

In 4 the axis is perpendicular to the V.P.

In 5 one face is inclined to the V.P. Draw the plan  $abcd$ , and project the elevation as shown.

2 Draw  $e'f'$  making  $30^\circ$  with  $XY$ . On it construct the elevation. Project the plan as shown.

### DEVELOPMENT

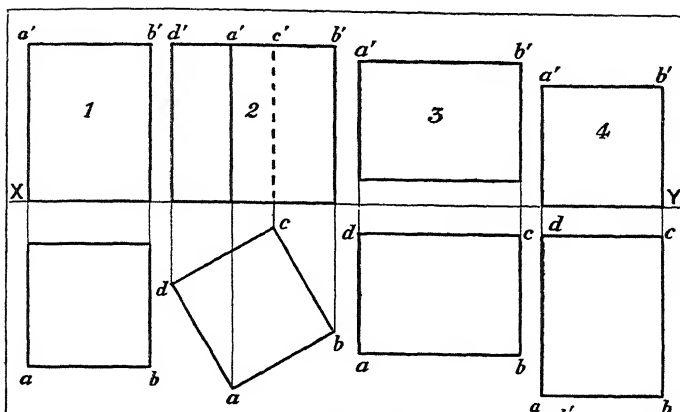
A drawing which shows the true shape of the complete surface of a solid all upon one plane, such as a sheet of paper or cardboard, is called a **development** of the surface. If the shape of this development be cut out, it can be refolded into a model of the solid. Development is an important part of engineering drawing, as the surfaces of structures made from plates, such as boilers, must be developed, in order that the plates may be marked off. The surface of a cube is composed of six equal squares, which may be set out as shown in Fig. 18. Cut out the shape, leaving strips as shown for glueing the sides together. Perforate or half cut through along the dotted lines, so that they will fold correctly, turn up the sides, fastening them together by the glued strips.

Fig. 19 shows the development of the surface of a square prism, which may be made into a model as explained in the previous paragraph.

Fig. 20 shows the development of the triangular prism. Strips for glueing up should be left on the sides of the triangles and on the long edge of one of the rectangles.

Other prisms may be developed similarly, by setting out the parallelograms which form their faces, together with the polygons forming their ends.

The student should make models of the solids he needs as explained in Figs. 18-20.



PROB 283.

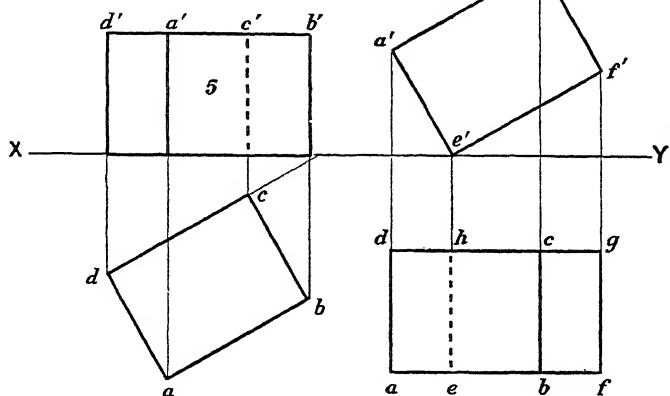
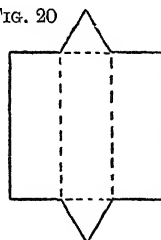
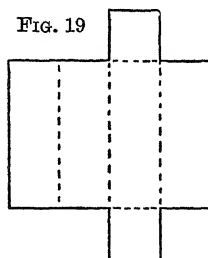
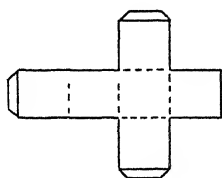


FIG 18

FIG. 19

FIG. 20



**ALTERATION OF THE GROUND-LINE.**—Attention is directed to this important principle at an early stage, as it is very desirable

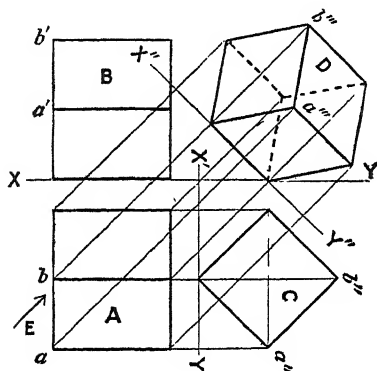


FIG 21

that the student should be accustomed to obtain projections at various angles by this method. It has already been pointed out that sometimes more than two views of an object are necessary to explain it clearly. Fig 21 shows four different views of a square prism standing with one edge in the H.P.

To illustrate this, take the piece of cardboard used to show the three co-ordinate planes in Fig 4. Place the model and draw the

plan A, the side elevation B, and the end elevation C, as shown in Fig 21. None of these views gives a clear idea of the appearance of the prism as a whole. To obtain this, look at the prism from the direction of the arrow E, when the view D is seen, showing both the end and side elevations in one figure.

To show the plane upon which D is to be projected, turn down plane C and draw  $X''Y''$  at right angles to E. Fold a piece of paper so as to form a hinge, glue it along  $X''Y''$ , and draw the view D. When turned down into the H.P., all four drawings will be seen on one surface.

It will thus be seen that the three ground-lines  $XY$ ,  $X'Y'$ , and  $X''Y''$  are all in the same H.P. Therefore any point of the prism, no matter in what position it may be viewed, is the same height above its respective ground-line. Thus  $b''$  and  $b'''$  will be the same elevation above  $X'Y'$  and  $X''Y''$  that  $b'$  is above  $XY$ .

Problems 284 and 285 illustrate this principle still further.

**PROBLEM 284.**—To draw the plan and elevation of a box 3" long, 2" wide, and  $1\frac{1}{4}$ " high, when its long sides are inclined at  $45^\circ$  to the V.P. Also a second elevation on  $X''Y''$ .

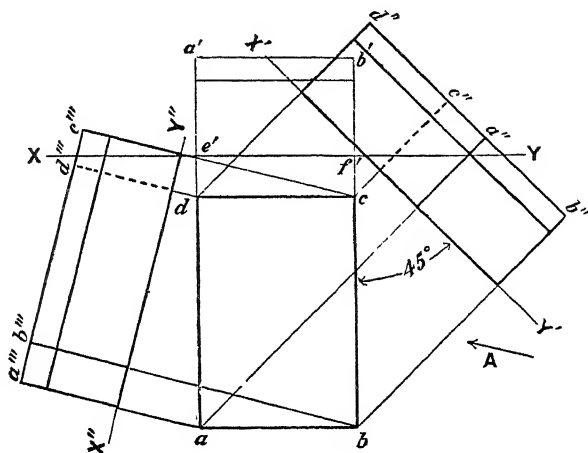
First draw the plan,  $abcd$ , and the elevation,  $a'b'e'f'$ , when the long sides of the box are perpendicular to the V.P. Now imagine a new V.P. making  $45^\circ$  with the long edge  $bc$ . This plane will intersect the H.P. in  $X'Y'$ , which becomes the new ground-line.

Project at right angles from each point of the plan to this new V.P., and set off the heights from the first elevation.

Any number of elevations may be obtained by imagining new vertical planes at right angles to the direction at which the box is viewed. For example, suppose a view is required from the direction

of the arrow-head **A**. Draw a new ground-line **X''Y''** at right

PROB 284



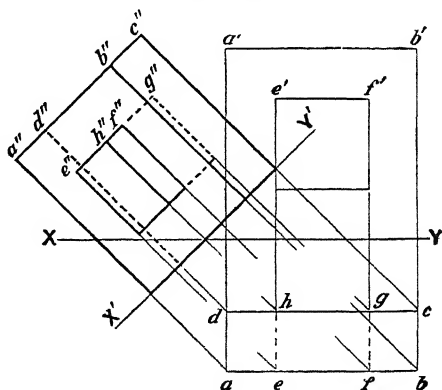
angles to **A** in any convenient position, and project as shown in the elevation on the left

**Note.**—Each point should be properly lettered as it is found.

**PROBLEM 285**—The plan and elevation of a square frame are given. Obtain an elevation when the face of the frame is inclined at  $45^\circ$  to the V.P.

PROB 285.

Draw a new ground-line **X'Y'**, making an angle of  $45^\circ$  with **XY**. If the paper be turned so that **X'Y'** faces the student, it will be seen that **ab**, the plan of the face of the frame, is inclined at  $45^\circ$  to the new V.P. Project at right angles to **X'Y'** from **a** and **b**, and make **a''** and **b''** the same height above



**X'Y'** that **a'** and **b'** are above **XY**. Now project from **c** and **d** for

the back, drawing the back edge from  $d$  in dotted line. For the opening, project from  $e$  and  $f$ , making the heights the same as in the first elevation. Now project from  $g$  and  $h$  for the back, and complete the figure, drawing the unseen edges in dotted line.

### SECTIONS

Plans and elevations of objects do not always furnish sufficient information concerning them. If, for example, the object be hollow, the arrangement of the inside parts is not shown clearly by dotted lines. In a building it is necessary to show the position and size of the floors, inner walls, etc. To do this, we imagine the object to be cut through by a plane, and the portion between the observer and the section plane to be removed, thus disclosing the interior. The cut is termed a **section**, and the cutting plane a **section plane**. At this stage, the sections dealt with will be chiefly those made by vertical planes.

To take a simple illustration, let  $a b c d$  (Fig. 22) be the plan of a small cupboard containing one shelf. Suppose it to be cut through by a vertical plane, of which  $H T$  is the horizontal trace, that is,  $H T$  is the line of intersection between the section plane and the horizontal plane. If we now imagine the portion of the cupboard in front of  $H T$  to be removed, then  $a' b' f' e'$  shows the appearance of the interior of the cupboard, and the position and manner in which the shelf is fixed. The solid portions cut through are indicated by *diagonal section lines*.

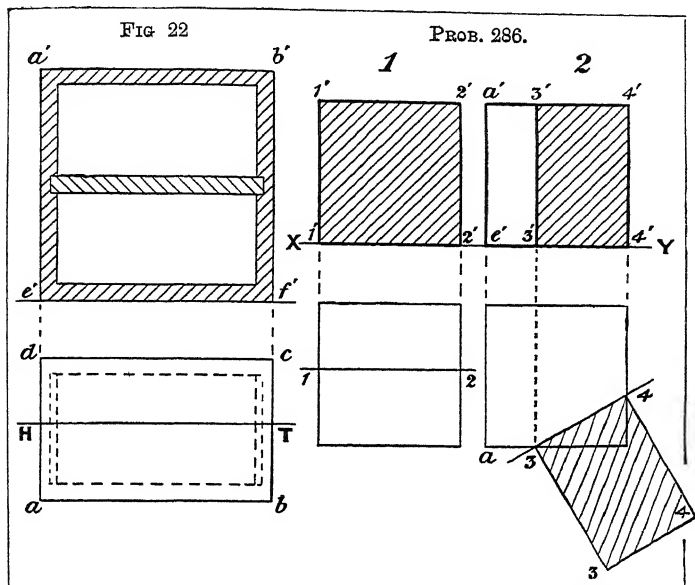
**PROBLEM 286.**—The plans of two cubes are given —1. Draw their sectional elevations when cut by the vertical planes 1 2 and 3 4 respectively. 2. Show the true shape of the section made by the plane 3 4. 3. Draw a sectional elevation of the cube on a plane parallel to 3 4.

The difference between a sectional elevation and the true shape of the section should be carefully noted. The **sectional elevation** shows the appearance of the object when viewed *at right angles to the V.P. of projection*; the **true shape of the section** is seen when viewed *at right angles to the section plane*.

1. Project the elevations of both cubes. As the section plane 1 2 cuts the first cube from side to side, obviously the sectional elevation will be the square  $1' 2' 2' 1'$ , the whole of which should be indicated by section lines, which should be drawn at an angle of  $45^\circ$ .

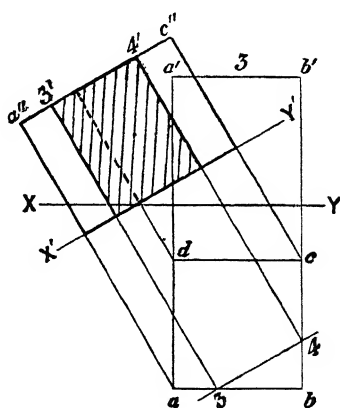
In the second cube, project from 3, giving the elevation of the section,  $3' 4' 4' 3'$ . The whole figure  $e' 4' 4' a'$  is the **sectional elevation**, that is, the representation of the section, together with

that portion of the solid which lies between the section plane, 3 4, and the V P.



2. The true shape of the section is obtained by rabatting or turning down the section plane, using 3 4 as a hinge, into the H P. Make  $44' = 4'4'$ . The rectangle 3 4 4' 3' shows the true shape. This may be also arranged as in Fig 3.

3. The sectional elevation, which shows the true shape of the section, is the view which is generally required. To obtain it, draw a new ground-line  $X'Y'$ , in any convenient position, parallel to 3 4. Project as shown in Fig 3, making  $a''$  the same height above  $X'Y'$  that  $a'$  is above  $XY$ . It is not necessary to project from  $b$ , as the portion of the solid in front of 3 4 is supposed to be removed. If any difficulty is experienced, a model cut out in soap or plasticene should be used.



**PROBLEM 287**—Given the plan and elevation of a hollow square block, cut by a vertical section plane 1 4. Draw an elevation showing the true shape of the section.

Draw a new ground-line  $X' Y'$  parallel to 1 4. Project from  $c$ , 1, 2, 3, and 4, and set off the elevation. The block is cut from 1 to 2 and from 3 to 4, hence these portions will be shaded in the elevation. Project from  $b$ ,  $g$ , and  $h$ , the edges from  $b$  and  $h$  are not visible, and are therefore indicated by dotted lines. It is unnecessary to project from any of the corners in front of 1 4, as they are supposed to be removed.

**PROBLEM 288.**—The plan and elevation of two stone steps, cut by a vertical plane 1 3, are given. Draw an elevation on a plane parallel to 1 3.

Draw  $X' Y'$  parallel to 1 3. First obtain the upper step by projecting from  $a$ ,  $e$ , and  $f$ . Draw  $a'' e''$  for the top of the step, making  $a''$  the same height above  $X' Y'$  that  $a'$  is above  $X Y$ . Now project from  $d$ , and draw the top of the lower step. The upper step is cut from 1 to 2, hence project to 1' and 2'. The lower step is cut from 2 to 3, therefore project to 3'. The shaded portion shows the true shape of the section because it is projected at right angles to the section plane.

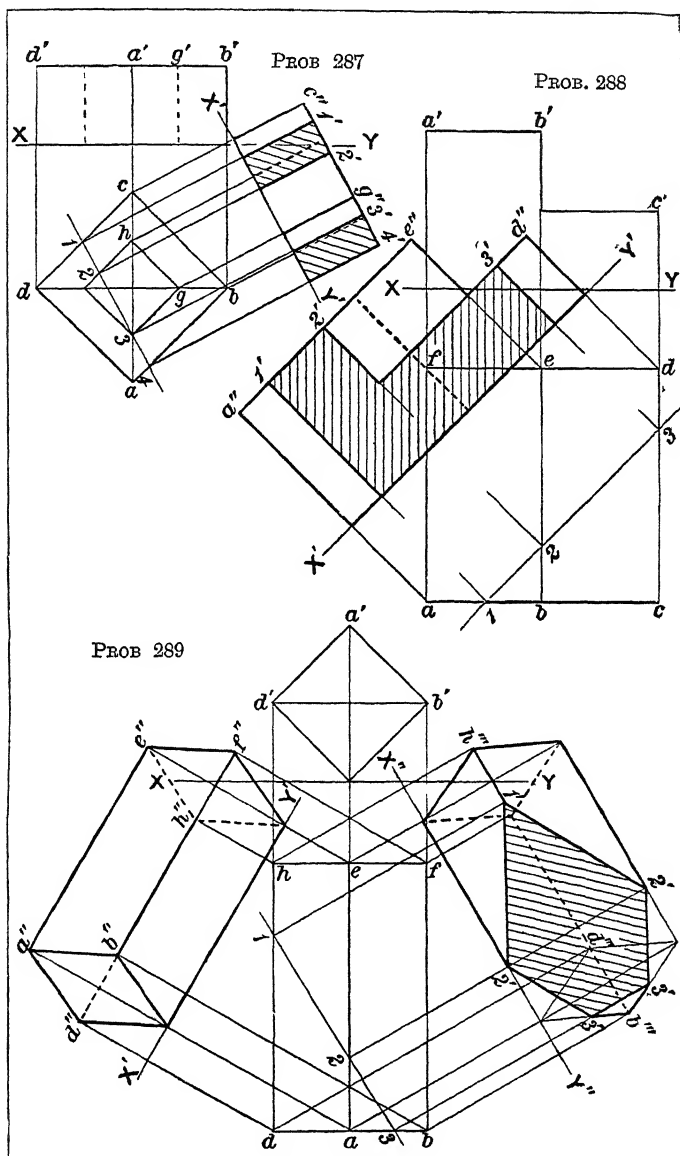
**PROBLEM 289.**—The plan and elevation of a square prism are given —1. Draw an elevation when the axis of the prism is inclined at  $60^\circ$  to the V.P. 2. Draw an elevation showing the true shape of the section made by the vertical plane 1 3.

1. Draw  $X' Y'$  making  $60^\circ$  with  $X Y$ . Project from  $a$  and  $e$ , making  $a''$  and  $e''$  the same height above  $X' Y'$  that  $a'$  is above  $X Y$ . Draw  $a'' e''$ , the top edge of the prism. Project from  $d$ ,  $b$ ,  $f$ , and  $h$ , and set off the heights from the end elevation as before. Draw the ends and complete the prism as shown.

2. Draw the new ground-line  $X'' Y''$  parallel to 1 3, and project an elevation of the prism as in the preceding exercise. Now find the points of section as follows—1 cuts the edge  $d h$ , therefore its elevation 1' must fall on  $d''' h'''$ . At 2 the section plane cuts the top and bottom edges of the prism, hence 2' 2'' will be the points in the elevation. At 3 the plane cuts through the end at 3' 3''. Join the points and shade in the section. Now thicken in the portions of the solid behind 1 3.

**Notes**—1. A careful study of the last three problems should enable the student to deal with any simple case of sections by vertical planes. The section plane should be taken in various positions so as to get a complete grasp of the principles involved.

3. Draw the figures to a large scale, and carefully letter each point as it is obtained.





## OTHER PRISMS

**PROBLEM 290.**—To draw the plan and elevation of an equilateral triangular prism. Side of equilateral base  $1\frac{7}{8}$ ", length of prism  $2\frac{5}{8}$ ".

Four positions are shown.

1. **Standing on its base with a rectangular face inclined at  $45^\circ$  to the V.P.**—Draw a line inclined at  $45^\circ$  to  $XY$ , and on it construct the equilateral triangle  $abc$ . From this project the elevation.

2. **With a rectangular face horizontal and an end parallel to the V.P.**—First obtain the elevation, which will be the equilateral triangle  $a'b'c'$ . From this project the plan  $abcd$ .

3. **With its axis horizontal and parallel to the V.P.**—First draw the plan and elevation as in 2. Draw  $X'Y'$  parallel to the axis. Project from the plan, making the height = the altitude of the triangle  $a'b'c'$ . Then  $abcd$  will be the plan, and  $a''c''f''d''$  the elevation.

This elevation might also have been obtained on  $XY$  by turning  $abcd$  with its axis parallel to  $XY$ . The advantage of the other method will be obvious to the student, as it does away with the necessity for moving the plan.

4. **With one face horizontal and its axis inclined at  $45^\circ$  to the V.P.**—Obtain the plan and elevation as in 2. Draw a new ground-line  $X''Y''$  inclined at  $45^\circ$  to the axis of the plan, then  $abcd$  will be the plan in the required position. For the elevation project from each point of the plan. As  $a, b, e, d$  are on the ground, it is only necessary to project their elevations to  $X''Y''$ .  $c'$  and  $f''$  will be the same height above  $X''Y''$  that  $c'$  is above  $XY$ .

**Note**—This problem should be carefully worked out by the student, as it furnishes the key to all similar projections of other prisms. In positions 3 and 4 repeat the end elevation and plan for each case separately, instead of projecting them both from 2, as there will be less liability of confusing the problems.

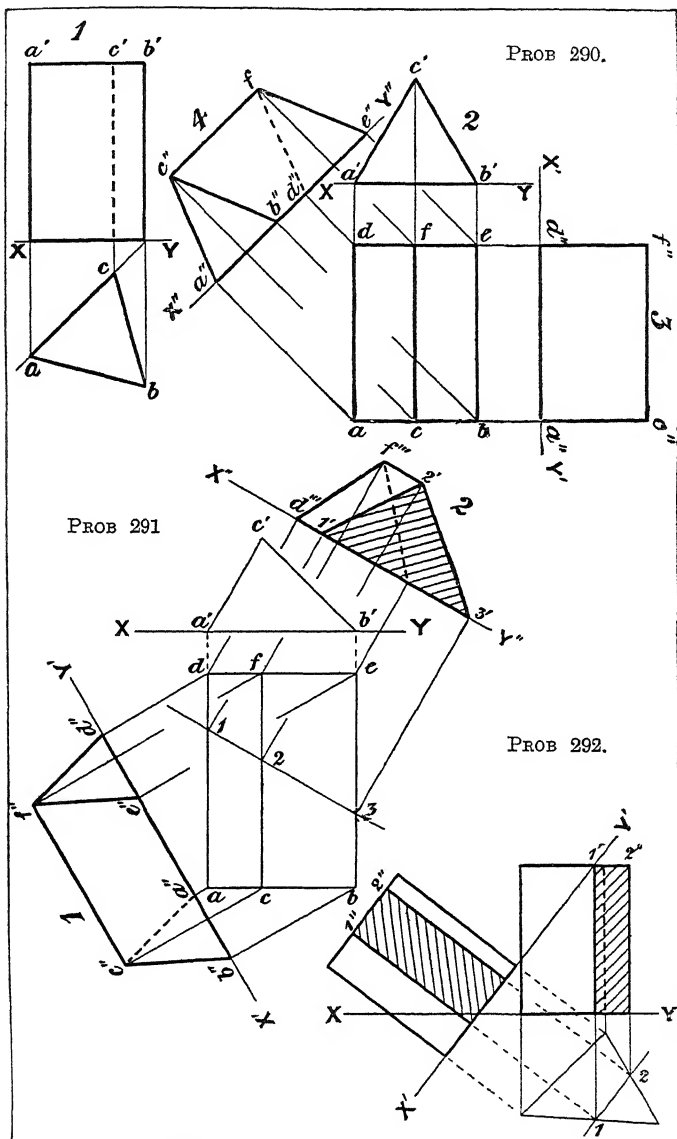
**PROBLEM 291.**—The plan and end elevation of a triangular prism are given. Draw a new elevation on  $X'Y'$ , also a sectional elevation on 13.

1. Project from each angle of the plan to  $X'Y'$ , making  $c''$  and  $f''$  the same height above  $X'Y'$  that  $c'$  is above  $XY$ .

2. For the sectional elevation draw a new ground-line  $X''Y''$  parallel to the section plane 13. Project from 1, 2, and 3. As 1 and 3 are on the H.P., project to  $X''Y''$ . Make  $2'$  the same elevation above  $X''Y''$  that  $c'$  is above  $XY$ . Now project from  $d, e$ , and  $f$ , for the end of the prism. Join  $2'$  and  $f''$ .

**PROBLEM 292.**—The plan of a triangular prism is given. Draw a sectional elevation on  $XY$ . Also an elevation showing the true shape of the section.

This may be readily followed from the figure. For the true shape, draw the ground-line parallel to 12.



**PROBLEM 293** —The plan of an equilateral triangular prism, cut by a vertical plane 1 3, is given. Draw a sectional elevation on  $XY$ .

First obtain the true shape of the end by constructing the end into the V P. This will be the equilateral triangle  $ABC$ . Project the elevation, obtaining the height from  $ABC$ .

To obtain the section, proceed as follows. Point 1 is on  $ad$ , then its elevation must be on  $a'd'$ . 2 is on  $cf$ , and its elevation will be on  $c'f'$ . Join 1' and 2'. Point 3 cannot be directly projected, as the plan and elevation are in the same straight line. Construct it into the V P as shown, when 3 3 on  $ABC$  will show how the section plane cuts the end of the prism. Obtain 3', and join 2' and 3.

**PROBLEM 294.**—The plan of an equilateral triangular prism, cut by a vertical section plane 1 3, is given. Draw a sectional elevation on a plane parallel to 1 3.

**Note** —In drawing sections of solids with oblique faces, it is better to project the complete plan or elevation of the solid first, and then obtain the section.

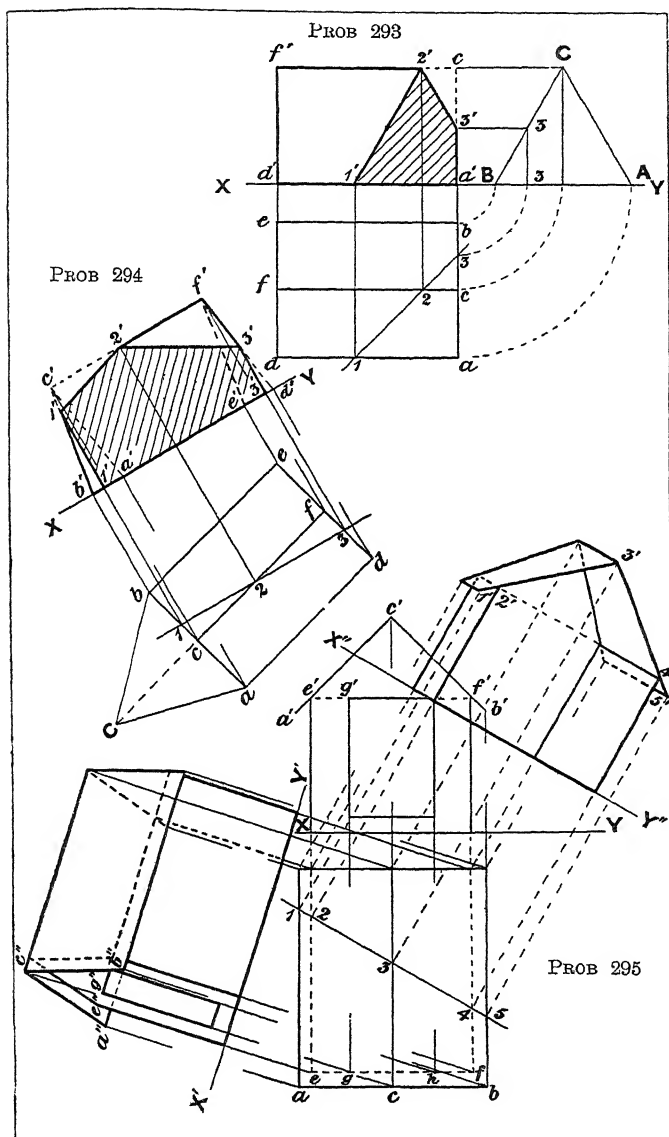
Draw  $XY$  parallel to 1 3. Project from each point of the plan, as in Problem 290. To obtain the height of the prism, rabatte the end into the H.P., when  $cC$  will be the elevation.

Now find the points of section. Point 1 is on the face, and its elevation will be the line 1' 1'. Point 2 is on  $cf$ , and its elevation will be 2' on  $c'f'$ . Join 1' and 2'. The elevation of 3 is the line 3' 3'. Join 2' and 3'. Notice that all lines lying between 1 3 and  $XY$  should be thickened in, as they represent the portion of the figure left, after the part in front of 1 3 is supposed to be removed.

**PROBLEM 295.**—The plan and elevation of a dog-kennel are given. Draw a new elevation on  $X'Y'$ , also a sectional elevation on 1 5. (Neglect the thickness.)

1. First project the roof as a triangular prism, making  $a'', b'', c''$  the same elevation above  $X'Y'$  that  $a', b', c'$  are above  $XY$ . Now project the sides of the kennel from  $e$  and  $f$ , and repeat for the back, showing by a dotted line where they meet the roof. Project the doorway from  $g$  and  $h$ .

2. For the sectional elevation project from 1, 3, and 5, obtaining 1', 3', and 5'. Join these points, and project the sides from 2 and 4. Project the end, and join with the elevation of the section. The student should follow this problem carefully, noting the hidden lines (dotted) in the various projections.



**PROBLEM 296** —To draw the plan and elevation of an equilateral triangular prism with one edge in the H.P. at an angle of  $30^\circ$  to the V.P.; one of its rectangular faces inclined to the H.P. at  $45^\circ$ .

Draw  $a'c'$  at  $45^\circ$  with  $XY$ , and on it construct the equilateral triangle  $a'b'c'$ . Project the plan as shown. We have now the plan and elevation of the prism when the axis is at right angles to the V.P. Draw  $X'Y'$  the ground-line of a vertical plane inclined at  $30^\circ$  to the edge  $ad$  of the prism. Project the edge  $ad$  to  $X'Y'$ . Now project the other two edges as shown, making  $b''c''$  the same height above  $X'Y'$  that  $b'$  and  $c'$  are above  $XY$ .

**PROBLEM 297** —Draw the projections of a hexagonal prism on the three co-ordinate planes, when one face is in the H.P. and one end parallel to the V.P. (See pages 128 and 134)

Draw the elevation  $a'b'c'd'e'f'$ , and project the plan as shown.

For the side elevation, project to  $YZ$ , and obtain the heights from the end elevation.

**PROBLEM 298.**—The elevation of a hexagonal prism is given. Obtain the sectional plan when cut by the horizontal plane  $1'2'$ .

Project as shown. Notice that it is not necessary to project from  $d'$  and  $e'$ , as the portion of the prism above the section plane is supposed to be removed.

**PROBLEM 299.**—A hexagonal prism has one face in the H.P. Draw its elevation when its axis is inclined at  $30^\circ$  to the V.P.

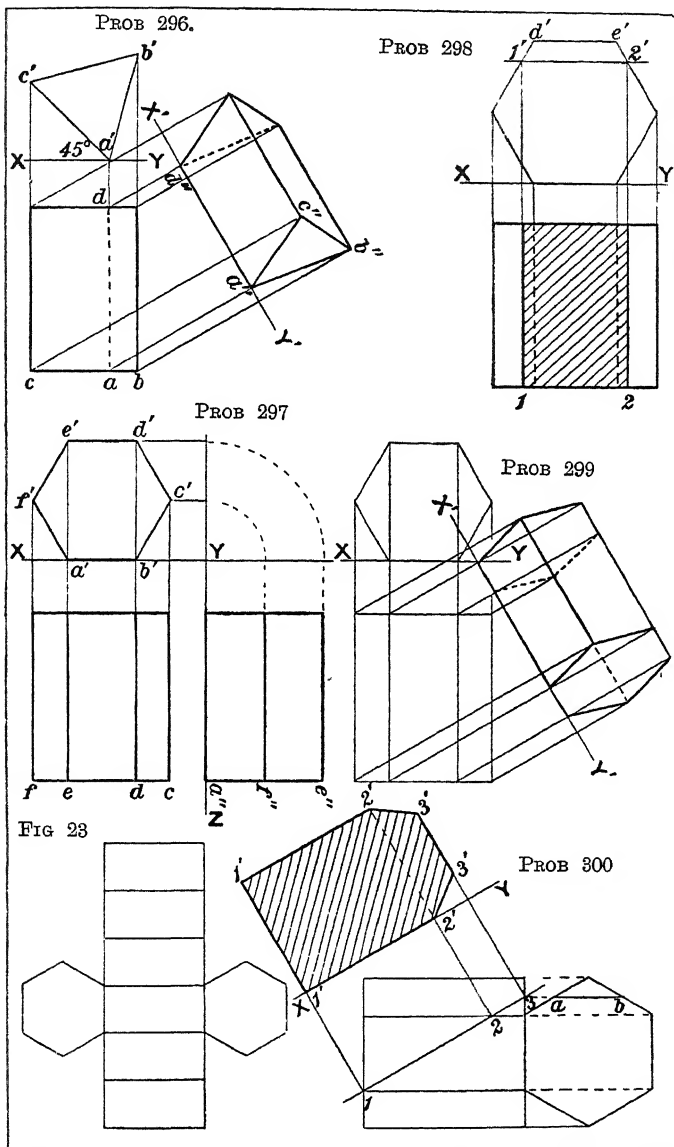
First draw the plan and elevation when its axis is perpendicular to the V.P. Now obtain a new ground-line making  $30^\circ$  with the axis of the prism.

The projection may be easily followed from the figure. The advantage of this method over that of moving the plan to an angle of  $30^\circ$  with  $XY$  has already been pointed out.

Figure 23 shows the development of the surface of the hexagonal prism. The other prisms should be treated similarly.

**PROBLEM 300.**—The plan of a regular hexagonal prism cut by a vertical plane, 13, is given. Obtain the true shape of the section.

It is first necessary to rabatte the end of the prism into the horizontal plane to obtain the heights. Draw  $XY$  parallel to the section plane. Project from 1, 2, and 3. The elevations of 1 and 2 will be equal to the height of the end. At 3, the section plane cuts through the edges of the end face. To obtain the elevation, project from 3 to the end, when  $ab$  will show how the section plane cuts the end. Make  $3'3'$  the same height above  $XY$  that  $a$  and  $b$  are above the H.P.



## PYRAMIDS

Figures 24, 25, and 26 show the plan and elevation of a square pyramid in three simple positions, which may be easily followed with the help of the model.

Figure 27 shows the development of the surface of the pyramid in Fig. 24. It is obtained as follows. From  $v$ , the centre of the base (Fig. 24), draw  $vV = e'v'$ , the altitude of the pyramid. Draw  $cV$ , the true length of the sloping edge of the pyramid. With centre  $O$  and radius  $cV$  describe an arc. On this arc set off the sides of the base  $ab$ ,  $bc$ ,  $cd$ , and  $da$ . Draw  $Oa$ ,  $Ob$ , etc. On  $cd$  construct the square base. Leave strips on the sides of the square, and on  $Oa$  for glueing up.

Other pyramids may be similarly developed.

**PROBLEM 301.**—Draw the plan and elevation of a square pyramid, when a triangular face lies in the H.P., and the axis is parallel to the V.P.

First draw the plan  $abcd$  and the elevation  $a'b'e'$ , when one edge of the base is parallel to the V.P. The diagonals  $ac$  and  $db$  represent the edges of the triangular faces. If we now turn the pyramid about the side  $bc$ , so that a triangular face lies on the H.P., we shall have the required elevation  $b'a''e''$ . Project as shown for the plan.

This method, though easy to follow, is cumbersome, and should not be resorted to in practice. *Changing the ground-line* is much quicker. This is shown on the right-hand side.

Draw the plan and elevation as before.

Instead of turning the elevation  $a'b'e'$  into the H.P., draw a new ground-line  $X_1Y_1$  along  $b'e'$ . If we now turn the paper,  $X_1Y_1$  will be the edge of the H.P., then  $a'b'e'$  will be the elevation of the pyramid when one triangular face is in the H.P.

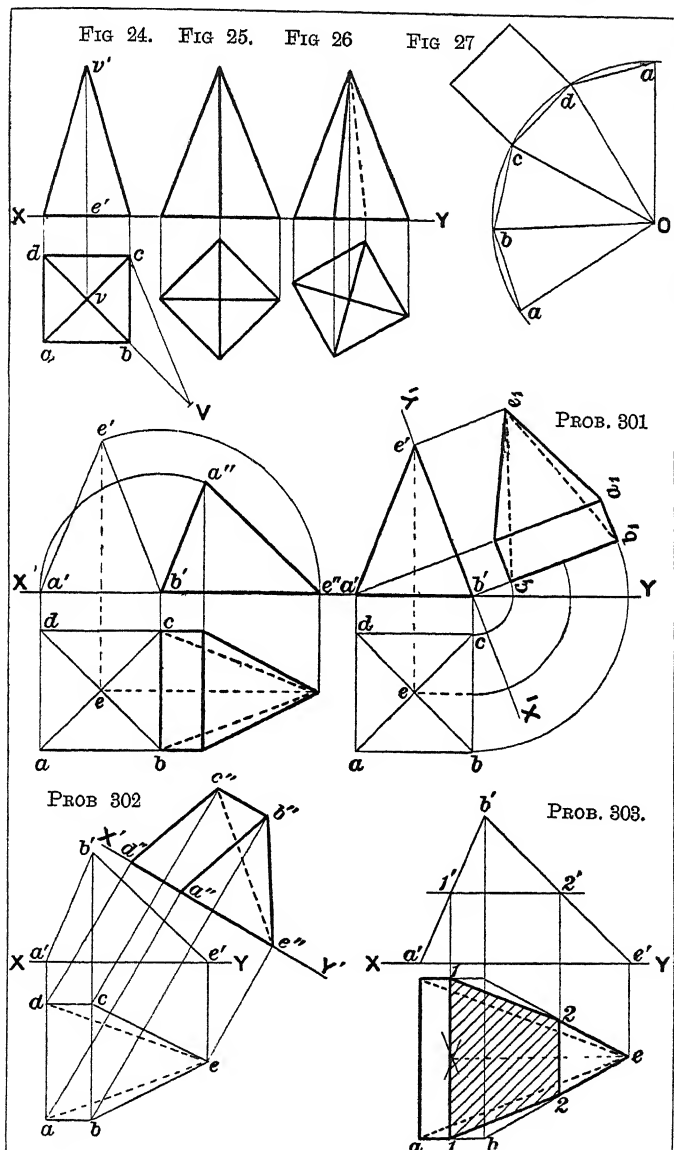
For the *plan*, project from  $a'$ ,  $b'$ , and  $e'$ . Set off  $b'c_1$ ,  $b'b_1$ , etc.  $= b'c$  and  $b'b$ , and complete as shown. Remember that  $c_1$  must be the same distance from  $X_1Y_1$  that  $c$  is from  $XY$ .

**PROBLEM 302**—The plan and elevation of a square pyramid are given. Draw a new elevation on  $X'Y'$ .

Project from each point of the plan. Points  $a$ ,  $d$ , and  $e$  are on the H.P.; their elevations will be  $a''$ ,  $d''$ , and  $e''$  on  $X'Y'$ .  $b''$  and  $c''$  will be the same height above  $X'Y'$  that  $b'$  is above  $XY$ . Draw  $a''b''$ ,  $b''c''$ , and  $c''d''$ . Join  $c''$  and  $b''$  with  $e''$ .

**PROBLEM 303**—Draw the sectional plan of the square pyramid of which  $a'b'e'$  is the elevation.

First project the plan of the pyramid, and then the section. The portion of the pyramid which is supposed to be removed is left in light line.





**PROBLEM 304**—Two elevations of a square prism cut by vertical planes 1' 2' are given. Draw new elevations, showing the true shape of the sections.

*Remember that a new elevation must be projected from the plan*

1 Obtain the plan  $abcd$ , and show the section plane, cutting the edges at 1, 2, 3, 4. Draw  $X'Y'$  parallel to the plan of the section plane, and project the elevation as shown

2 As before, obtain the plan, and show where the section plane cuts the edges at 1, 2, 3. Draw the new ground-line parallel to 1 3. Project the elevation. Make 2'' the same elevation above  $X''Y''$  that 2' is above  $X'Y'$

**PROBLEM 305** —The plan and elevation of a square pyramid cut by a vertical plane, 1 4, are given. Draw a sectional elevation on 1 4.

Draw a new ground-line parallel to 1 4, and project the new elevation of the pyramid. Now project from each point of the section 1 and 4 are in the H P, and their elevations will be on  $X'Y'$ . 2 is on  $ed$ , then 2' will be on  $e'd''$ . Similarly, 3' will be on  $e''a''$ . Complete the elevation as shown.

**PROBLEM 306**—Draw the plan and elevation of an equilateral triangular pyramid, when one triangular face is horizontal, and when the axis is parallel to the V.P.

First draw the plan and elevation when standing with its base on the H P. The plan will be the equilateral triangle  $abc$ , placed so that  $bc$  is perpendicular to  $XY$ . If we now imagine the pyramid turned upon  $bc$ , until the face  $BCD$  is in the H P, then the axis of the pyramid will be parallel to the V P. Project the elevation  $a'b'd'$ .

Now proceed as in Problem 301, by drawing a new ground-line  $X_1Y_1$  through  $b'd'$ . Then  $a'b'd'$  will be the elevation, the face  $b'c'd'$  being in the H P. Project the plan  $c_1b_1d_1$  as shown.

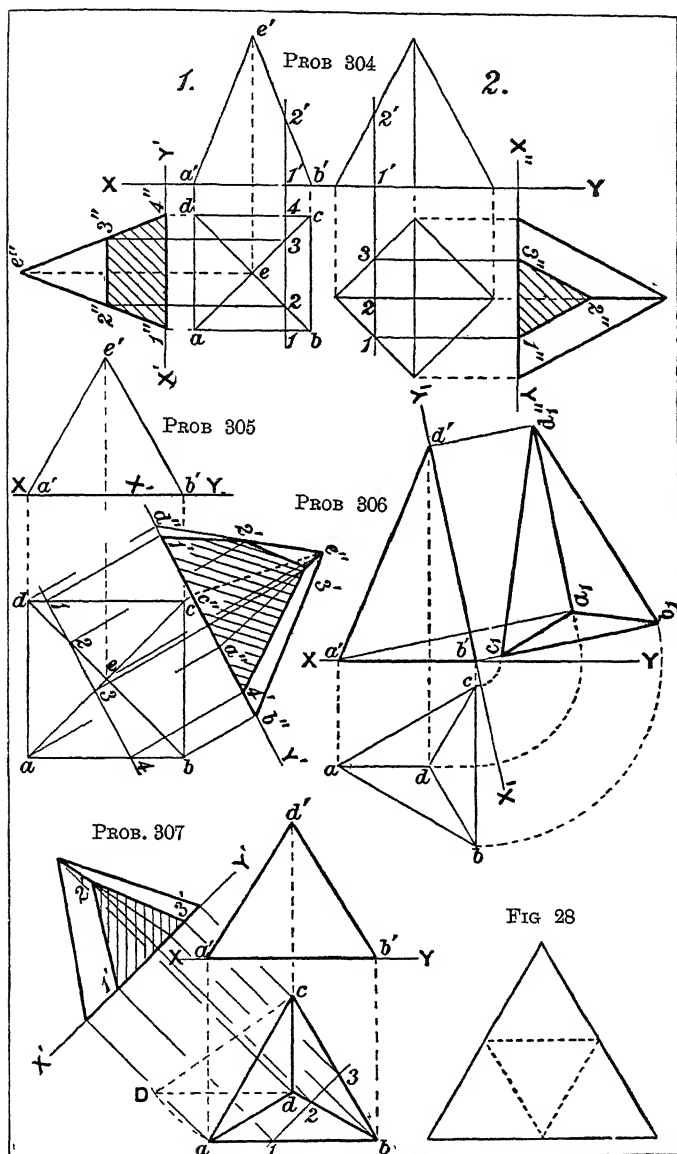
**PROBLEM 307**—Draw the plan and elevation of a tetrahedron—

- 1 When one face is horizontal, and one edge parallel to the V P—
- 2 Draw a sectional elevation on 1 3.

The tetrahedron is a triangular pyramid, each of its four faces being an equilateral triangle. Its development is shown in Fig 28.

For the plan, draw the equilateral triangle  $abc$ . For the elevation, project from  $a$ ,  $b$ , and  $c$  to  $XY$ . To obtain the altitude, draw  $dD$  perpendicular to  $dc$ . As all the edges of the solid are equal, make  $cD = ca$ . Then  $cD$  represents  $CD$  rabatted about its plan  $cd$  into the H P.  $dD$  will be the altitude.

2 Draw a new ground-line parallel to 1 3. Project the tetrahedron, obtaining the height as before. Obtain the points of section as shown.



**PROBLEM 308.**—A right hexagonal pyramid stands with its base on the H P, and one of the edges of the base inclined at  $20^\circ$  to the V P. Draw its plan and elevation when cut by a horizontal plane midway between its base and its vertex.

The hexagon  $abcdef$ , with  $cd$  making  $20^\circ$  with  $XY$ , will be the plan of the pyramid. Project the elevation as shown. Draw the section plane midway between the vertex and the base. From the points  $1'$  and  $6'$  where the plane cuts the edges  $v'f'$  and  $v'e'$ , project to the plans of those edges. From  $1$  and  $6$  draw parallels to the sides of the base, and complete.

**Note**—The lower portion of a pyramid formed by cutting off the top is called a *frustum*.

**PROBLEM 309.**—The elevation of a right hexagonal pyramid is given. Obtain the plan, also draw an elevation showing the true shape of the section, when the pyramid is cut by the vertical plane  $1'2'$ .

1. Project from  $a'$ ,  $b'$ , and  $c'$ . Take any point  $e$  and draw  $ed$  and  $ef$  at  $30^\circ$  with  $XY$ . Draw  $da$  and  $fc$  parallel to  $ed$  and  $ef$ , and complete the plan.

2. The new elevation must be projected from the plan, therefore obtain the plan of the section plane. Draw  $X'Y'$  parallel to  $22$ . Project the new elevation of the pyramid and find where the section plane cuts the edges. Notice that  $1''$  will have the same elevation above  $X'Y'$  that  $1'$  has above  $XY$ .

**PROBLEM 310**—Draw the plan and elevation of a right hexagonal pyramid when one triangular face lies on the H P., and the axis is parallel to the V.P.

Draw the plan  $abcdef$  and the elevation  $f'b'v'$  when the pyramid is standing on the H P. By referring to the model it will be seen that the plan must be arranged with  $bc$  at right angles to the V.P., so that when turned about  $bc$ , the face  $VBC$  will rest on the H P. with the axis of the pyramid parallel to the V.P.

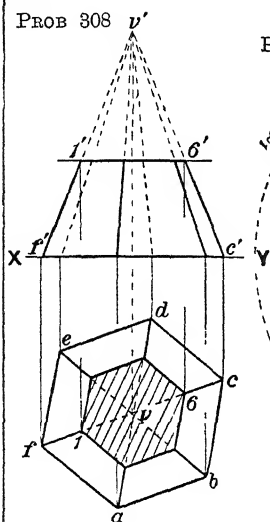
Draw a new ground-line  $X_1Y_1$  through  $b'v'$ , then if the paper be turned it will be seen that  $b'f'v'$  will represent the elevation of the pyramid when one face is in the H P., and the axis is *parallel* to the V.P. Project from  $f'$ ,  $a'$ ,  $b'$ , and  $v'$  for the plan as shown, remembering that  $a_1$ ,  $b_1$ ,  $c_1$ ,  $d_1$  are the same distance below  $X_1Y_1$  that  $a$ ,  $b$ ,  $c$ ,  $d$  are below  $XY$ .

**PROBLEM 311.**—The elevation,  $a'c'v'$ , of a hexagonal pyramid having its axis parallel to the V P is given. It is cut by a horizontal plane  $1'3'$ . Obtain the sectional plan.

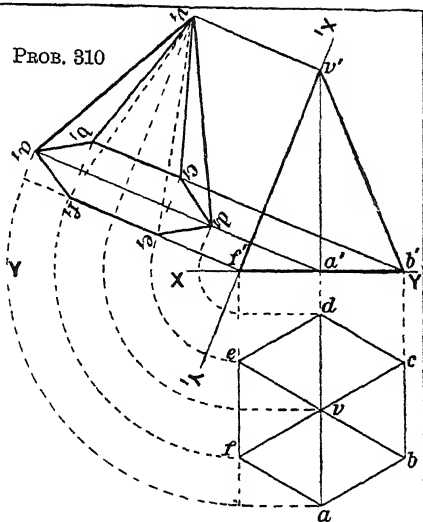
It will be necessary to obtain the true shape of the base by rabatment it into the V.P. Project from  $a'$ ,  $b'$ ,  $c'$ . From  $b'$  draw lines to meet the projectors from  $a'$  and  $c'$  at angles of  $60^\circ$ . Make  $AA = A'b'$ , and complete the hexagon.

Project the plan of the pyramid, making  $ba$ ,  $aa$ ,  $ab = a'A$ ,  $AA$ ,  $AB$ . For the section, project from  $1'$  to  $av$ ,  $av$ , from  $2'$  to  $bv$ ,  $bv$ , and from  $3'$  to  $cv$ ,  $cv$ . Join the points thus obtained, and complete.

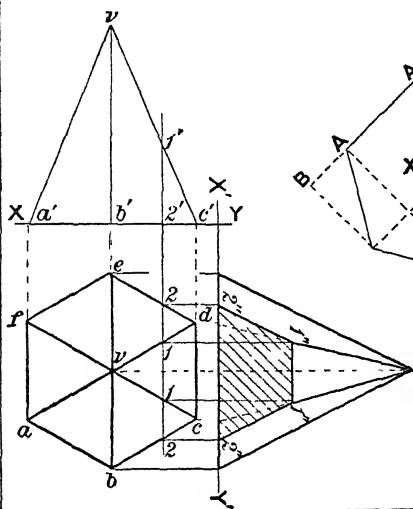
PROB 308



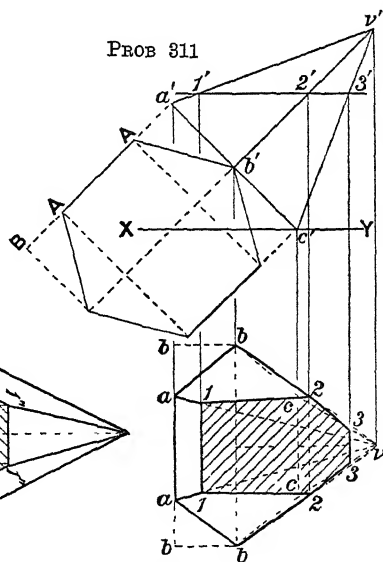
PROB. 310



PROB 309



PROB 311



**PROBLEM 312**—To construct the plan and elevation of an octahedron:—1 When one of its axes is vertical. 2 When one of its triangular faces is in the horizontal plane

**Note**—The octahedron has eight faces, each of which is an equilateral triangle. It is formed of two square pyramids placed base to base, and all its edges are equal. It has three diagonals, all of equal length.

1 Draw  $ad$  at any given angle to  $XY$ . On it construct a square and draw the diagonals. This will be the plan. For the elevation draw a projector from  $e$ , and make the axis  $e'e'$  equal to  $ac$ . Bisect  $e'e'$  by  $a'c'$  and draw projectors from each angle of the plan. Join the points  $a', b', c'$ , and  $d'$ , with the points  $e', e'$ .

2. As each face is an equilateral triangle, draw  $abc$ ; this will be the plan of the bottom face. Describe a circle about the triangle, and draw the hexagon  $ae$ . Draw  $de, ef, fd$ .

Project the elevation of the face  $adb$ . To obtain the height, with centre  $b'$  and radius equal to an edge of the solid, as  $ab$ , cut the perpendicular from  $d$  in  $d'$ . Join  $d'$  with  $a'$  and  $b'$ . From  $d'$  draw  $d'e'$  parallel to  $XY$  to meet the projector from  $e$ . Join  $e$  and  $b'$ .

**Note**—The student should make an octahedron to illustrate these positions. If  $ab$  (Fig 29) be the edge of the solid, develop eight equilateral triangles as shown, on cardboard or thick paper. Cut out the shape of the development, and fold the edges of the triangles along the dotted lines.

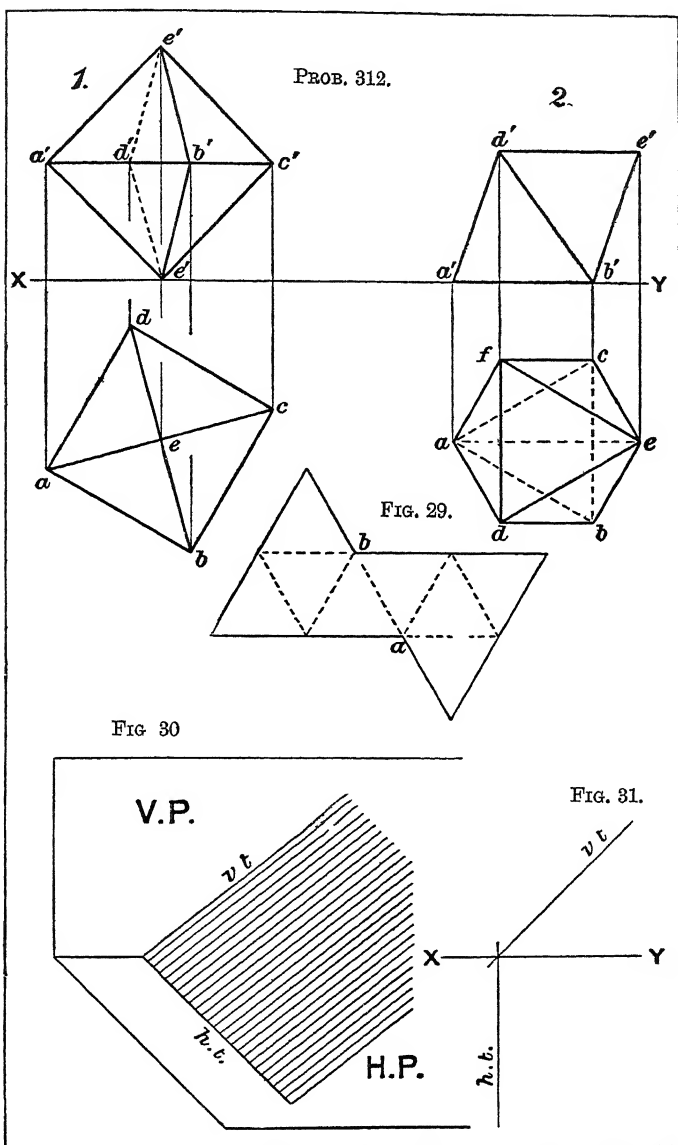
## PROJECTIONS WHEN THE FACE OF THE FIGURE IS INCLINED TO THE HORIZONTAL PLANE

We have so far dealt mainly with Plans and Elevations of figures standing with a face in the H.P., and with Sections made by vertical and horizontal cutting planes. Projections inclined to the V.P. have already been dealt with by alteration of the Ground-Line (see page 134)

In representing surfaces inclined to the two co-ordinate planes, it is more convenient to *first* assume that the plane upon which the surface rests is perpendicular to the V.P., and inclined *only* to the H.P. Such a plane is often spoken of as a *singly* or *simple inclined plane*. Fig 30 shows a pictorial representation of this plane, which may be extended indefinitely on both sides of the two co-ordinate planes

The lines where this plane meets the two co-ordinate planes are called its *traces*, and the plane is usually represented by them, as shown in Fig. 31, where the vertical trace (*v.t.*) shows the inclination to the H.P., and the horizontal trace (*h.t.*) is perpendicular to the V.P.

It is obvious that the elevation of any surface lying on this plane will be in the vertical trace. If other elevations, inclined to the V.P., are required, they may be easily obtained by altering the ground-line.



**PROBLEM 313.**—To draw the plan and elevation of an isosceles triangle (base  $1\frac{1}{2}''$ , altitude  $2''$ ) when the base is in the H.P., and the plane of the triangle is perpendicular to the V.P., and inclined at  $40^\circ$  to the H.P.

**Note.**—To illustrate this problem fold a piece of paper and fasten it with a pin to the H.P. with the crease perpendicular to the V.P. If a triangle be drawn on the paper it may be arranged in the positions mentioned in the problem.

**First Method**—Draw the traces of a plane inclined at  $40^\circ$  to the H.P. As the base of the triangle is in the H.P., it must be on the horizontal trace of the plane in which the triangle lies. Draw the triangle  $a b c$ . This will be the plan of the triangle when the inclined plane is rabatted into the H.P., and  $b' c'$  will be its elevation. If now the plane be rotated on its horizontal trace until it is inclined at  $40^\circ$ , then the point  $C$  will move along the arc to  $c'$ , and  $b' c'$  will be the required elevation of the triangle. Project from  $c'$  to meet  $c c$ , the parallel to  $X Y$ . Then  $a b c$  will be the plan of the given triangle.

Other projections at different angles may be obtained by drawing new ground-lines. Thus  $a'' b'' c''$  shows a new elevation of the triangle on  $X' Y'$ . This elevation is inclined to both H.P. and V.P.

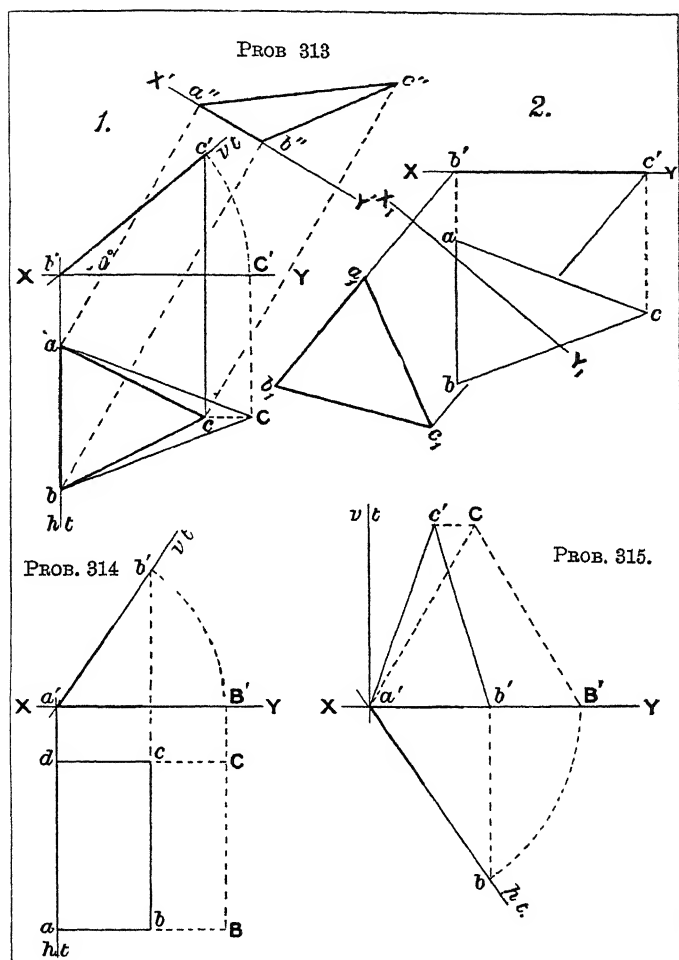
**Second Method.**—This problem may also be solved by changing the ground-line. Draw the plan  $a b c$  and the elevation  $b' c'$  of the triangle when lying with its surface in the H.P. Draw  $X_1 Y_1$  in any convenient position making  $40^\circ$  with  $X Y$ .  $X_1 Y_1$  may be regarded as the intersection of a new horizontal plane with the V.P. As a new plan is required, remember that it must be projected from the elevation. Project from  $b'$  and  $c'$ . Make  $a_1, b_1$ , and  $c_1$  the same distance below  $X_1 Y_1$  that  $a, b$ , and  $c$  are below  $X Y$ . Then  $a_1 b_1 c_1$  and  $b' c'$  will be the required plan and elevation, and will be found to correspond exactly with  $a b c$  and  $b' c'$  in the first solution.

**PROBLEM 314.**— $a b c d$  is the plan of a square with the edge  $a d$  in the H.P. Find the inclination of the square to the H.P.

Through  $a d$  draw the horizontal trace of the plane containing the square. If this plane be turned into the H.P., the plan of the square will be shown in its true shape by the square  $a B C d$ , and its elevation by the line  $a' B'$ . If the plane be now rotated on its horizontal trace, until  $B'$  meets the projector from  $b$ , the elevation of  $b$  will be obtained. Draw the vertical trace from  $b'$  through  $a'$ . The angle  $b' a' B'$  will be the required inclination.

**PROBLEM 315.**—The elevation  $a' b' c'$  of an equilateral triangle standing in a vertical plane is given. Find its inclination.

Draw  $v t$ , the vertical trace of the plane containing the triangle. If the triangle  $a' b' c'$  be rabatted into the V.P., its elevation will



be the equilateral triangle  $a' B' C'$  If  $a' B'$  be now rotated until  $B'$  falls on the projector from  $b'$ , the horizontal trace of the plane containing the triangle will be found, and the inclination to the V P will be the angle  $B' a' b$ .

**Note.**—Compare this solution with that of Problem 314.



**PROBLEM 316** — Draw the plan and elevation of a right triangular prism 2 5'' long, with equilateral ends of 1 5'' edge, when one of the rectangular faces is inclined at  $40^\circ$  to the H.P., and one of its short edges is perpendicular to the V.P. Also show the form of a section made by a H.P. passing through a point 0 5'' below the highest point

Draw  $vt$  and  $ht$ , the traces of a plane inclined at  $40^\circ$  to the H.P. The rectangle  $dABe$  will be the plan of the prism when one rectangular face is in the H.P., and  $d'A'$  will be the elevation of  $dA$ . Turn  $d'A'$  into the vertical trace  $d'a'$ . For the height of the prism rabatte the end, of which  $de$  is the plan, into the H.P. This will be the equilateral triangle  $deF$ . Make  $a'c'$  equal to the altitude of this triangle, and complete the elevation. Now project each end of the elevation, and from this complete the plan.

For the section find a point 0 5'' below  $c'$ . Draw the section plane  $1'2'$ , and project as shown

**PROBLEM 317** — The plan,  $fedcv$ , of a right hexagonal pyramid is given. The edge,  $ab$ , is in the H.P. Draw an elevation on X.Y.

Through  $ab$  draw the horizontal trace of the plane containing the base of the pyramid. Turn the base about the horizontal trace into the H.P. This will be the hexagon  $abCDEf$ , the plan of the base when horizontal. Project to X.Y., giving  $E'a'$ , the elevation of the base when in the H.P. Rotate  $a'E'$  until it meets the projector from  $e$  in  $e'$ . Through  $e'$  draw the vertical trace of the plane upon which the pyramid stands. Project from  $f'$  and  $v$  for  $v'$ , and complete the elevation.

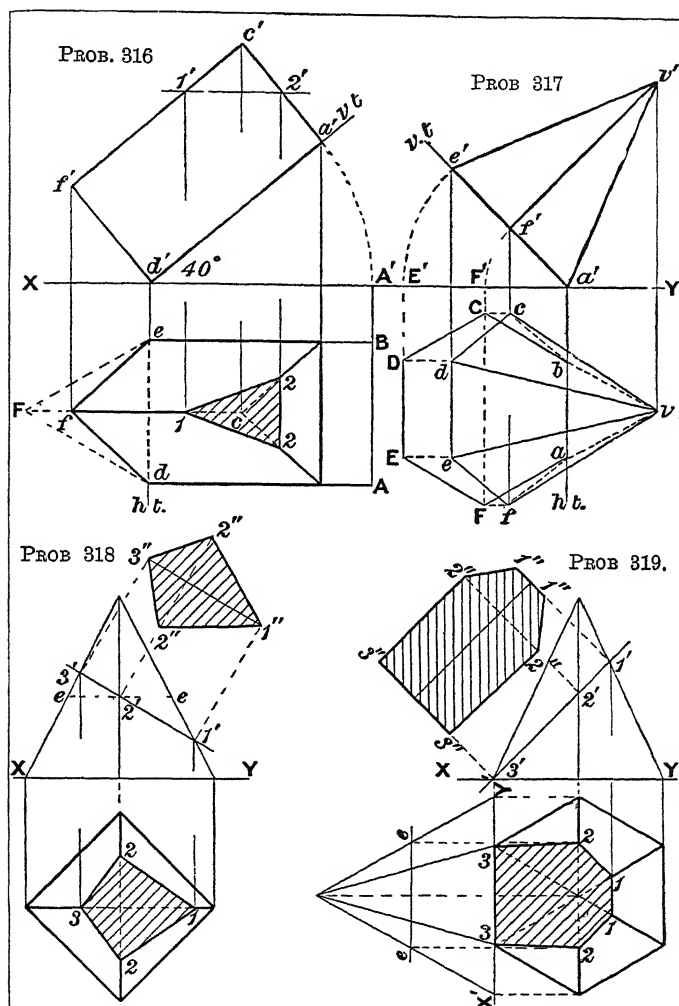
**PROBLEM 318.** — The elevation of a square pyramid cut by an inclined plane  $1'3'$  is given. Draw the sectional plan, and the true shape of the section.

Draw the plan, and project from  $1'$  and  $3'$  for  $1$  and  $3$ . It is evident that  $2'$  cannot be projected directly to the plan, as the elevation and plan are in a vertical line, as, however, the diagonals of the base are at right angles, a horizontal line  $ee$  drawn through  $2'$  will give the width of the section at that point. Make  $22 = ee$ , and complete as shown.

The true shape is obtained either by rabatting the elevation into the V.P., using the diagonal  $1'3'$  as a hinge, or by rotating it into the H.P. The former method is used here as being more convenient. Make  $2''2'' = 22$ .

**PROBLEM 319.** — The elevation of a hexagonal pyramid cut by an inclined section plane  $1'3'$  is given. Obtain the sectional plan, and the true shape of the section.

Project the plan of the pyramid and obtain  $11$  and  $33$ . To obtain the width of the section at  $2'$ , it is necessary to project a side



elevation on  $X' Y'$  Draw  $e e$  the same height above  $X' Y'$  that  $2'$  is above  $X Y$  Make  $2 2 = e e$ , and complete

For the true shape make  $1'' 1'', 2'' 2'', 3'' 3'' = 1 1, 2 2, 3 3$  respectively

## PROJECTION OF CURVED BODIES

## THE CIRCLE

In projecting a curve, it is usual to determine the projections of a number of points in it, and then draw the curve by freehand through these points.

It is not necessary to show the projections of the circle when parallel to either of the co-ordinate planes, or when perpendicular to both of them. When inclined, one or both of the projections may be ellipses.

**PROBLEM 320.**—A circle is in a vertical position with its surface inclined at  $45^\circ$  to the V.P. Draw its plan and elevation

Draw the elevation  $A'B'$  and the plan  $ab$  when the surface is parallel to the V.P. Divide the circumference into any number of equal parts (8 or 12 most convenient), and mark the plans  $c, d,$  and  $e$  of the points  $C', D', E'$  on  $ab$ . Draw  $X'Y'$ , the ground-line of a vertical plane, inclined at  $45^\circ$  to  $ab$ . If we now imagine the paper turned so that  $X'Y'$  is horizontal, then  $ab$  will be the plan of a circle inclined to  $X'Y'$  at  $45^\circ$ . From each point in the plan project as shown, making each point the same height above  $X'Y'$  as the corresponding point is above  $XY$ . The ellipse drawn through the points thus obtained will be the required elevation.

**PROBLEM 321.**—A circle lies in a plane inclined at  $45^\circ$  to the H.P. Draw its plan and elevation.

**First Method.**—Draw the traces of a plane inclined at  $45^\circ$  to the H.P. The circle  $AB$  is the plan, and the line  $A'B'$  is the elevation when the circle is in the H.P. If we now turn each point of this elevation into the vertical trace, then  $A'b'$  will be the required elevation, and the ellipse  $Ab$  the plan.

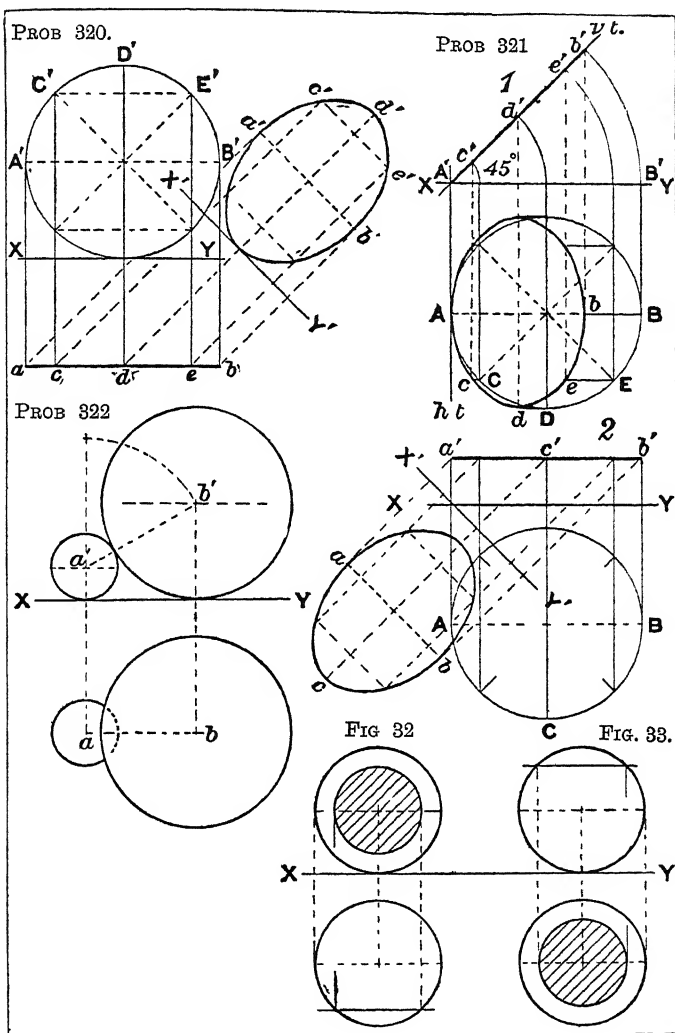
**Second Method.**—Draw the plan  $AB$  and the elevation  $a'b'$  when the circle is in or parallel to the H.P. Draw  $X'Y'$  inclined at  $45^\circ$  to  $a'b'$ . Then  $a'b'$  may be regarded as the required elevation. Project from  $a'b'$  for the plan, which will be the ellipse  $ab$ .

## THE SPHERE

**PROBLEM 322.**—Two spheres of 3" and 1" diameter respectively are lying in contact with each other on the H.P. The line joining their centres is parallel to the V.P. Draw the plan and elevation

As the line joining the centres of the spheres is parallel to the V.P., the elevation will be two circles touching each other

With a radius of 0.5" describe a circle on  $XY$ . This will be the elevation of the smaller sphere. Draw a parallel to  $XY$  at a distance of 1.5". From centre  $a'$  with a radius of 2" ( $1.5'' + 0.5''$ ) describe an arc cutting the parallel in  $b'$ . From  $b'$  describe a circle having a radius of 1.5", giving the elevation of the larger sphere.



For the plan, project from  $a'$  and  $b'$ . Draw  $ab$  parallel to  $XY$ . From centres  $a$  and  $b$  describe circles

Figures 32 and 33 show sections of the sphere made by vertical and horizontal planes respectively.

**PROBLEM 323.**—The plan of a sphere cut by a vertical section plane 15 is given. Draw the sectional elevation.

Project the elevation of the sphere. Describe a semicircle on 15, this will represent half the shape of the section when turned into the H P. Divide it into four equal parts, and from each point draw perpendiculars to 15, giving the widths for the elevation. Project from 1, 2, 3, 4, 5, and on each side of  $a'b'$  set off 22, 33, etc., giving the points 2', 3', 4'. Through these points draw the ellipse and complete the figure.

**PROBLEM 324.**—A sphere is cut by an inclined plane 1'5'. Draw its sectional plan.

The solution, which is similar to that employed in Problem 323, may be easily followed from the figure.

### THE CYLINDER AND CONE

Figures 34 and 35 show plans and elevations of the cylinder and cone in simple positions, and explain themselves.

**PROBLEM 325.**—A cylindrical roller, 2' long and 1 5' diameter, has its axis inclined at  $30^\circ$  to the V P. Draw its elevation. Scale 1" to 1' 0"

This is an application of Problem 320, and may be easily followed. First draw the elevation and plan of the roller when its axis is perpendicular to the V P. Now draw the ground-line  $X'Y'$ , making  $30^\circ$  with the axis. Project each end as in Problem 320, and complete the figure.

**PROBLEM 326.**—A cone has the diameter of its base 2 5'', and its altitude 3''. Draw its plan and elevation when lying on its side with its axis parallel to the V P.

First draw the plan and elevation when its base is in the H P. If we now draw a new ground-line  $X'Y'$  along the side of the cone, and imagine the elevation turned so that  $X'Y'$  is horizontal, then  $a'b'v'$  will be the elevation of the cone when lying on its side. For the plan, draw the axis parallel to  $X'Y'$  and project as in Problem 325.

**PROBLEM 327.**—A cone has its axis parallel to the V P, and its base inclined at  $60^\circ$  to the H P. 1. Draw its plan and elevation. 2. Draw an elevation when its axis is perpendicular to the V P.

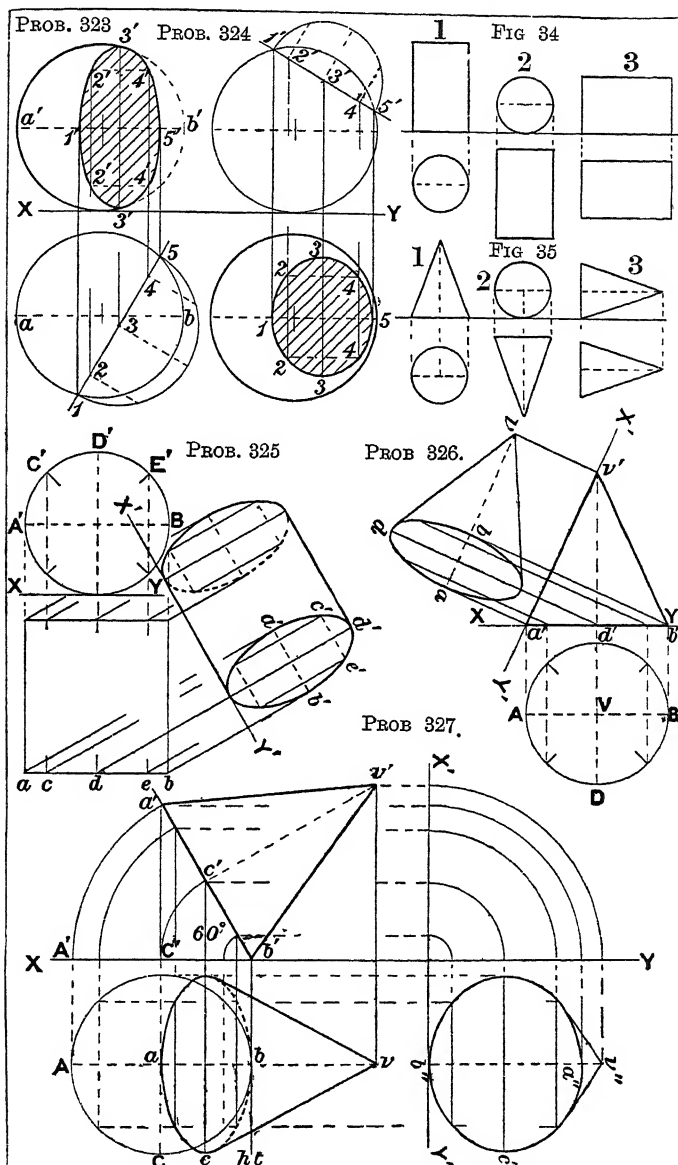
1. Draw the vertical and horizontal traces of a plane inclined  $60^\circ$  to the H P. (Probs 313 and 321) The circle  $A b$  will be the plan, and the line  $A'b'$  will be the elevation of the base of the cone when standing on the H P. Rotate the circle into the vertical trace, set off the altitude of the cone, and complete the elevation.

For the plan, project from each point as shown, and draw tangents from  $v$  to the sides of the ellipse.

2. Draw the new ground-line  $X'Y'$  perpendicular to the axis. As a new elevation is required, project from the plan, obtaining the elevation of each point from the first elevation.

**Notes.**—1 A semicircle described upon  $a'b'$  will also give half the shape of the base of the cone, from which the measurements for the ellipse  $a b$  may be obtained.

2. A cylinder with its axis inclined may be projected similarly.



**Sections of the cylinder.** When the cylinder is cut by a plane perpendicular to the axis, the section is a circle as in Fig. 36

When the section plane is parallel to the axis, the section is rectangular, as in Figs. 37 and 38. To obtain the width of the section in Fig 38, it is necessary to rabatte the end into the V P and transfer  $aa$  to the plan ( $11 = aa$ )

When the section plane is inclined to the axis, the section is elliptical.

**PROBLEM 328**—The elevation of a cylinder cut by the plane  $1'4'$  is given. Draw the sectional plan, and the true shape of the section.

The plan of the cylinder will be a circle. Project from  $4'$  for the section.

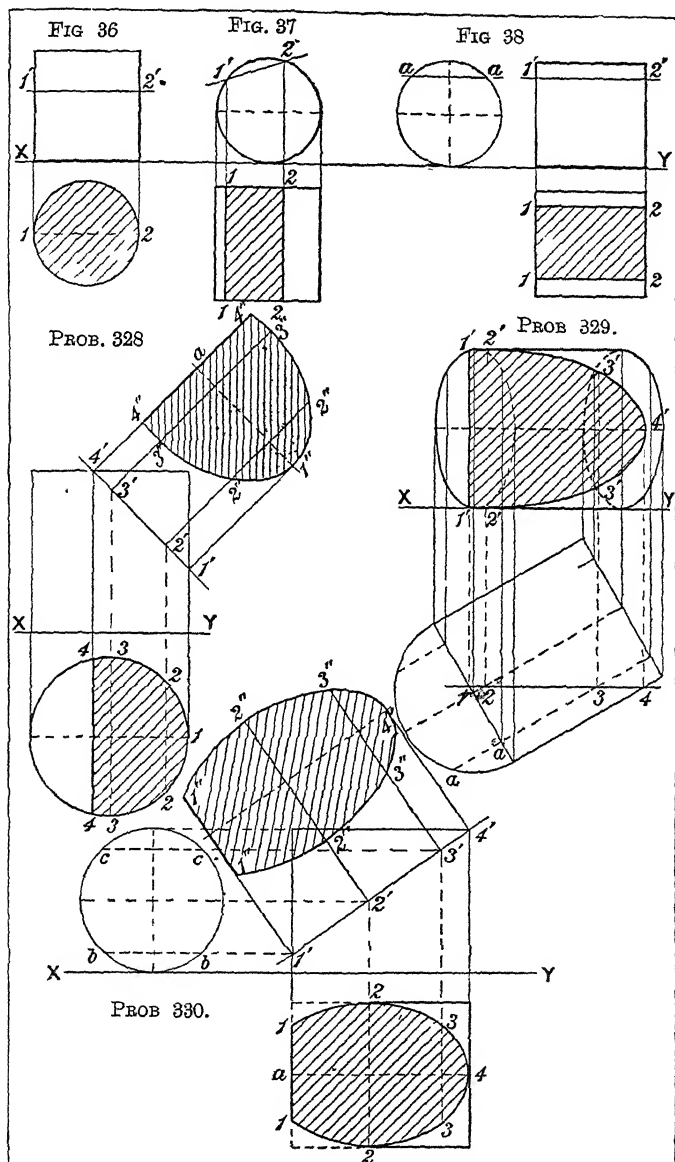
The true shape of the section will be a segment of an ellipse. Draw  $1''a$  parallel to  $1'4'$ , and project from  $1'$  and  $4'$ . Make  $4''4'' = 44$ . Draw  $33$  through the centre of the plan and obtain the elevation  $3'$ . Project from  $3'$ , and make  $3''3'' = 33$ , giving the greatest width of the section. To obtain other points in the curve, take points  $22$  in the plan, and project, making  $2''2'' = 22$ . Through the points thus obtained draw the curve.

**PROBLEM 329.**—The plan of a cylinder cut by a vertical plane  $14$  is given. Obtain its elevation.

First obtain the elevation of the cylinder as in Problem 325. Half only of the true shape of the end is shown, from which the measurements for the ellipses may be obtained. Project from  $4$  to the centre line of the elevation for  $4'$ , from  $1$  to the face giving  $1'1'$ . From  $2$ , the point where the section plane cuts the axis, project to the elevation for  $2'2'$ . If the ordinate  $aa$  of the semicircle be produced to cut the section line in  $3$ , two more points in the curve may be obtained. Project from  $3$ , and set off the distance  $aa$ , on each side of the centre line for the points  $3'3'$ . Through the points obtained draw the curve, and complete

**PROBLEM 330**—The elevation of a cylinder cut by an inclined plane  $1'4'$  is given. Draw the sectional plan, and also the true shape of the section

Project the plan of the cylinder, which will be a rectangle, and draw the axis  $a4$ . Rabatte the end into the V.P. to obtain the width of the section. Set off  $11 = bb$ , the width where the section plane cuts the end. Project from  $2'$ , the widest part of the section, for  $22$ . To obtain other points on the curve, take any point  $3'$  and project, making  $33 = cc$ . Through the points obtained draw the curve, and complete. The true shape is obtained as in Problem 328





There are five different figures formed (conic sections) when the cone is cut by section planes —

When the cutting plane is parallel to the base, the section is a circle, as in Fig 39

If the cutting plane passes through the apex and any part of the base, the section is a triangle, as in Fig 40, where the true shape of the section and the sectional plan are shown.

The other three sections are the ellipse, the parabola, and the hyperbola, which have previously been explained on page 110

**PROBLEM 331** —The plan of a cone cut by a vertical section plane 17 is given. Find the elevation

Draw the elevation of the cone, and project points 1 and 7 of the section to *XY*. Next obtain the height of the section. With centre *O* and radius *O4* describe a circle (Half only is shown, to avoid confusion) From *a*, the point where the circle cuts *Od*, project to the elevation for *a'*. Through *a'* draw a horizontal line representing the elevation of the circle, cutting the axis in *4'*, and giving the highest point of the section. To obtain other points in the curve, mark 2 and 3, and describe circles through them from *O*. Project from *b* and *c*, where these circles cut *Od*, and draw the elevations at *b'* and *c'*. Projectors from 2, 3, 5, and 6 to meet these lines will give the required points of the curve. Draw the curve, which is a hyperbola, and complete.

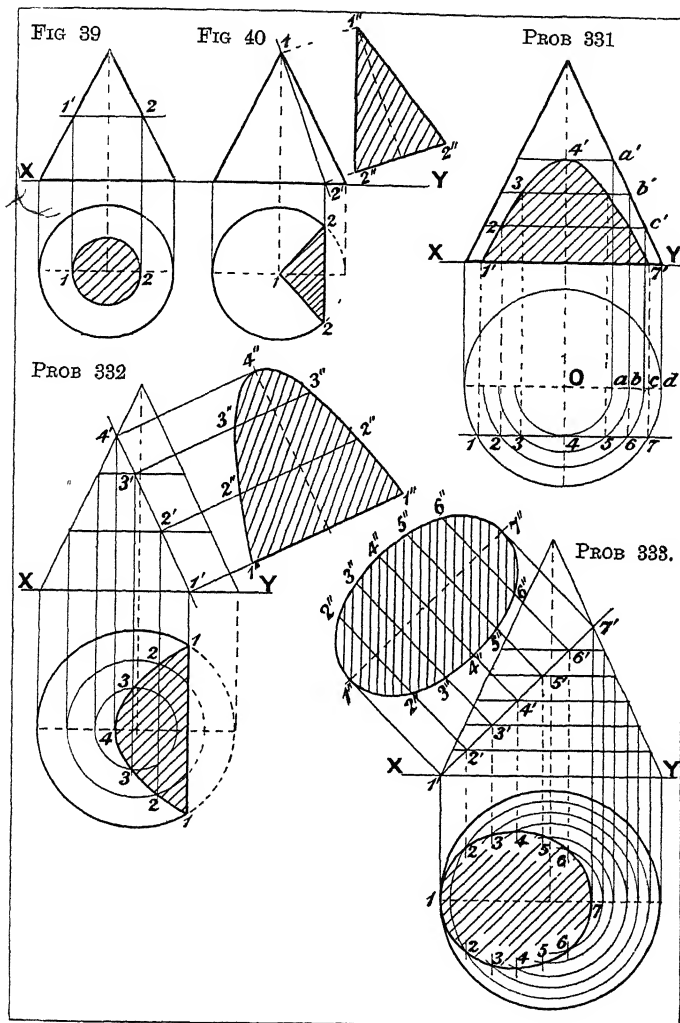
**PROBLEM 332.**—The elevation of a cone cut by a section plane 1'4' parallel to its side is given. Obtain the sectional plan and the true shape of the section.

Draw the plan of the cone, and find points 11 and 4 of the section. To find other points in the curve, take any number of points, as 2' and 3', and draw horizontal section lines through each point. Obtain the circles on the plan of which these lines are the elevations. Project from 2' for points 22, and from 3' for 33. Through the points obtained draw the curve.

For the true shape, project at right angles to the section line from 1', 2', 3', and 4'. Draw a centre line, and make 1''1'', 2''2'', 3''3'' equal to 11, 22, 33 of the plan. The curve drawn through these points will be a parabola.

**PROBLEM 333.**—The elevation of a cone cut by a section plane 1'7' is given. Draw the sectional plan, and the true shape of the section

Draw the plan of the cone, and find points 1 and 7. Take any number of points, 2', 3', etc., and imagine horizontal sections passing through them. The plans of these sections will be circles. Project from 2' to the plan of the horizontal section passing through it for 22, from 3' to the next circle, etc. Through the points obtained draw the curve



For the true shape, draw  $1''7''$  parallel to  $1'7'$ , and project from each point of the section line. Make  $2''2'' = 22$ , etc. Draw the curve through the points obtained. The curve in both plan and true shape will be an ellipse.

**PROBLEM 334.**—A cone lying on its side is cut by a plane  $1' 4'$  parallel to the ground-line. Draw the plan.

Draw the plan as in Problem 326, and project for points 1 1 and 4 4. Assume other points in the section, as  $2'$  and  $3'$ , and draw sections through them parallel to the base, as before. Draw half the true shape, and erect perpendiculars to the sections at  $2'$  and  $3'$ . Project from  $2'$  and  $3'$ , and set off distances equal to  $2' 2'$  and  $3' 3'$ , on each side of the centre line of the plan, for points 2 2 and 3 3. Draw the curve through the points thus found.

### APPLICATIONS OF SOLID GEOMETRY

**PROBLEM 335.**—The plan and elevation of a lamp shade standing with its smaller end on the H.P. are given. Draw a fresh plan when the shade is lying with its side in the H.P., and with its axis parallel to the V.P.

This is an application of Problem 326. Draw  $X' Y'$  passing through a side,  $a' b'$ , of the shade. If this be now regarded as the edge of the H.P., then  $a' b' c' d'$  will represent the elevation of the shade when lying on its side. Divide the circles in the plan into equal parts and project to the elevation in  $e' f' g'$ , as shown. Project from  $b' e' f' g' c'$ , at right angles to  $X' Y'$ . Draw the axis  $b d$  parallel to  $X' Y'$ . On either side set off the widths, making  $f f = F F$ ,  $g g = G G$ , etc. Draw the larger ellipse, and repeat similarly for the smaller end. Tangents to the ellipses give the sides of the shade.

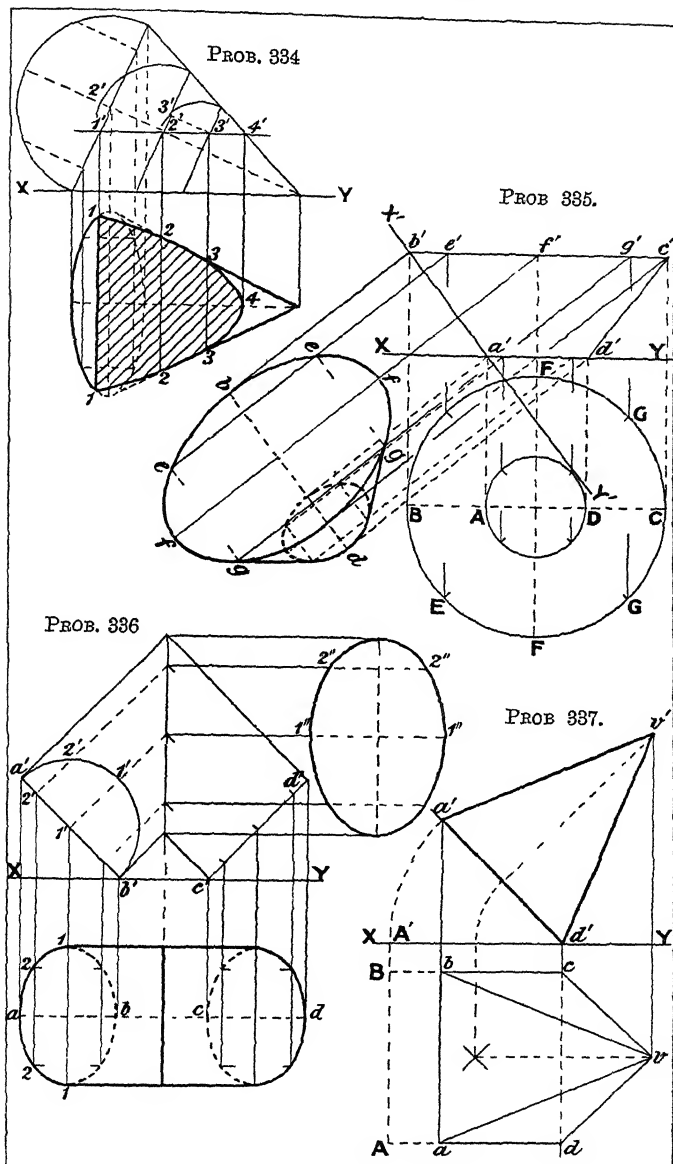
**PROBLEM 336.**—The elevation is given of an elbow formed by two pieces of cylindrical piping. Draw the plan, and also the true form of the intersection of the pipes. Neglect the thickness of the material.

Project the plan of the elbow as shown.

The true form of the intersection will be an ellipse, projected as in Problem 330.

**PROBLEM 337.**—The plan of a square pyramid is given. Draw an elevation on  $X Y$ .

First obtain the plan and elevation of the base when the pyramid is standing on the H.P. This will be the square  $A B C d$ , and the line  $A' d$  (Problem 314). Rotate the elevation  $A' d$  until it meets the projector from  $a$  in  $a'$ . Draw the vertical trace of the plane upon which the pyramid stands. Project from  $v$  to meet the elevation of the altitude, and complete the figure.



**PROBLEM 338**—The roof-plan of a building is given, to a scale of 10' to 1". Draw its elevation upon a vertical plane parallel to  $XY$ , assuming that the height of the walls throughout is 20', and that the slope of the roof throughout is at  $45^\circ$  to the horizontal

Project from each of the corners for the elevation of the walls. To obtain the height of the roof, imagine a vertical section through  $h$  rabatted into the H.P., then  $hH$  will be the required height. Project from  $h$  and  $l$ , set off  $hH$ , and complete the main roof. Similarly,  $eMf$  represents the true shape of the gable. Project from  $m$ , and set off the height  $mM$ , giving  $m'$ . Project from  $g$ , and complete the figure.

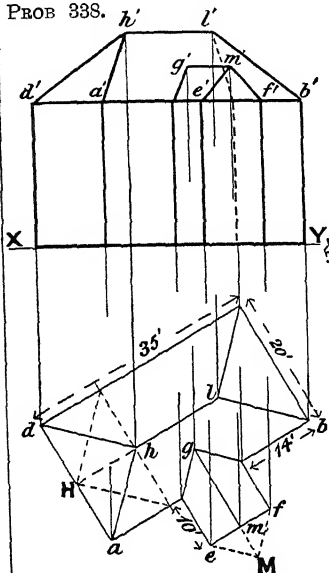
**PROBLEM 339.**—Two similarly shaped mouldings  $P$  and  $Q$  meet at  $MM$ . A cross section of the moulding  $Q$  is given. Show the shape of the section at the joint.

Project from each corner of the moulding to the plan of the section  $MM$ . Draw  $XY$  parallel to the section-plane  $MM$ , and project at right angles from each point on  $MM$ . Make 1', 2', 3', etc., the same height above  $XY$  that 1, 2, 3, etc., are above  $ss$ , and complete the figure, which will show the true shape of the section at the joint  $MM$ .

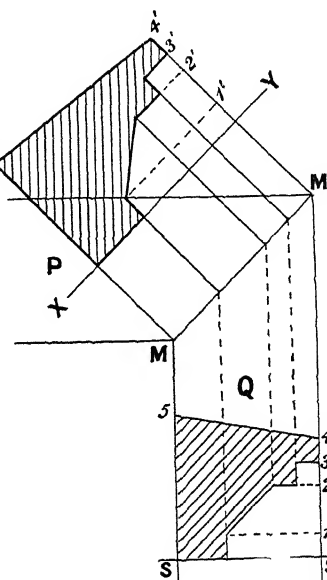
**PROBLEM 340**—The front elevation and section of an equilateral pointed arch-opening in a wall are given. Two steps are also shown. Draw another elevation when the plane of the wall is inclined at  $45^\circ$  to the  $VP$ .

First obtain the plan of the opening and the two steps, making  $bf = 12$ , and  $fc = 23$ . As a new elevation is required it must be projected from the plan. Draw  $X'Y'$  inclined at  $45^\circ$  to  $XY$ . Project from  $a$ ,  $b$ , and  $v$ , and make  $a''$ ,  $b''$ ,  $v''$  the same height above  $X'Y'$  that  $a' b' v'$  are above  $XY$ . To obtain the curve, take other points, as  $g'$  and  $h'$ , the plans of these points will be  $g$  and  $h$ . Project for  $g''$ ,  $h''$ . Through these points draw the curve. Obtain the back lines of the arch by projecting from  $d$ ,  $l$ ,  $c$ , etc. For the edge of the first step, project from  $e$  and  $f$ , obtaining the height from the front elevation. The edge of the second step is on the line of the back of the arch, set off the height, and complete the figure.

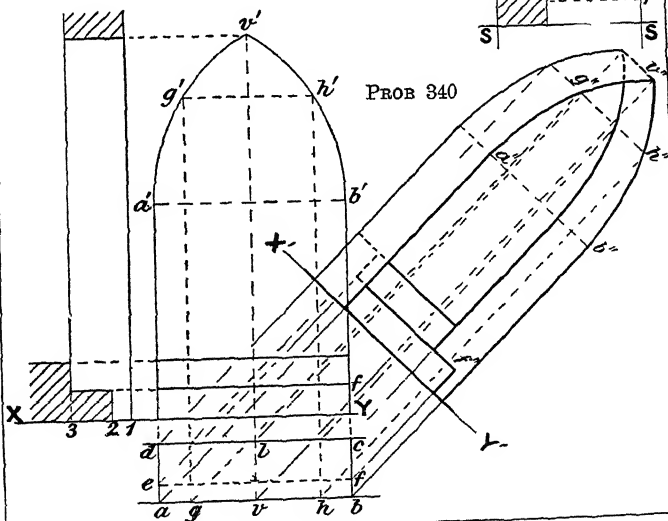
PROB 338.



PROB 339.



PROB 340



**PICTORIAL OR METRIC PROJECTION**

This conventional method of projection is of great convenience, especially in the representation of rectangular objects, as the general appearance is seen at a glance. For example, Fig 41 shows the pictorial projection of a box, the horizontal and vertical lines are drawn correctly to a full-size scale, the oblique lines are drawn at an angle of  $45^\circ$  to a half-size scale. This half-size scale for the oblique lines is usually adopted to avoid the distorted appearance that the object would present were these lines drawn to their full size.

Fig 42 is the projection of a half-lap joint, the two pieces of wood are separated to show the fitting.

Fig 43 shows the projection of a mortise and tenon joint.

**PROBLEM 341.**—Three views are given of part of the corner of a frame, the halves being joined by a dovetailed notch. Draw the pictorial representation in a manner similar to that employed for the cube, in which lines parallel to  $oy$  and  $oz$  are drawn horizontally and vertically to a scale of full size, and lines parallel to  $ox$  are drawn by using the  $45^\circ$  set-square, and to a scale of half-size.

Draw the front view with dimensions equal to those on **A**. From each corner draw lines at  $45^\circ$ , making  $ab$  equal to half the width on **B**. Complete the figure, showing the unseen edges by dotted lines.

**Note**—It is not necessary to copy the diagrams. The dimensions should be taken directly from the figures.

**PROBLEM 342**—Two views of a tusk tenon joint are shown. Represent the portion **A** (removed from **B**) in pictorial projection.

Draw the front view, taking the dimensions from **A**, and proceed as in the previous problem.

**PROBLEM 343.**—Represent the given hexagonal nut in pictorial projection.

Draw the face as before. The construction may be readily followed from the figure.

FIG. 41.

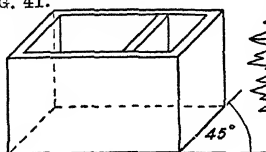


FIG 42

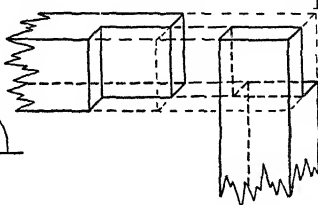
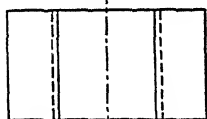
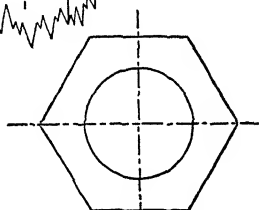
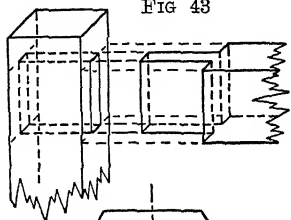
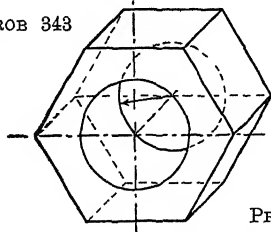


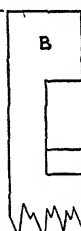
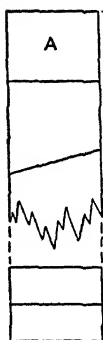
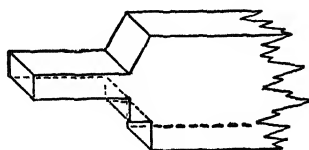
FIG 43



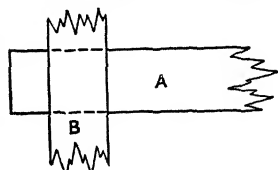
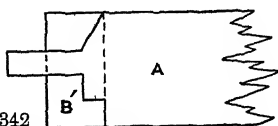
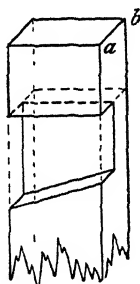
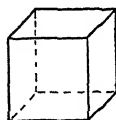
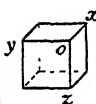
PROB 343



PROB. 342



PROB 341



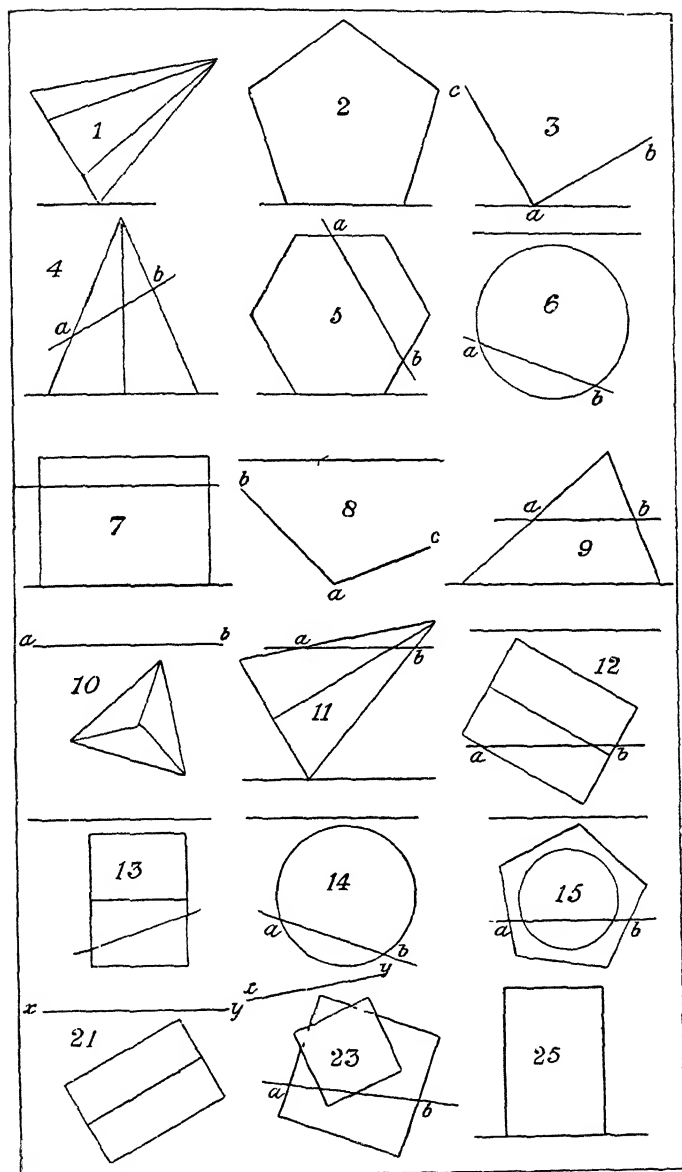


## EXERCISES

## CHAPTER XV

**Notes**—1 Any Figures relating to these Exercises will be found on the opposite page Copy the given figures, at least twice the given size

- 1 The elevation of a right hexagonal pyramid is given. Determine its plan
- 2 Draw the side elevation and plan of a pentagonal prism, 3" long, of which the given figure is the end view.
3. The lines  $ab$ ,  $ac$  are the elevations of two circles Obtain their plans.
4. The elevation of a square pyramid cut by a plane  $ab$ , is given. Draw its plan
- 5 The end elevation of a hexagonal prism,  $1\frac{1}{2}$ " long, cut by a plane  $ab$ , is given. Draw the sectional plan
- 6 The plan of a sphere is given Determine the sectional elevation made by a vertical plane passing through  $ab$
7. The elevation of a cylinder cut by a horizontal plane is given Draw its plan
8. The lines  $ab$ ,  $ac$  are the plans of two unequal equilateral triangles Draw the elevation.
- 9 The elevation of a square pyramid, lying on one of its triangular faces, with its axis parallel to the vertical plane, is given. Find the sectional plan made by a plane cutting the pyramid through  $ab$ .
10. The plan of a tetrahedron is given Find an elevation on  $ab$ .
11. The elevation of a right square pyramid is given. Determine its sectional plan when cut by the plane  $ab$
- 12 Draw the sectional elevation of the square prism when cut by a vertical plane  $ab$ .
- 13 The plan of a cube cut by a vertical plane is given. Find its elevation.
14. The plan of a cone is given. Find the elevation of the section made by the vertical plane  $ab$  (*Prob* 331)
- 15 The given figure represents the plan of a pentagonal prism, 1" in height, surmounted by a cylinder 2" in height Draw the elevation when cut by a vertical plane through  $ab$
16. A pyramid having for its base a square of 2.5" sides, and its axis 3.25" long, rests with one face on the horizontal plane. Draw its plan, and a sectional elevation on a vertical plane, represented by a line bisecting the plan of the axis and making  $60^\circ$  with it. (*Sc*) (*Prob*. 301)
- 17 Draw the plan of the above pyramid when its base is inclined at  $47^\circ$ , and one edge at  $27^\circ$ . (*Sc*)
18. Draw the plan of a right pyramid whose base is a hexagon of 1.25" side, and its axis 3.25", when it stands upright on a horizontal plane Give the section made by a vertical plane which cuts off half one edge and a quarter of the next, measuring downwards from the vertex (*Sc*.)
- 19 Draw the plan of a cube 3" edge, in any position, as long as no edge is horizontal. (*Sc*)
- 20 A pentagon,  $A B C D E$ , side 1.75", revolves upon the line joining  $A$  with the centre of the opposite side, till its plane becomes inclined to the vertical plane at  $50^\circ$  Draw its plan.
- 21 The plan of an equilateral triangular prism is given. Draw its elevation on  $X Y$ . (*Sc*)
22. A rectangle, length 2.5", breadth 1.75", is the plan of a square. Determine the inclination of the plane of the square (*Sc*) (*Prob*. 314.)
23. The plans of two cubes are given, the smaller resting on the larger. Draw an elevation on  $xy$ , and a section on  $ab$ . (*Sc*)
- 24 A square prism, 3" long and 1.5" square, lies with one of its side edges horizontal, and one of its sides inclined at  $30^\circ$  Make an elevation on a vertical plane which is at  $50^\circ$  to the axis. (*Sc*)
- 25 The given rectangle represents the elevation of a square. Determine the inclination of its plane to the vertical plane. (*Sc*.)



## CHAPTER XVI

**THE APPLICATION OF GEOMETRY TO THE SETTING OUT OF SCHEMES OF ORNAMENTAL PATTERNS, THE CONSTRUCTION OF UNITS OF PATTERNS, THE SPACING OF SURFACES FOR DECORATIVE PURPOSES, AND TO THE CONSTRUCTION OF ARCH FORMS, TRACERY, AND MOULDINGS**

THE importance of geometry as the groundwork of design can hardly be over-estimated. All patterns which require repetition must be arranged upon a geometrical foundation. The triangle, square, diamond, hexagon, octagon, and other polygons, with the circle and the ellipse, are the chief forms used in geometrical ornament, and when repeated form good patterns.

*Natural forms*, such as leaves and plants, however, give a much greater and more varied field of treatment. In using natural forms for ornament they must be arranged and adapted according to the nature of the material in which they are to be expressed, and also so as to properly cover the shape which is to be ornamented. For example, the treatment of a plant, used as the *motive* or principle for decorated ironwork, would be very different to its treatment when used for curtain or wall-paper patterns; in the first case the treatment would be more rigid and severe to be in keeping with the metal used, while in the latter it would be more flowing and less formal. This adaptation of natural forms is what is termed *conventional* treatment.

A knowledge of the application of geometry is essential in architecture and window tracery, in wall decoration, in mosaics, parquetry, tiles, floor-cloths, carpets, and rugs; in inlaid wood-work or marquetry, in pottery and coloured glass-work; in metal-work and jewellery; and in many other crafts.

The examples given are necessarily only suggestive of the many varieties of patterns which may be formed, and the student should not rest content with merely copying the given figures (*which should be done to a larger scale*), but should vary the treatment and invent fresh patterns of his own.

For further information the student should consult such works as Lewis Day's "Ornamental Design," Jackson's books on "Design," and Meyer's "Handbook on Ornament."

Most geometrical ornament may be arranged in one of the three following classes—

1 When the ornament is continuous, as in **bands** or borders, such as Figs. 1-19, 39, 43, 57, etc

2 When the ornament is repeated in flat patterns over an unlimited surface as in **diapers**, such as Figs 33, 59, 74, etc

3 When the ornament fills an enclosed space as in **panels**, such as Figs. 112-155, etc.

## EXPLANATION OF TERMS USED

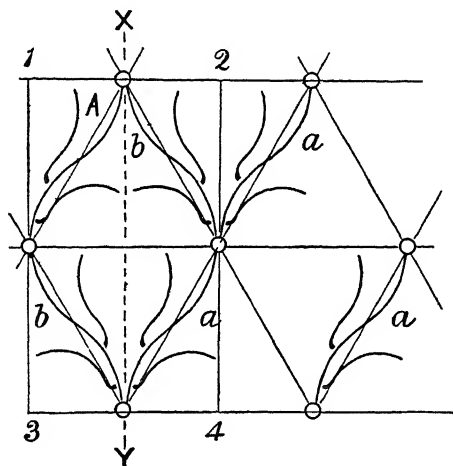
A few terms of frequent occurrence are explained here —

**Repetition** means a succession of the same form. It is the basis of decorative art, but becomes monotonous when not varied; hence the reason for the introduction of the square panels in the fret bands, Figs 13 and 15. In Fig. A the line *A* is repeated at *a, a, a*, while it is reversed at *b, b*; that is, the same form is repeated in an opposite direction, or *contrasted*.

The repetition of any form on its axis produces **symmetry**, one of the most important principles used in producing ornament, thus the repetition of the curved lines on the axis *X Y* produces the like-sided ogee form contained in the rectangle 1 2 3 4.

It is very important that the student should clearly understand what is meant by the **unit of design**, or the "unit of repetition," as it is often termed. In Fig. A the "unit of design" is the ornament

FIG. A



included in the rectangle 1 2 3 4; this ornament when repeated forms an "all-over" pattern.

**Note.**—On many of the following patterns the extent of the unit is indicated by dotted lines.

The construction lines used in the setting out of geometrical patterns for parquetry (inlaid woodwork for flooring), mosaics (patterns composed of stone, glass, clay, etc.), floor-cloths, carpets, window-glazing, wall-decoration, ceilings, etc., are arranged to form a **network**, so as to secure accuracy both of construction and repetition. The nets most frequently used are those formed of squares and of equilateral triangles.

## THE SQUARE NET

The square network may be arranged in two positions — First, as in Fig B, where horizontal lines are crossed at right angles by vertical lines, or as in Fig C, where the lines are drawn at an angle of  $45^\circ$

The best way to draw the first network is to rule horizontal lines at equal distances apart with the T-square, and cut these lines with a line at  $45^\circ$ . Through the points where this line cuts the parallels, rule the vertical lines with the set square. This secures both accuracy and quickness

FIG B

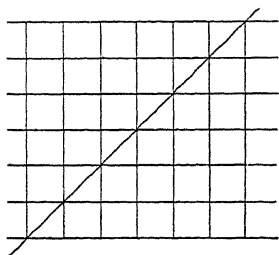
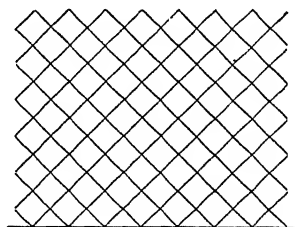


FIG C



For the second net, set off equal divisions on either a horizontal or a vertical line, and draw lines at  $45^\circ$  on each side of the points of division

This second arrangement gives the **diamond** or **lozenge**. Upon these two nets an infinite variety of patterns may be formed

Figs. 1–19 show a number of simple straight line bands. Figs 5, 6, 11, 12, 13, 14, and 15 are Greek frets. Fig. 16 is a Chinese fret. Fig 10 is a plait. Figs. 17 and 18 are Arabian or Moresque interlacement bands. Fig 19 is the Gothic nail-head ornament. The monotony of Figs 13 and 15 is broken by introducing square panels

Some of the more common terms used in connection with the laying out of ornament can be simply illustrated by means of the network.

If each square be filled by a repetition of the same pattern, a **diaper** is formed, Fig. 20.

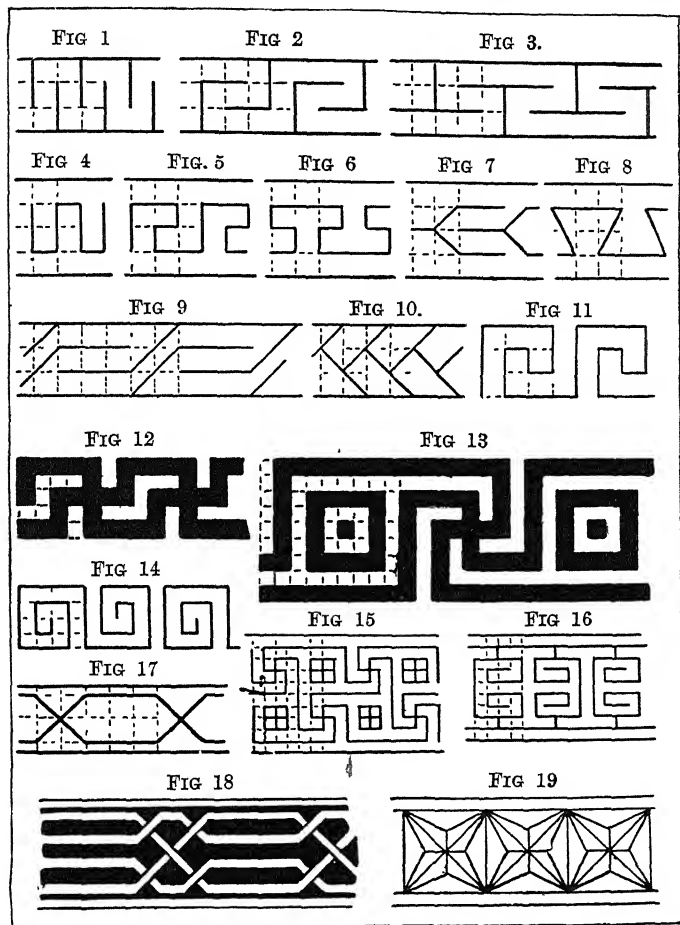
If the alternate squares only be filled the arrangement is called **chequering**, Fig 21

When larger intervals are left between the filled up spaces the method of setting out is called **spotting**. Sometimes smaller units are filled in between the spots, this is what is known as **powdering**. Fig 22 a a, etc., indicate the latter process

If the pattern be arranged in rows with spaces between, the method is called **striping** (Figs 23, 24). Wide stripes are called **bands**, narrow ones **lines**.

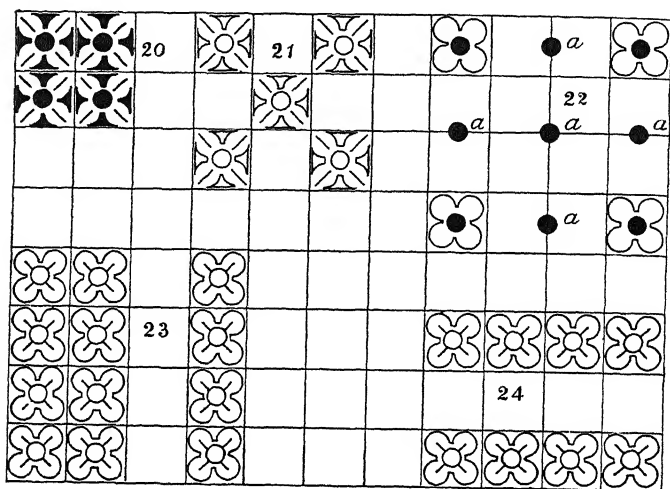
The term **diaper** is so frequently used that a more detailed

description is given. Strictly speaking, the term was applied to those repeated patterns in which the unit of design was confined



within a geometrical form, as in Fig 20. Afterwards developments were made to relieve the monotony of the pattern, and portions of the unit were allowed to run out of one form into the other, as in Fig. 25; or the boundary lines of the geometrical form were ornamented, as in Fig. 26. These arrangements are known as "all-over" patterns.

Sometimes the formal geometrical shapes are entirely omitted, as in many wall-paper patterns, producing what are sometimes



called "free all-over" patterns. It must, however, still be remembered that the pattern is designed upon a geometrical foundation

FIG. 25.

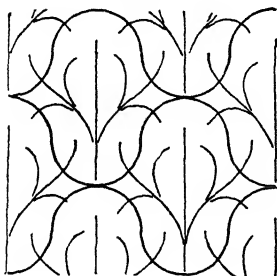
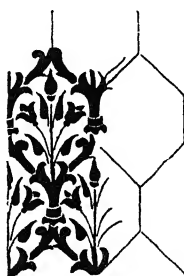


FIG. 26



Figs 27-34 are all formed on a square net; Figs 27-31 by filling groups of squares. Fig 32 is a plaid formed by "lining" in two directions. Figs. 33 and 34 are more difficult fret diapers, the pattern of which shows much clearer when filled in as shown. The unit of repeat is indicated by dotted lines. In drawing complicated frets the student should first mark in such parts as *a*, which recur at regular intervals; then such lines as *b*, which fall midway between

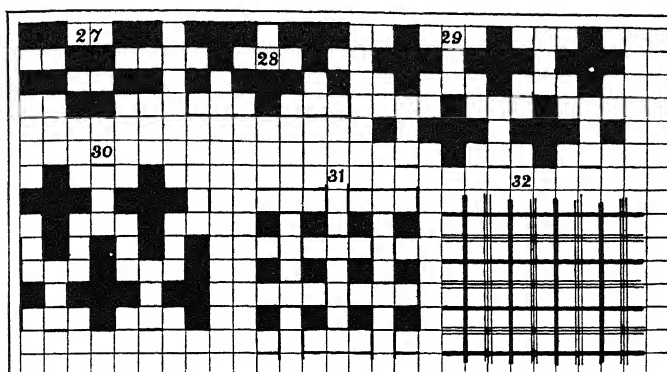
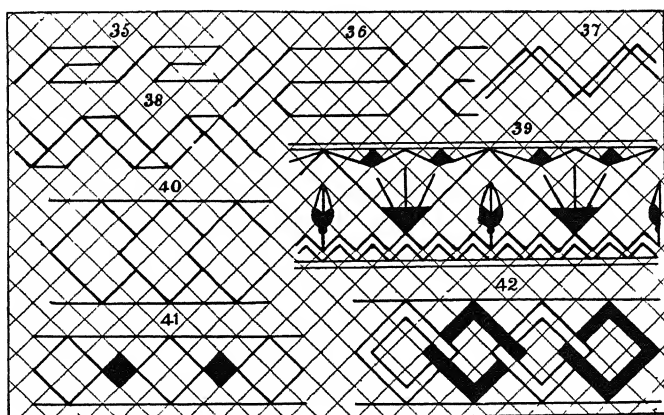
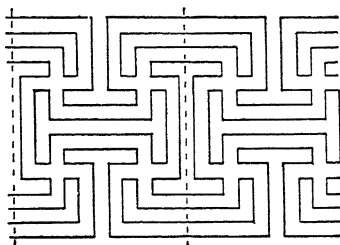
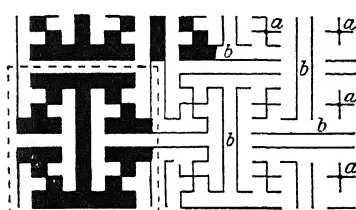


FIG 33

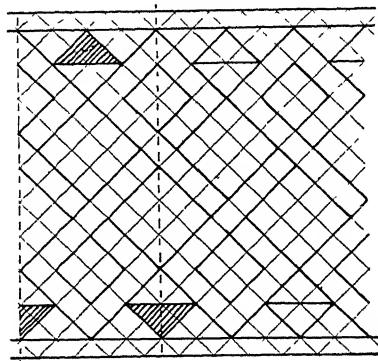
FIG 34





Figs. 35 and 36 are Arabian frets. Figs 37 and 38 illustrate the **zigzag** or **chevron**. Fig 39 is a variation on the zigzag similar to an Egyptian border, the Egyptians used the zigzag to represent water. Figs. 40, 41, and 42 are borders of overlapping, alternating,

FIG 43



and interlacing squares. Fig 43 is a more complicated interlacement band; the unit of repetition is indicated by the dotted lines.

### COMBINED SQUARE AND DIAMOND NETS

A combination of the square and diamond nets gives octagonal forms which are not, however, equal sided. The method of inscribing a regular octagon in a square is shown in Figs 52 and 53. In Fig. 52 the octagon has its alternate angles in the sides of the square, in Fig. 53 its alternate sides coincide with the sides of the square. It will be noticed that the octagon will not repeat by itself when placed side by side, quadrangular spaces being left between.

FIG. 44.

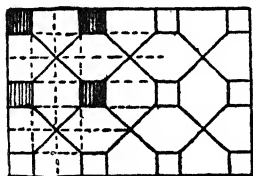
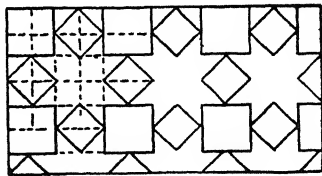


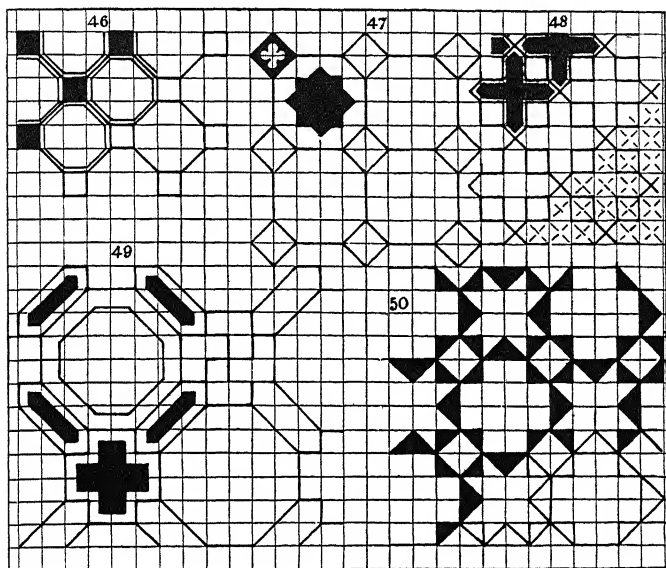
FIG. 45



Figs 44-51 are simple diapers formed on the net. Fig. 51 is an octagonal band.

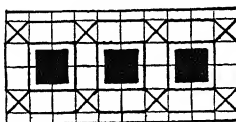
Fig 52 is a diaper formed by intersecting octagons. Fig. 53 is a diaper of regular octagons and squares.

A network formed by producing the sides of intersecting squares is used for many Moorish patterns. First draw a square diamond



net, then describe a circle about one of the squares, and obtain the intersecting square as shown. Produce the sides of the second

FIG 51



square, giving the other lines for the net. Fig. 54 shows a diaper on this net. The spaces may be filled up in a variety of ways.

### THE TRIANGULAR NET

If we draw the lines of the net crossing at an angle of  $60^\circ$ , we obtain a rhombus. Cross this net by lines passing through the wider angles of the rhombus and we get the equilateral triangle. The net may be arranged in two positions, as in Figs. 55 and 56. It is the most useful net for the designer, as it is the readiest basis upon which "drop" patterns may be formed. The equilateral

FIG 52

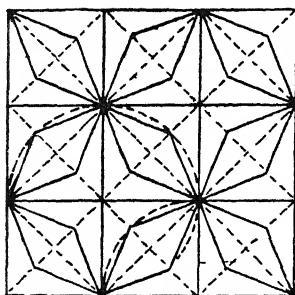


FIG 53

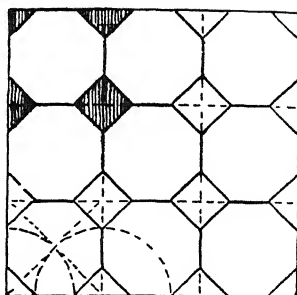


FIG 54

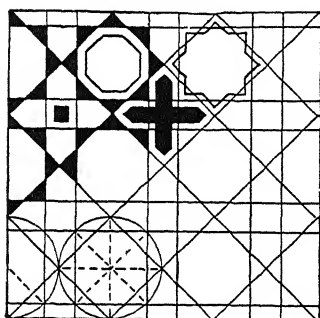


FIG. 55.

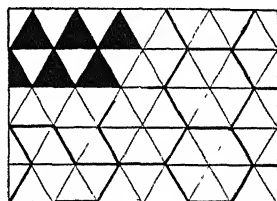


FIG. 56

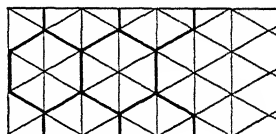


FIG 57.

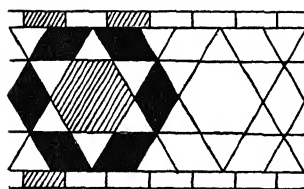


FIG 58

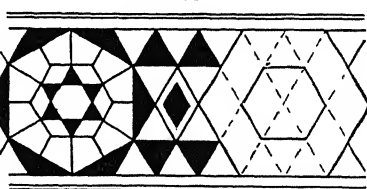


FIG. 59.

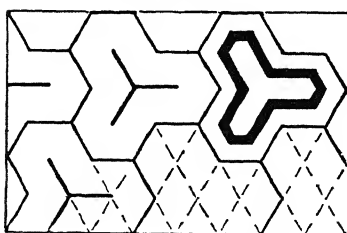
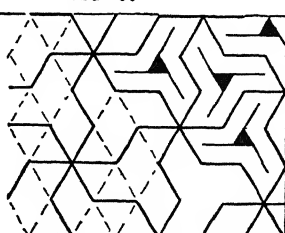


FIG 60



triangle, the rhombus, and the hexagon, and other shapes formed from them, all make perfectly fitting diapers upon the net

FIG. 61.

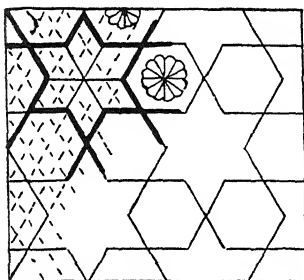


FIG 62

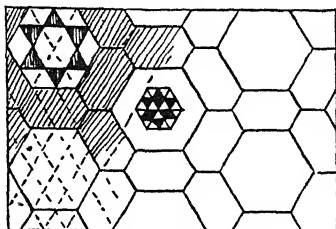
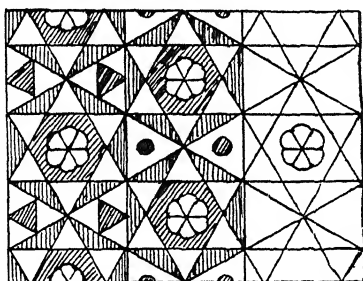


FIG 63



Figs 57 and 58 are bands suitable for textile Mosaic borders.

Figs. 59-62 are diapers based on the hexagon Fig. 63 is a marquetry pattern from an Indian box.

### THE CIRCLE

When curves are introduced the designer's field becomes still more fruitful. Figs 64-73 are bands based on the circle, Figs.

FIG 64.

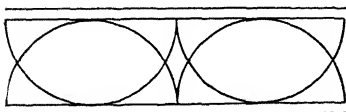
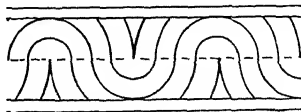


FIG 65.



68-71 are various forms of the *gulloche*, a pattern largely used by the Assyrians and Greeks ; Figs 72 and 73 are "scale" bands In Fig 73 the treatment of a corner is shown In designing a border,

set out the corner first, the ornament should be arranged so as to tend to bind the borders together.

The simplest diaper formed from circles is when they are

FIG 66

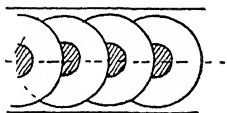


FIG. 67

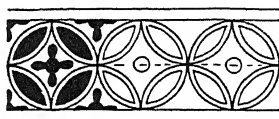


FIG 68

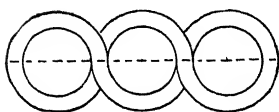


FIG 69.

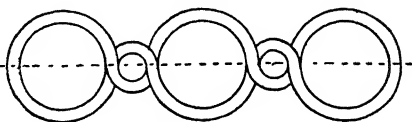


FIG. 70

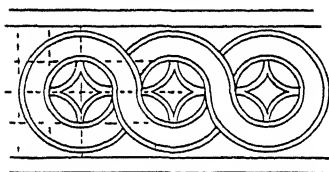


FIG 71

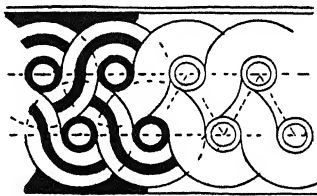


FIG 72

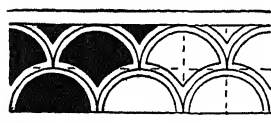


FIG 73

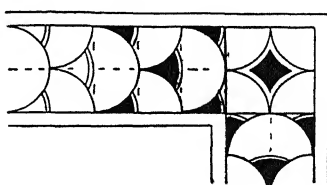


FIG 74

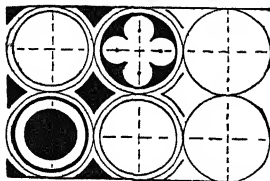
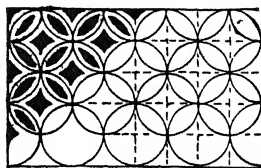


FIG 75.



arranged on the square net, with their edges touching, as in Fig. 74. When the circles intersect, much more elaborate patterns are

FIG 76.

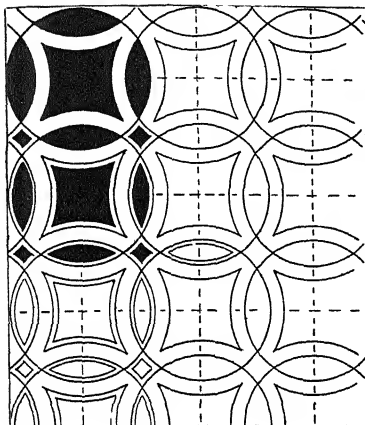


FIG 77

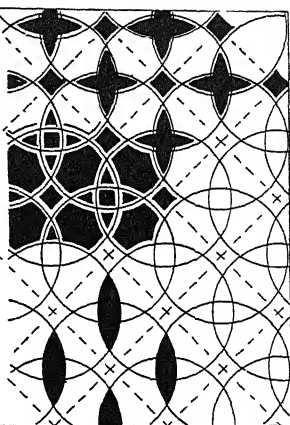


FIG 78



FIG 79

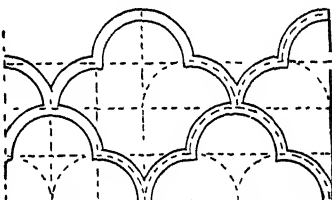


FIG 80

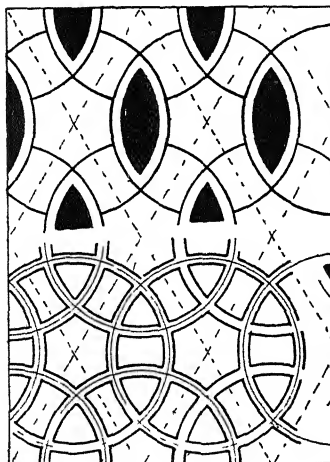
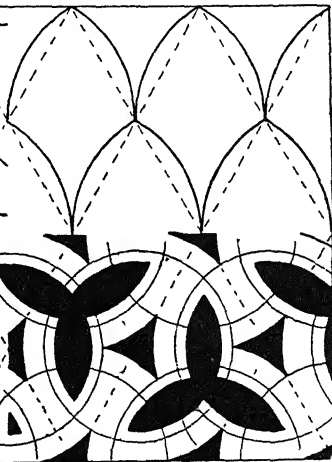


FIG. 81.



obtained. Figs 75, 76, 77, show diapers formed by intersecting circles on square and diamond nets. These patterns may be enriched by the addition of ornament to the spaces. Figs. 78 and 79 are scale diapers.

FIG. 82

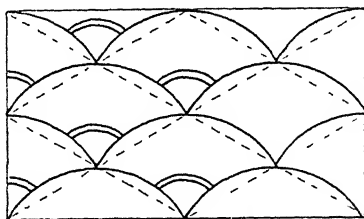


FIG. 83

FIG. 84.

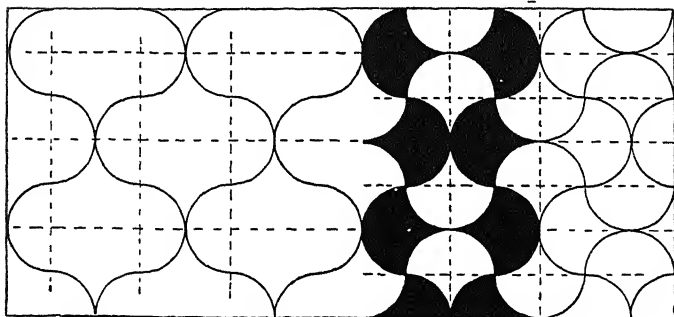
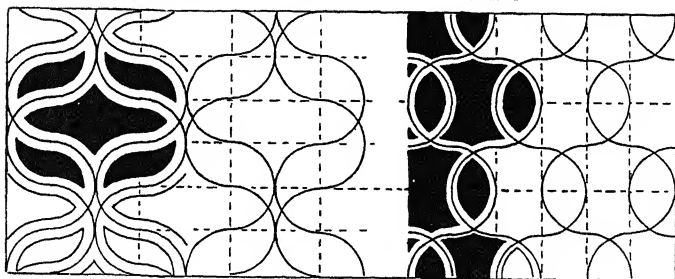


FIG. 85

FIG. 86

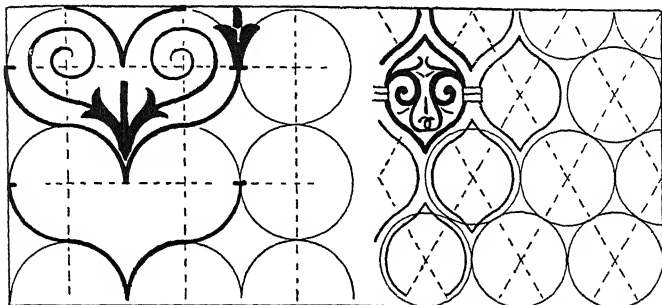


Figs 80-82 are arranged on the diamond net. In Fig. 80 the alteration of the radii of the circle will produce a variety of patterns. Different methods of filling the spaces are shown. Figs. 81 and 82 are scale diapers.

Figs 83-88 are diapers formed from ogee curves, Fig 84 is a double ogee; Figs 85 and 86 are interlacing ogees set out on an oblong foundation Fig 87 is based on circles arranged on a

FIG 87

FIG 88



square net, while in Fig 88 the circles are on a triangular net. The spaces may be enriched either by geometrical or by conventionalized natural forms.

### COMBINATIONS OF THE STRAIGHT LINE AND CIRCLE

Combinations of straight and curved lines afford a still greater variety of patterns. Figs. 89-96 show various band motives, Figs

FIG 89

FIG 90

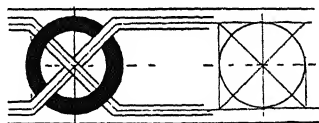
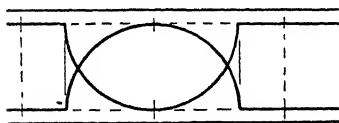


FIG 91

FIG 92

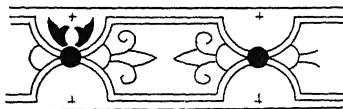
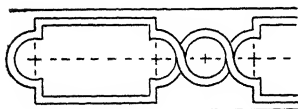
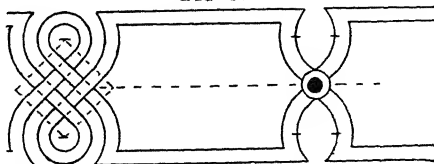


FIG 93





93 and 94 show combinations of tangential straight lines with curves ; Figs 90, 91, 93, and 94 are interlacing bands , Figs 95 and

FIG 94

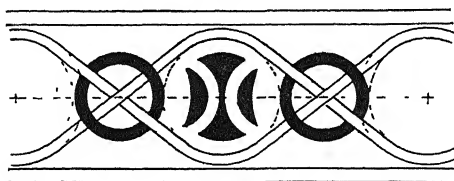


FIG 95.

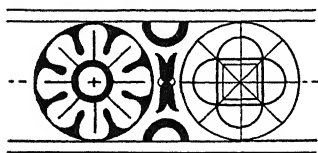


FIG 96.

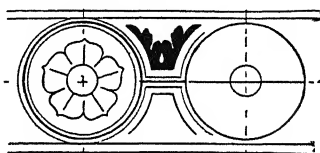


FIG. 97.

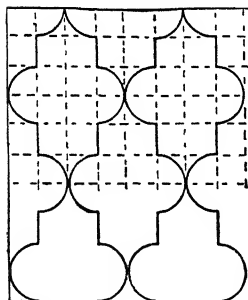


FIG 98.

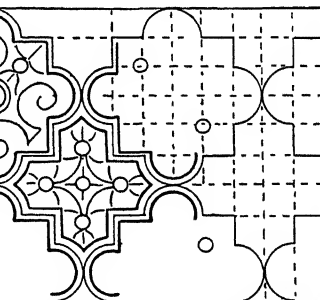


FIG 99

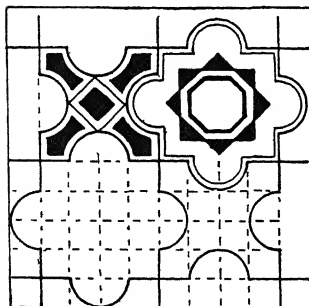
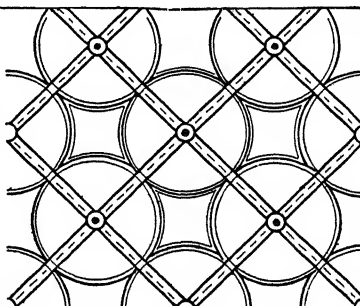


FIG 100.



96 are rosette bands Different methods of filling the rosette are suggested

Figs 97-100 show diapers composed of straight lines and circles Fig 100 is arranged upon a square diamond net, figs 98 and 100 are suitable for the spacing of ceilings

The student must be accustomed to the *analysis of the pattern* as well as to the building of it up, that is, when a pattern is given, to be able to construct the geometrical foundation upon which it is based. This may be done by noticing the lines upon which any single feature recurs.

### COMBINATIONS OF FREE CURVES WITH GEOMETRICAL FORMS

In the following examples, free curves, chiefly plant forms, are used in combination with the more formal geometrical forms It has already been pointed out that a much greater variety of more beautiful designs may be obtained by the use of free curves, such as those suggested by the growth and outlines of leaves and flowers

In the repeating borders which follow, the lines of the setting-

FIG 101

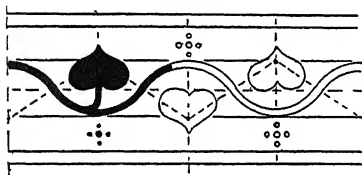


FIG 102

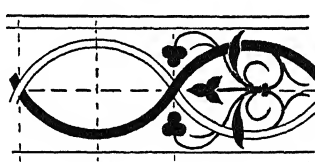


FIG 103

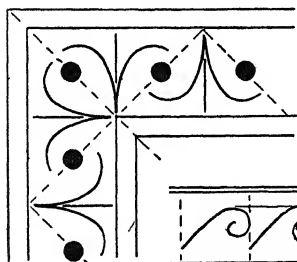


FIG 104.

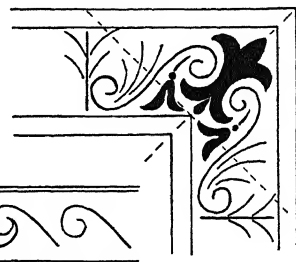


FIG 105

out are shown. The chief difficulty is the corner, which should be dealt with first. As a rule, it is better to ornament it more fully than the other part of the border, and to use lines which tend to hide the mitre line, and which add an appearance of strength to the joint

Figs 101-105 show various borders The necessary construction

lines, which must be set out first, are clearly shown. The centres for the circular arcs, in Figs 101 and 102, are shown by the small dots. In Figs 103 and 104 two corners are shown. Where the lines are arranged, as in Fig 105, it is necessary to reverse the direction in the centre of the border.

Figs 106 and 107 show the setting-out of circular borders.

FIG 106

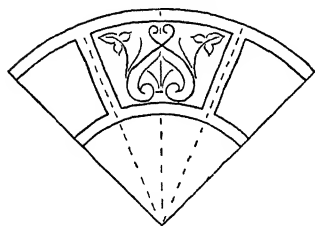


FIG 107.

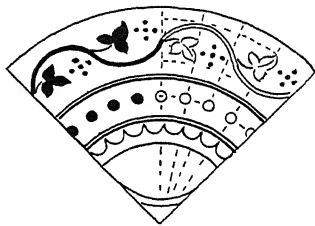


FIG 108

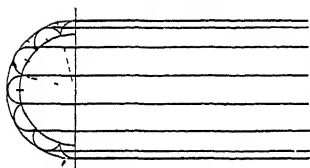


FIG. 109

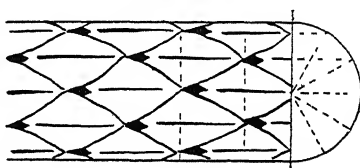


FIG 110

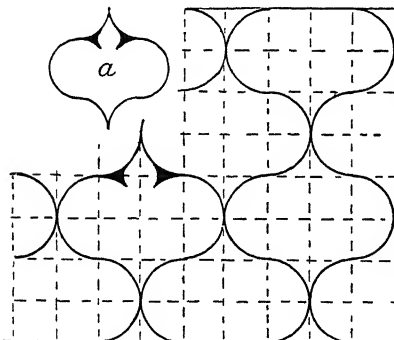
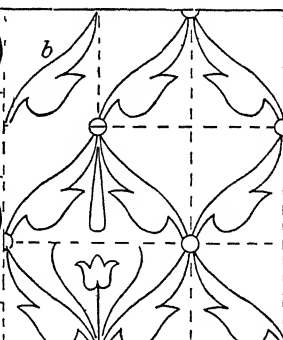


FIG. 111



Large mouldings, like the torus moulding on the bases of pillars, are often ornamented in various ways. Figs. 108 and 109 show two simple examples and the constructions necessary to draw them.

Figs 110 and 111 show how an "all-over" pattern may be formed.

In Fig 110, *a* is the given unit. It is evident that the figure is formed of semicircles described from the angles of squares. Draw

the square net and construct the pattern. The spaces may be further enriched with ornament.

In Fig 111 the given leaf-form, *b*, by repeating and reversing, forms a diaper. It will be seen that the figure can be enclosed in an oblong. First draw the rectangular net and form the pattern as in Fig A, page 175.

**Note.**—These patterns should be drawn much larger, and tracing paper may be used for the repeats.

### ENCLOSED ORNAMENT

The preceding figures furnish examples of continuous ornament, as in *bands*, and of repeating ornament, as in “all-over” arrangements, such as *diapers*. The third division includes the spacing and ornamenting of enclosed figures, and may be termed **panel ornament**. The chief forms to consider are the square, the oblong, the triangle, the lozenge, and the polygons, of which the hexagon and the octagon are the most important.

### THE SQUARE

Figs 112–123 indicate some of the ways in which the surface of the square may be spaced. The leading lines for the subdivisions

FIG 112

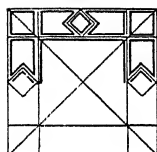


FIG 113

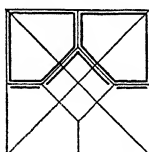


FIG 114

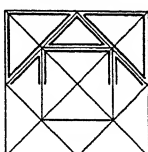


FIG 115

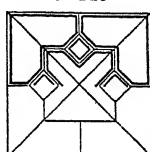


FIG 116

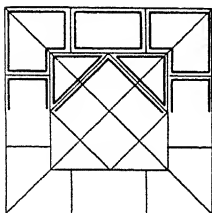


FIG 117

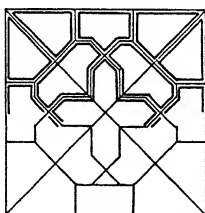


FIG 118

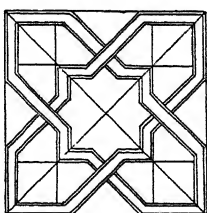


FIG 119

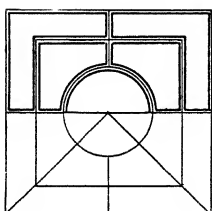


FIG 120

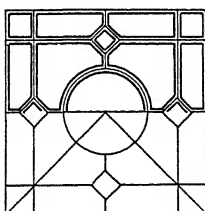
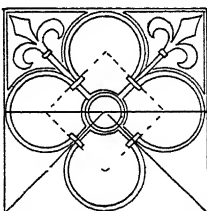


FIG 121.



are the diagonals and the diameters. In Fig 122 the square is subdivided into sixteen squares, and then lines are drawn from the middle points of the sides at an angle of  $60^\circ$

FIG 122

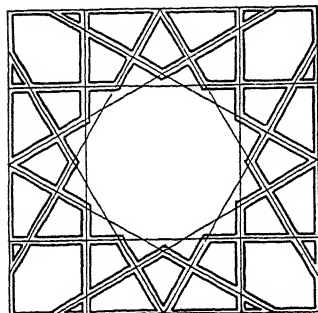
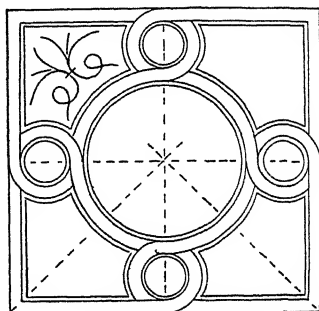


FIG 123



### THE CIRCLE

Under this heading will also be included sectors, stars, etc. The circle is spaced better, as a rule, by using curved lines, or a combination of curved and straight lines, particularly when applied

FIG. 124

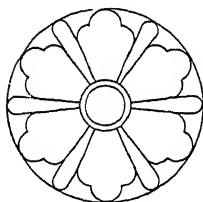


FIG 125

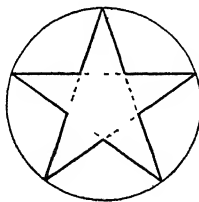


FIG 126.

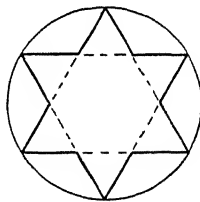


FIG 127

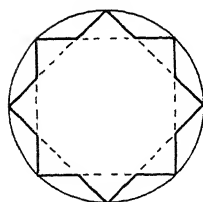


FIG 128

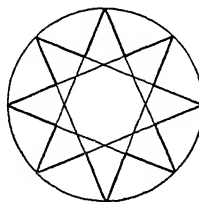
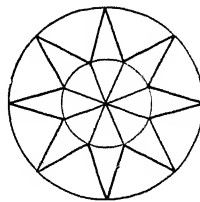


FIG 129



to tracery, as shown on page 204. The problems dealing with the inscription of circles in Chapter XI. form the bases of the constructions in most cases.

By dividing the circumference of the circle into equal parts and drawing radii, sectors are formed, which are the foundation of rosettes, as in Figs 95, 96, and 124. If the points of subdivision be joined the inscribed polygon is formed, as in Probs 81-86, page 40. By joining every second or third point various stars are obtained, as in Figs 125-128. Three forms of the eight-pointed star are shown, other stars may be formed in a similar manner.

FIG 130

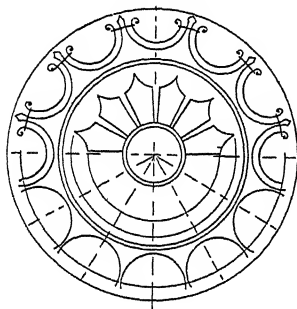


FIG 131

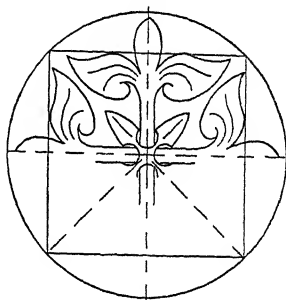


FIG 132

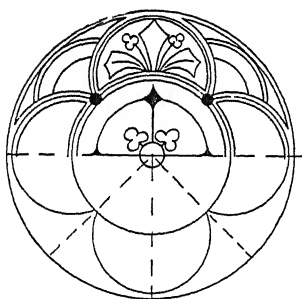
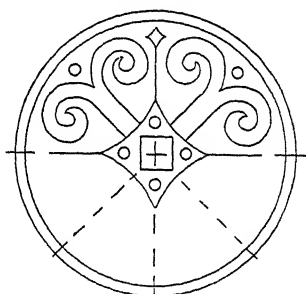


FIG. 133



Circular panels may be ornamented in various ways —by dividing into circular bands, as is often done with plates, salvers, etc., each band being separately ornamented as in Figs 106, 107, and 130; by inserting regular geometrical figures (squares, polygons, etc.), as in Fig 131, by using arcs, as in Fig 132, or they may be treated in a similar manner to the rosette, as in Fig 133.

### THE OBLONG

This is, perhaps, the most common form of all. Ceilings, floors, walls, doors, book-covers, and many other objects take this shape. The relative proportions of the sides differ so much that the spacing

is necessarily very varied. The diagonal lines are not often used for dividing the surface, as the mitre line of the corner alone gives

FIG 134

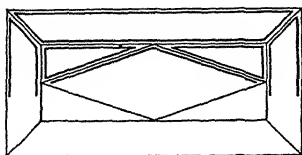


FIG 135

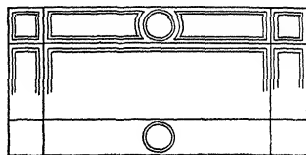


FIG 136

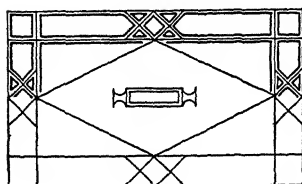


FIG 137

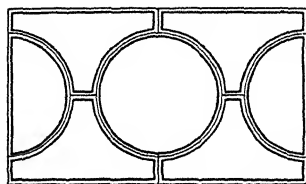
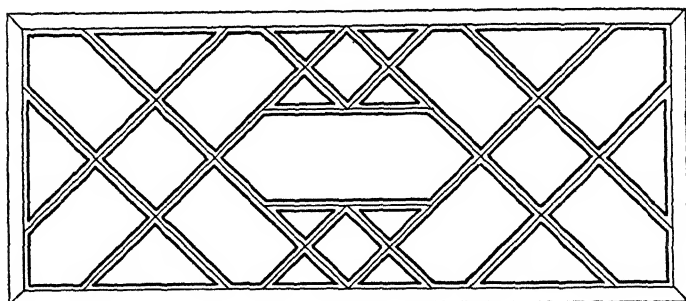


FIG 138



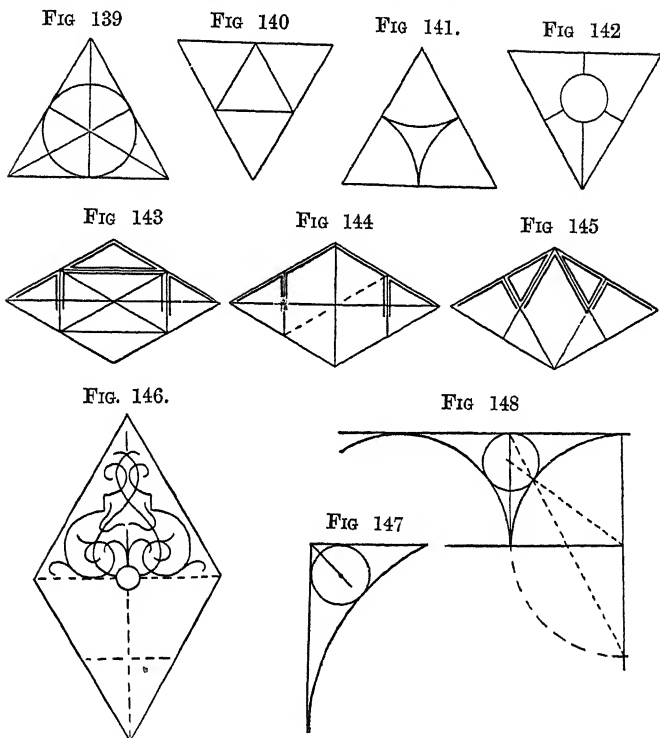
an equal width of border all round. Figs. 134-138 show simple subdivisions of the figure.

### THE TRIANGLE, THE LOZENGE, THE SPANDREL, AND THE ELLIPSE

The triangle and lozenge are not commonly used. Figs. 139-145 show subdivisions of these figures. When the rhombus (lozenge) is not subdivided, the ornament is grouped about the diagonals as in Fig. 146.

The spandrel is the irregular triangular space between the outside of an arch and the right angle enclosing it as in Fig. 147; or the space between two arches and the line touching them as in

Fig 148. It often contains an inscribed circle, the centre of which is found by applying *Problem 150*, as shown.



The surface of the ellipse may be divided by elliptical bands, similarly to Figs 107 and 130, or circles may be inserted as in Fig 123.

# REGULAR POLYGONS

The polygons most commonly used in ornament are the octagon and the hexagon. The diagonals and diameters intersecting give a great variety of subdivisions, of which Figs 149-152 are examples. Star-shaped polygonal figures are shown among the examples on the square and circle. The hexagonal bands and diapers also suggest many suitable spacings of the hexagon. Figs 153 and 154 show the simplest subdivisions obtained by using the diagonals and diameters respectively. Fig. 155 shows a division based on the six-pointed star.



FIG 149.

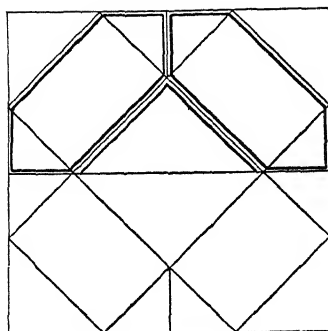


FIG. 150

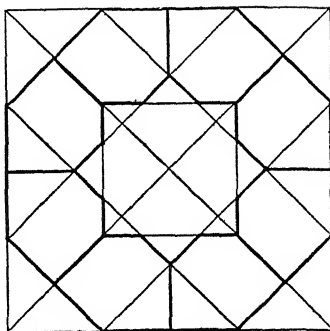


FIG 151.

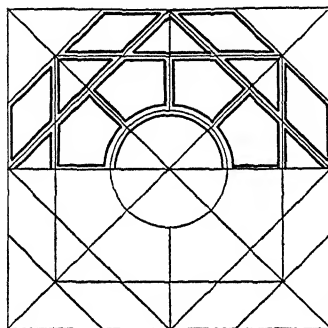


FIG. 152.

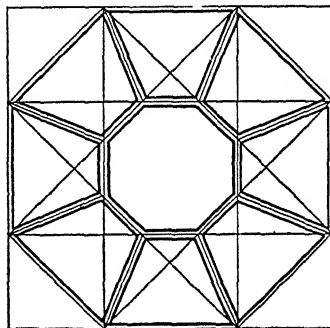


FIG 153.

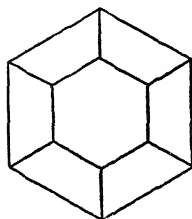


FIG 154

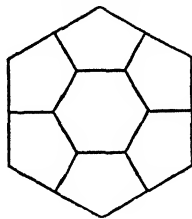
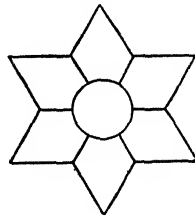


FIG. 155.



### WALL SURFACE

Wall surface is usually divided horizontally into five divisions, namely, the **plinth**, **dado**, **wall-vail**, **frieze**, and **cornice**, with borders and mouldings between, as in Fig. 156 Each of these divisions is ornamented in a different manner; the dado being the

supporting part, as it were, has a more formal treatment, while the upper portion is usually decorated with a freer pattern. Sometimes vertical divisions, such as pilasters, are added. The spaces between are then usually decorated as enclosed ornament.

### CONSTRUCTION OF PATTERNS TO A LARGER OR SMALLER SCALE

This is easily done by applying *Problems 117-119*. Suppose, for example, it is required to make an enlarged copy of a unit of the given border, Fig 157, making its height  $1\frac{1}{4}"$ .

The unit would be the portion of the pattern between the lines  $aa$  and  $bb$ . Draw the centre line, and set off the border line (half of  $1\frac{1}{4}"$ ) on each side. Construct a proportional scale, making  $AA = 1\frac{1}{4}"$  (the required height), and  $Aa = aa$  (the given height).

Set off  $A2 = 12$  and  $A3 = 13$ . Draw parallels giving the required dimensions for the enlarged unit. With radius  $A2'$  describe the circle, and with radius  $A3'$  mark the boundary lines of the unit. Any other necessary dimensions may be found on the scale in a similar manner. The circle may be divided into five sectors by the help of the dividers, or by *Problems 86, 81, or 108*.

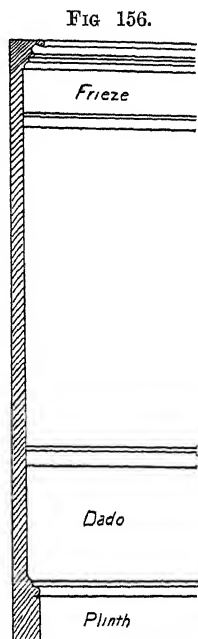
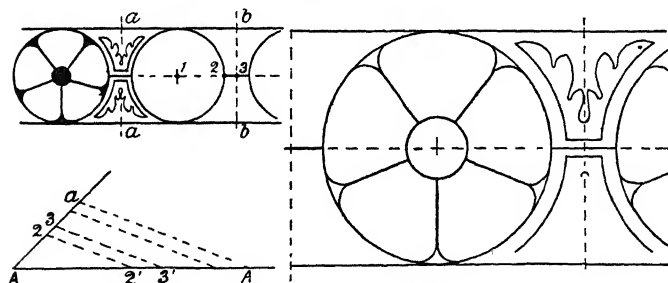
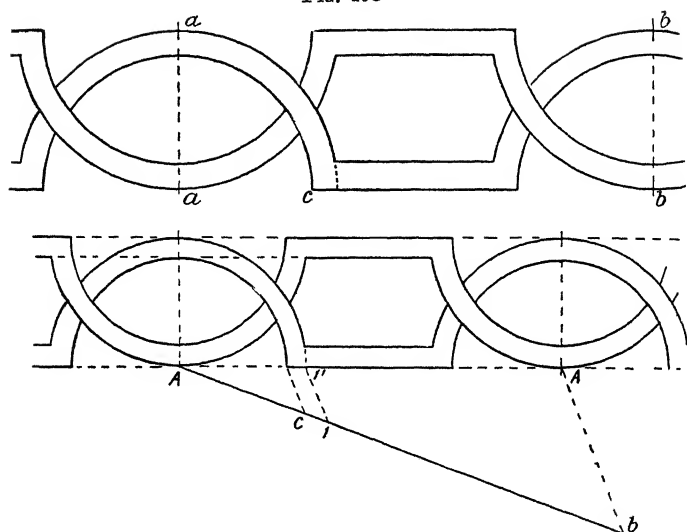


FIG 157.



In Fig. 158 the border is reduced so that the unit  $aa$  to  $bb$  is reduced to  $2"$ . Make  $AA = 2"$ , and draw  $Ab = ab$ . For the required height set off  $Al = aa$ , and obtain  $Al'$ . Set off  $Ac = ac$ , and obtain the radius for the smaller semicircle as shown. The rest of the construction may be easily followed.

FIG. 158



## ARCHES

The curves of arches may be produced in such a number of ways, that it is only possible to show typical cases. The alteration of the position of the foci from which the curves are struck provides a great variety of forms. A few problems are worked out from given data; in the case of the other figures the position of the centres (C), the lengths of the radii, and the dotted construction lines, sufficiently indicate the method to be followed.

The distance from A to B (Fig 159) is the **span** of the arch. A and B are termed the **springing points**. D is the **crown** and CD the **height** of the arch. The separate stones are called **voussoirs**, and the central stone (a) is the **key-stone**. The outside of the voussoirs is the **extrados**, or **back**, and the inside is the **intrados**, or **soffit**.

Some of the most typical forms of arches are—

The **semicircular arch** Fig 159

The **stilted arch**, which stands upon upright lines Fig 160

The **segmental arch** described from one centre Fig 161

The **segmental arch** described from two centres Fig 162

The **horse-shoe or Moorish arch** Fig 163

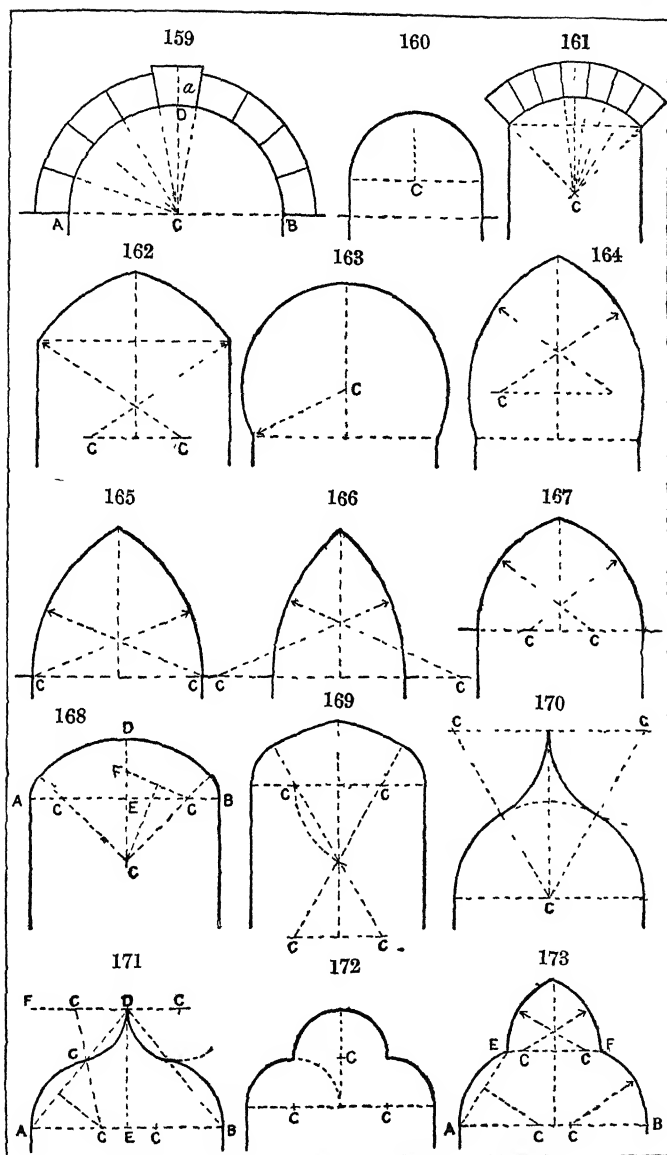
The **pointed horse-shoe or Moorish arch** Fig 164

The **equilateral arch** Fig 165

The **lancet arch** Fig 166 The centres lie without the span.

The **obtuse arch** Fig 167. The centres lie within the span.

The **three-centred arch** Fig 168.



To construct this arch, the span  $AB$  and the height of  $ED$  being given. Make  $AC$ ,  $BC$ , and  $DE$  all equal. Join  $F$  and  $C$ , and bisect. Where the bisecting line meets  $DE$  produced will be the third centre. From the points  $C$ ,  $C$ ,  $C$  describe arcs.

The depressed Tudor arch described from four centres Fig. 169

The ogee arch described from three centres Fig. 170.

The ogee arch described from four centres Fig. 171.

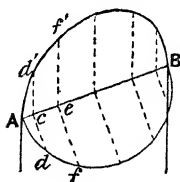
Given span  $AB$ , and height  $ED$ . Draw  $DF$  parallel to  $AB$ . Join  $DA$  and  $DB$ . Mark any point  $G$  where the arcs unite. Bisect  $AG$  for centres on span. Join  $CG$ , and produce to meet  $FD$  for the other centres.

The round-headed trefoil. Fig. 172

The pointed trefoil Fig. 173. Obtain  $AE$  and  $EF$ , and bisect  $AE$  for the centres on  $AB$ .

The elliptic arch Fig. 178. The points between the voussoirs are obtained by drawing normals to the curve as in *Problem 261*. For the outer line make the normals equal, and sketch in the curve.

The rampant arch has one springing point higher than the other.



To construct it, join  $A$  and  $B$ . Describe a semicircle and draw perpendiculars from any number of points to the circumference. From the same points draw vertical lines. Make  $cd' = cd$ ,  $ef' = ef$ , etc.

## MOULDINGS

Mouldings are very important architectural features. They are used to improve the appearance of the angles, projections, arches,

FIG. 174

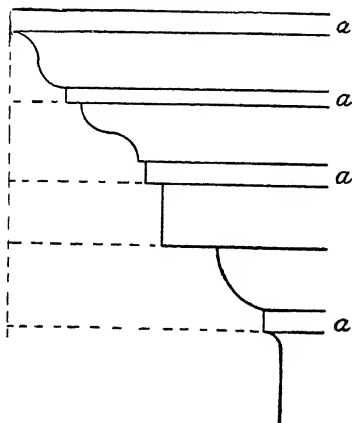
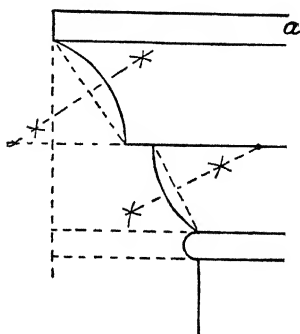


FIG 175



doors, windows, etc., of buildings. The profile of mouldings varies according to the style of architecture. The Greeks and Romans

used eight forms, namely the *fillet*, the *cyma recta*, the *cyma reversa*, the *cavetto*, the *ovolo*, the *bead*, the *scotia*, and the *torus*. These they frequently enriched with either carving or colour. Figs 174 and 176 show the cornice and base respectively of the Roman Ionic order with the mouldings in position, those marked *a* are fillets. Fig 175 shows another cornice with the *cavetto*, *ovolo*, and *bead*. As these curves are not quadrants like Figs 183 and

FIG. 176

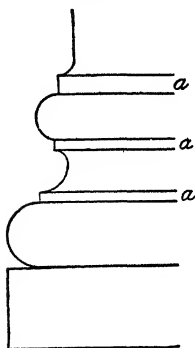
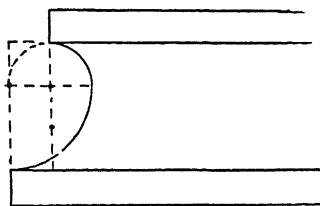


FIG. 177



184, the method of obtaining the centres is shown. Fig 177 is an enlarged drawing of the *scotia* in Fig 176, showing how the centres of the arcs are obtained.

I **Roman mouldings** are formed from arcs of circles.

Fig 179 is a *fillet* or *band*, a narrow flat member generally separating other mouldings.

Fig 180 is an *astragal*, or *bead*.

Fig. 181 is the *torus* moulding, used in the bases of columns, and frequently ornamented with the *guilloche*.

Fig 182 is the *scotia*, a sunken moulding at the base of a column.

The centres for the quadrants must be in one line. (See also Fig 177.)

Fig. 183 is the *ovolo* or *quarter-round*, often encircled with the egg-and-dart ornament.

Fig 184 is the *cavetto*, the reverse of the *ovolo*.

Figs 185–187 show three forms of the *cyma recta*.

In Fig 185,  $ab = ac$ ; and in Fig 186,  $ab$  is greater than  $ac$ . In both figures the centres lie in the parallel drawn through the middle point of  $bc$ . In Fig. 187, the point  $d$  is not the centre of  $bc$ , and the centres are obtained as shown.

Figs 188–190 show three forms of the *cyma reversa*.

In Fig 188,  $ab = ac$ ; in Fig. 189,  $ab$  is less than  $ac$ . The centres of both figures lie in the parallel to  $ab$  through the middle point  $d$ . In Fig 190, the point  $d$  is not in the centre of  $bc$ . Figs 185–190 are also known as *ogee* mouldings.

II. **Grecian mouldings** are usually formed from conic sections—the ellipse, parabola, and hyperbola—and are therefore much more varied and graceful in contour.

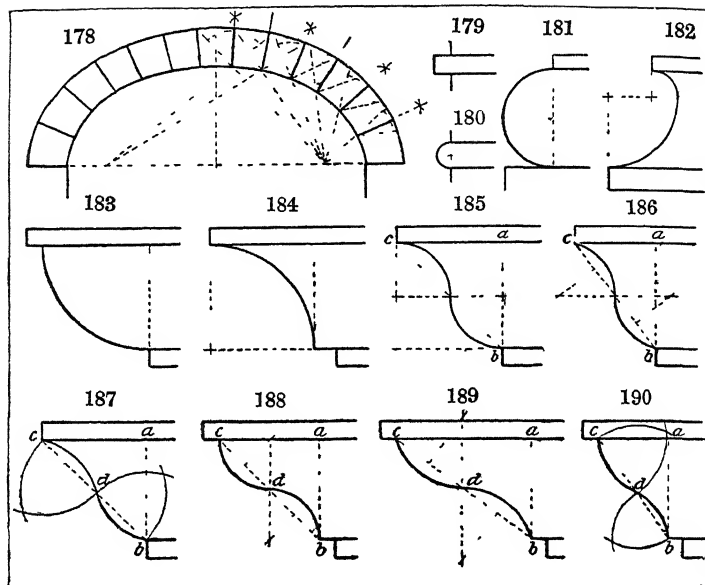


Fig 191 is the echinus, or ovolo.

Given  $ab$  and  $bc$ , also  $d$ , the point where the moulding turns. This turn is called a *quirk*. Divide  $ae$  and  $ed$  each into any number of equal parts, and draw the parabolic curve as in Problem 265.

Fig 192 is another echinus. Here the curve is the hyperbola.

Given  $ab$  and  $bc$  as before. Make  $df$  any convenient length, the alteration of the position of  $f$  changes the character of the curve

Fig. 193 shows the cavetto formed by the quarter of an ellipse

Fig 194 is the scotia formed from a semi-ellipse.

Given  $fe$ , the depth of the curve. Draw  $eh$  parallel to  $cd$ , and construct the semi-ellipse in the parallelogram  $ch$ .

Figs 195 and 196 show the *cyma recta*.

In Fig. 195,  $ab$  is greater than  $bc$ . The construction can easily be followed

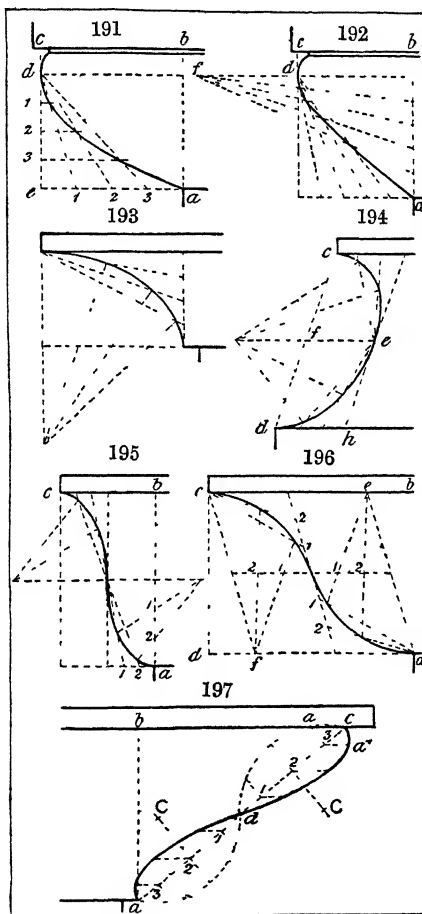
In Fig 196,  $ab$  is less than  $bc$ . Make  $ce = cd$ . Join  $e$  and  $a$ . Draw  $cf$  parallel to  $ae$ , and proceed as in the last figure. The curve lies within the parallelogram  $aecf$ .

The *cyma reversa* is similarly constructed to the *cyma recta*.

If Figs. 195 and 196 be turned at right angles, they will show the *cyma reversa*.

Fig 197 shows Nicholson's method of obtaining any ogee curve.

Join  $a$  and  $c$ . On  $dc$  and  $ad$  describe arcs, from any convenient centres as  $C$ . Take any number of points, 1, 2, 3, etc. From these points draw perpendiculars to the arcs. From the same points draw parallels to  $bc$ . Set off on these parallels, distances = the perpendiculars at the same point, thus  $3a' = 3a$ , etc

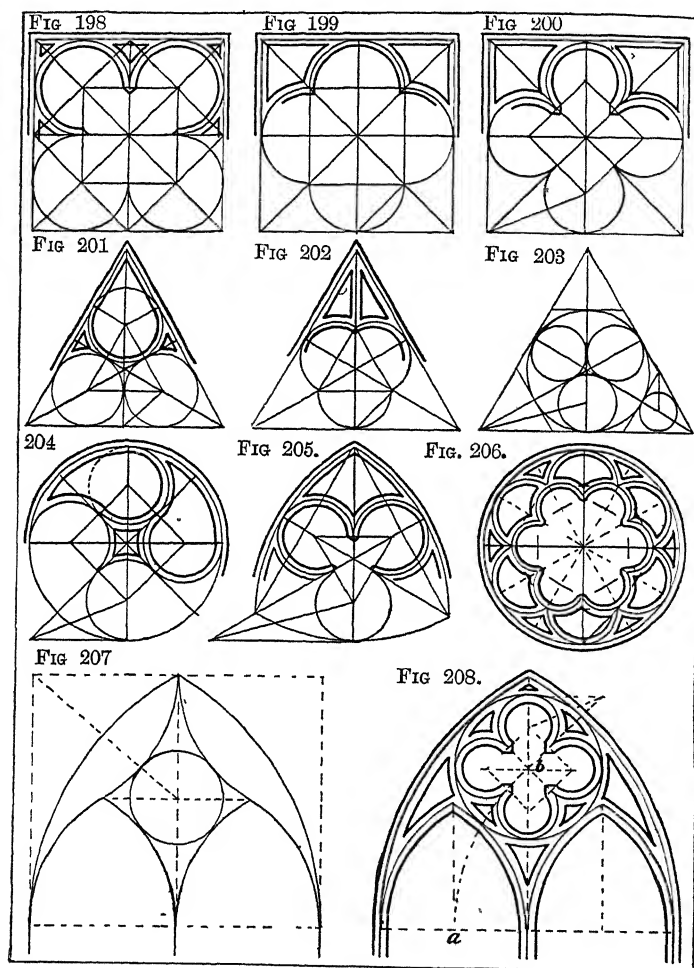


## TRACERY

Figs 198-208 show the application of Geometry to simple tracery. The light centre lines form the foundation, and portions only of the figures are completed, so that the construction lines may



be seen Draw the figures to a much larger scale For further information the student is referred to works on architecture.

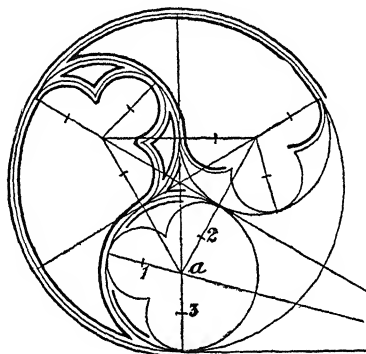


Figs. 198-200 show quatrefoils, in 199 and 200 the same construction will apply to quatrefoils in circles. All the figures are based upon problems in Chapter XI. Figs 207 and 208 show two arrangements of the leading lines for two-light windows. The

centre for the circle in Fig 208 is obtained by cutting the height with the arc  $ab$ .

Fig 209 shows the construction of a more elaborate example of tracery. First inscribe three circles in the circle. The centres for the inner foils are shown on the figure. Centre 1 lies on the line

FIG 209



used to find the centre of the inscribed circle, 2 and 3 are equidistant from the centre  $a$ . The foils in this case are not tangential to one another, as tangential arcs would not combine so agreeably with the curves of the inscribed and enclosing circles.

### EXAMINATION PAPERS

The following Examination Papers of the Board of Education not only furnish suitable tests for the student, but also provide a number of valuable illustrations of the application of Geometrical Drawing to Ornamental Design, etc

APRIL 28, 1906

6 to 7 30 p.m.

## GEOMETRICAL DRAWING.

*NB—The diagram referring to question 6 is to be pricked through or accurately transferred to the paper. Failing this, marks will be deducted.*

## SECTION A

Read the general instructions

YOU MAY ATTEMPT THREE QUESTIONS ONLY, WHICH MUST BE SOLVED ACCURATELY, USING INSTRUMENTS

The constructions must be strictly geometrical, and not the result of calculation or trial

*All lines used in the constructions must be clearly shown*

Set squares may be used wherever convenient. Lines may be bisected by trial

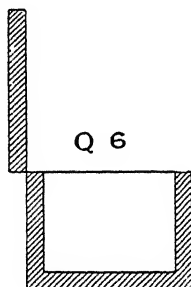
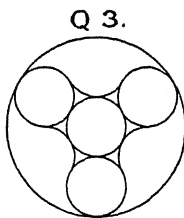
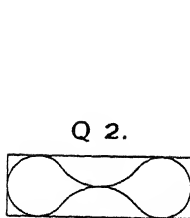
1 A drawing is made to a scale of 2' to 1" ( $\frac{1}{2}$ ). Make a scale for this drawing to show metres (up to 4), decimetres, and (diagonally) centimetres. Figure the scale properly, and show by two small marks on it how you would take off a distance of 3 metres 7 centimetres. You may assume a metre to be equal to 39"

1 metre = 10 decimetres = 100 centimetres

\*2 Draw a figure similar to that in the given diagram, making the rectangle  $4\frac{1}{2}" \times 1\frac{1}{2}"$  Determine clearly all points of contact between circles (22)

\*3 Copy the given diagram, making the radius of the enclosing circle  $1\frac{1}{2}"$ . The radii of all the other circles and arcs are equal. Determine all points of contact (20)

(18)



4 Construct a square of 2 8" side. Within it inscribe a rectangle having its four angles in the four sides of the square, and its sides parallel to the diagonals of the square. One side of the rectangle is to be twice as long as the adjacent side (18)

5 Draw a line **AB**, 3 75" long. Find a point **C**, 1 2" from the line **AB**, and so placed that the lines **CA** and **CB** form a right angle (14)

\*6 The diagram shows the cross-section of a box having its lid open at right angles. Make a plan of the box, the outside length being 2 1/2". Show also an elevation on a vertical plane making 60° with the long edges of the box. Only visible lines need be drawn (18)

7 A hollow sphere of 3 1/2" outside diameter, formed of material 1/4" thick, is cut by a plane which passes 1" from the centre of the sphere and is inclined at 60° to the horizontal (20.)

## SECTION B

Read the general instructions

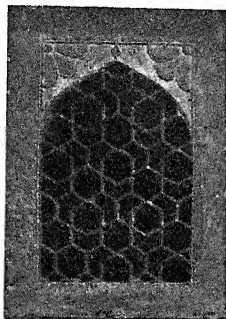
YOU MAY ATTEMPT TWO QUESTIONS ONLY

All freehand work employed in this section must be neat and careful, and its intention must be made quite clear

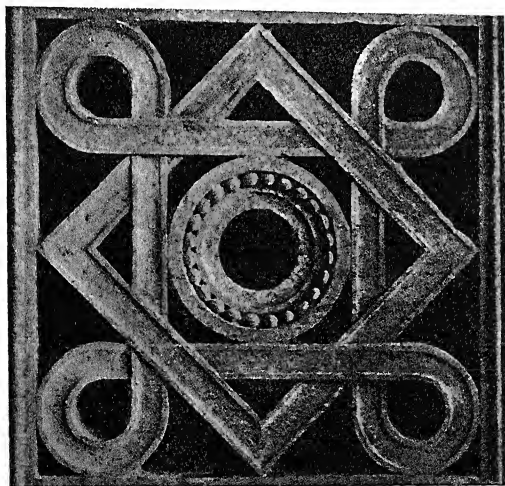
*All constructions must be clearly shown*

\*8 Show clearly the construction you would use in setting out the pattern of the given open-work panel, ignoring the surrounding framework. Your drawing should be made about four times the size of the diagram. Mark distinctly what you consider the unit of repeat (20)

Q. 8.



Q. 10.



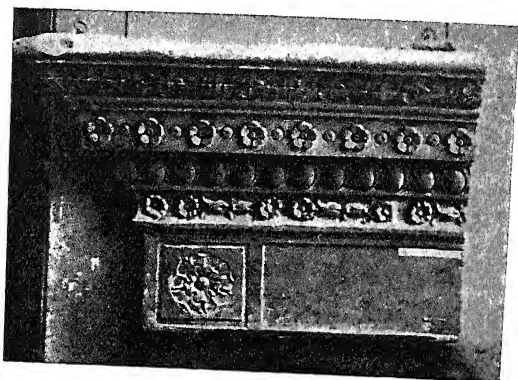
9 A pavement is to be formed of square and regular octagonal tiles, each of 3" edge. Draw the pattern with instruments, to a scale of  $\frac{1}{4}$  full size, showing clearly your method of work. Draw at least four tiles of each kind. Indicate plainly what you consider the unit of repeat (16)

\*10 Show what geometrical constructions you think desirable in drawing the given panel (18)

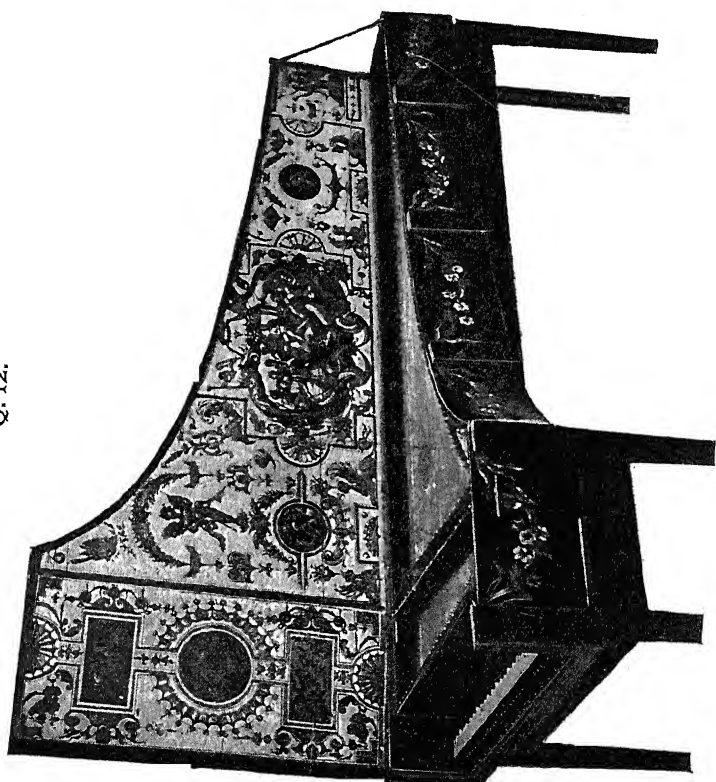
\*11 Draw an approximate section of the given moulding, at least twice the size of the diagram. Straight lines should be ruled, but curves may be drawn either freehand or with compasses. All "enrichments" or decoration of the parts should be disregarded. (16)

\*12 Make an approximate sketch plan, and front and side elevations, of the harpsichord shown, indicating your methods of projection. None of the painted ornament should be drawn (18)

Q. 11.



Q. 12.



APRIL 27, 1907.

6 to 7 30 p m.

## GEOMETRICAL DRAWING

*N B —The diagrams referring to questions 6 and 7 are to be pricked through or accurately transferred to your drawing-paper, if you attempt these questions. Failing this, marks will be deducted*

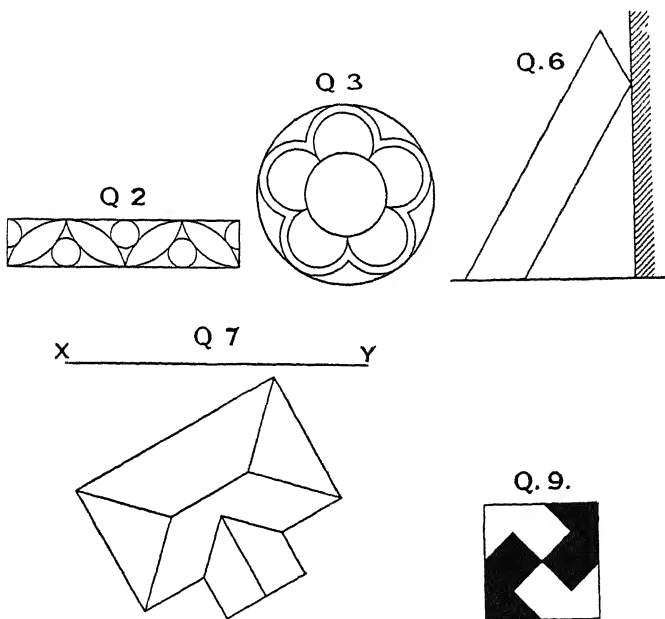
## SECTION A

Read the general instructions

IN THIS SECTION YOU MAY ATTEMPT THREE QUESTIONS ONLY, SHOWING YOUR KNOWLEDGE BY THE USE OF INSTRUMENTS. The constructions must therefore be strictly geometrical, and not the result of calculation or trial

*All lines used in the constructions must be clearly shown*

Set squares may be used wherever convenient. Lines may be bisected by trial



1 Make a *plain* scale, one-seventh of full size, by which feet, inches, and half-inches may be measured up to 3 feet. The scale must be properly finished and figured, and is not to be "fully divided", i.e. inches and half-inches are not to be shown throughout the whole 3 feet. Make a reduced drawing, to this scale, of the sheet of drawing paper on which you were working (22)

\*2 Draw *half* the given figure, making the half-length  $2\frac{1}{4}$ " and the height 1" Show all your constructions, and all points of contact between circles (20)

(N B —The diagram is not drawn to scale)

\*3 Copy the given figure, making the radius of the outer circle 1 5" The outer cinquefoil is to be formed of *semicircles*, the inner one of tangential arcs (18)

4 About a circle of 1 3" radius describe a quadrilateral **ABCD** having the following dimensions :—

$$AB = 3'', BC = 3\frac{1}{2}''$$

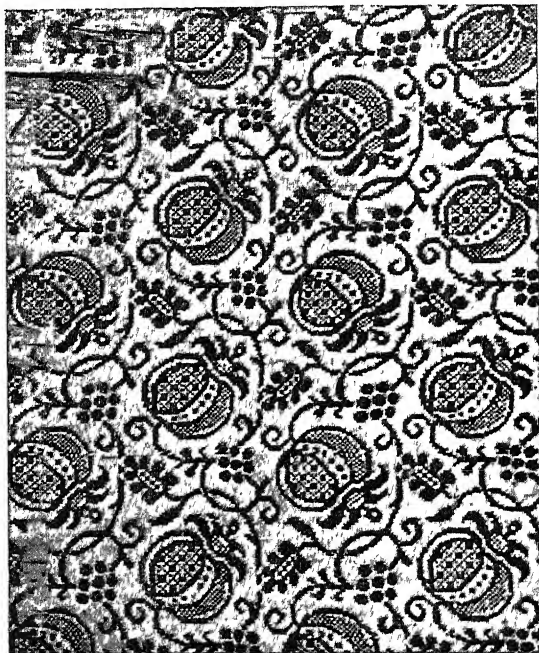
The angle **BCD** is  $82^\circ$

Show all the constructions you employ, and determine the points of contact Measure carefully and write down, the remaining sides and angles of the figure (16)

5 Find the shape of a sheet of paper such that when it is folded across the middle each half-sheet shall have the same shape as the whole sheet Make the longest side 3" long (18)

\*6 The diagram shows the side elevation of a shallow, circular, metal bath, resting on the ground and against a wall Draw *either* the plan of the bath, or its elevation on a vertical plane parallel to the wall (20)

Q. 8.



\*7. The roof-plan of a building is given, to a scale of 10' to 1" Draw its elevation upon a vertical plane parallel to the line *xy*, assuming that the height of the walls throughout is 10', and that the slope of the roof throughout is at  $45^\circ$  to the horizontal Only visible lines need be shown. (18)

### SECTION B.

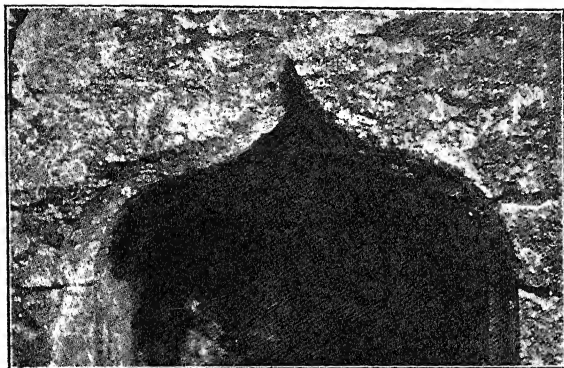
Read the general instructions.

IN THIS SECTION YOU MAY ATTEMPT TWO QUESTIONS ONLY

All freehand work employed in this section must be neat and careful, and its intention must be made quite clear

*All constructions must be clearly shown*

Q. 10.



Q 11.





\*8 Draw, with instruments, a system of construction lines on which the given repeating pattern can be set out. Then sketch in the main features of the design sufficiently to show the value of your construction. Mark very clearly what you consider the unit of repeat.

\*9 The diagram is copied from an old Moorish tile. Set out accurately with instruments the pattern of which you suppose this tile was intended to form part, showing at least four tiles. The dark portions should be indicated by slight shading.

\*10 Make a careful approximate copy of the arch shown. Use arcs of circles for the curves, and show their points of contact. Make your drawing the same size as the diagram.

\*11 Draw an approximately accurate elevation of the given semi-circular arch, showing the arrangement of the voussoirs or arch-stones. Make your drawing about twice the size of the diagram.

\*12 Make a sketch elevation showing the principal parts of the chalice, ignoring the smaller mouldings. Draw also the plan, or half-plan, of the foot of the chalice, showing your methods of setting out.

Q. 12.



MAY 2, 1908

6 to 7.30 p m

GEOMETRICAL DRAWING

SECTION A

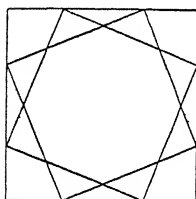
Read the general instructions

IN THIS SECTION YOU MAY ATTEMPT THREE QUESTIONS ONLY, SHOWING YOUR KNOWLEDGE BY THE USE OF INSTRUMENTS. The constructions must therefore be strictly geometrical, and not the result of calculation or trial.

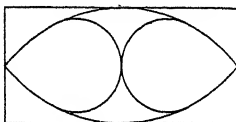
All lines used in the constructions must be clearly shown

Set squares may be used wherever convenient. Lines may be bisected by trial.

Q 2



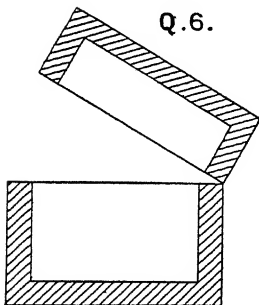
Q 3



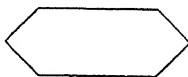
Q 5



Q.6.



Q 8



1. Construct a diagonal scale two-thirds ( $\frac{2}{3}$ ) of full size, by which inches and hundredths of an inch can be measured up to 6 inches. Figure the scale properly, and show by two small marks on it how you would take off a distance of 5 17''

Draw, to this scale, an equilateral triangle of 4 82'' side, and write down the length of its altitude (22)

\*2 The diagram shows a regular 8-pointed star, resulting from the inscription of two squares within an outer square. Draw the figure, making the outer square of 2 1/2'' side (16)

\*3 Make an enlarged copy of the diagram, using the following dimensions — The radius of each of the two smaller arcs is to be 1'', and that of each of the two larger ones 3 1/4''. Show how you determine every point of contact (18)

(N B —The diagram is not drawn to scale)

\*4 Describe a circle of  $\frac{1}{4}$ " radius. With any point on the circumference of this circle as centre, describe a circle of 2" radius. Draw the curve every point on which is equidistant from these two circles. Find at least eight points on the curve. (20)

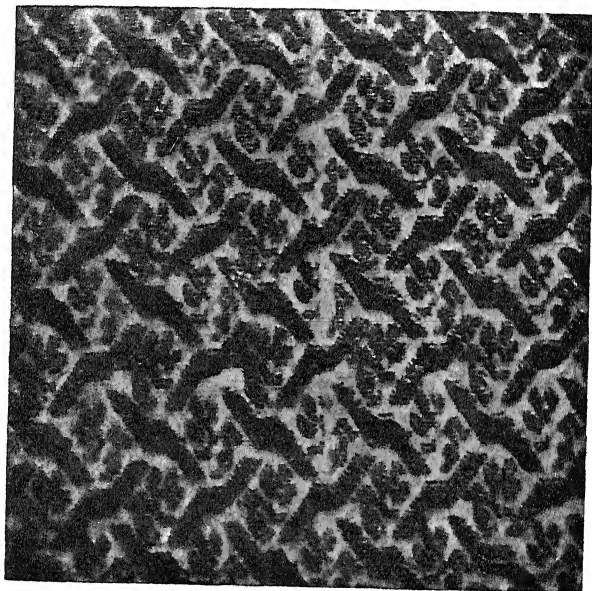
\*5 Copy the given section of a moulding, enlarging the total height to 3", and the other dimensions in proportion. The centres of the arcs are shown. (20)

\*6 The diagram shows the cross-section of a box with a hollow or "caddy" lid, which is open at an angle of  $30^\circ$ . The outside length of the box is to be 2.25".

Draw (a) the plan, (b) the *front* elevation of the box. Only the visible lines need be shown in each case. (18)

\*7. A cylindrical pipe, outside diameter  $2\frac{1}{2}$ ", inside diameter 2", is cut across by a plane making  $45^\circ$  with the axis of the cylinder. Draw the true form of the section. (20)

### Q 9.



### SECTION B.

Read the general instructions

IN THIS SECTION YOU MAY ATTEMPT TWO QUESTIONS ONLY

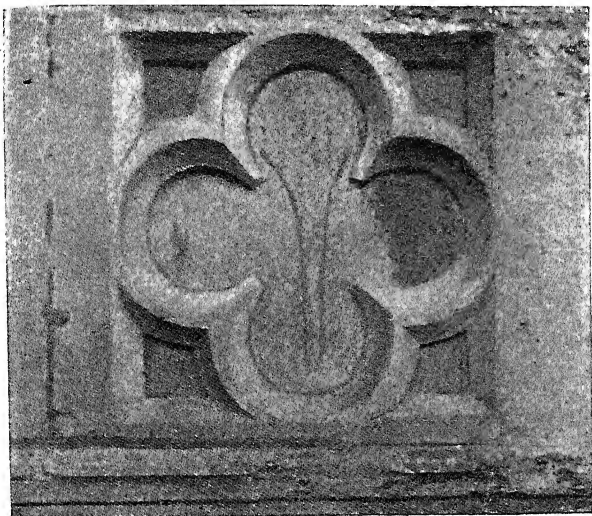
All freehand work employed in this section must be neat and careful, and its intention must be made quite clear

*All constructions must be clearly shown*

\*8 The given form can be combined with squares to produce a repeating diaper pattern. Work out, using instruments, two ways in which this can be done. Mark very clearly the unit of repeat in each case, and indicate what you consider the most practical method of obtaining a number of repeats. (18)

\*9 Draw neatly, with instruments, to about twice the scale of the diagram, the framework on which you would set out the given pattern. Mark very clearly what you consider the unit of repeat. Sketch enough of the pattern to make your intention plain and show not less than six repeats of the framework. (18.)

Q 10.

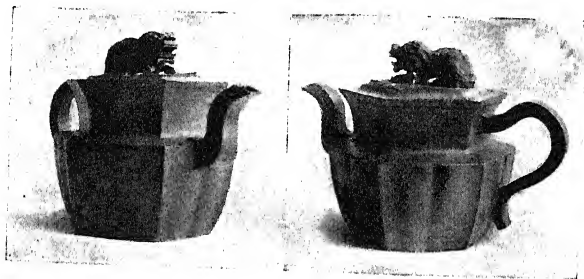


Q.11.



- \*10 Copy the quatre-foil panel as exactly as you can, using instruments throughout. Make the dimensions of your drawing twice those of the diagram. (16)
- \*11 Show a geometrical framework on which the given tile border could be set out. Sketch in about half a unit of repeat to show the application of your method. (20)
- \*12 Two views are given of a Japanese teapot. Make a careful approximate sketch plan and side elevation. The animal on the lid need not be drawn. (18)

Q 12



MAY 1, 1909.

6 to 7.30 p m.

# GEOMETRICAL DRAWING

## SECTION A

Read the general instructions

IN THIS SECTION YOU MAY ATTEMPT THREE QUESTIONS ONLY, SHOWING YOUR KNOWLEDGE BY THE USE OF INSTRUMENTS. The constructions must therefore be strictly geometrical, and not the result of calculation or trial.

*All lines used in the constructions must be clearly shown*

Set-squares may be used whenever convenient. Lines may be bisected by trial.

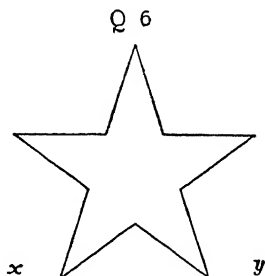
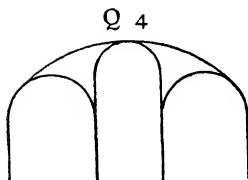
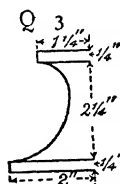
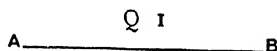
\*1 The given line **AB** represents a length of 15 centimetres. Construct a scale by which decimetres, centimetres, and millimetres can be measured up to 3 decimetres. Figure the scale properly, and show by two small marks on it how to take off on it a distance of 263 millimetres.

By means of the scale draw a circle of 125 millimetres' radius, and in it place a chord 189 millimetres long. Write down in millimetres the distance of this chord from the centre of the circle.

(1 metre = 10 decimetres, or 100 centimetres, or 1000 millimetres) (20)

\*2 Construct a regular decagon or figure of 10 sides, each side being 1" long. Within it inscribe five equal circles, each touching two of the others and one side of the decagon.

(N.B.—If a protractor is used for measuring an angle, such use must be clearly shown.) (20)



\*3 The curve of the given "scotia" moulding is made up of two quadrants or quarter circles. Draw the figure from the given dimensions. (16)

(N.B.—The diagram is not drawn to scale.)

\*4. The diagram shows the leading lines of a window composed of three semicircular-headed "lights" included under a three-centred arch. The side lights are to be each 3' wide, while the middle light is to be 2' wide and to have the centre of its semicircle 2' above those of the other two. Parts of the side semicircles also form part of the enclosing arch. Draw the figure to a scale of 2' to 1". Show clearly how you obtain all points of contact. (20)

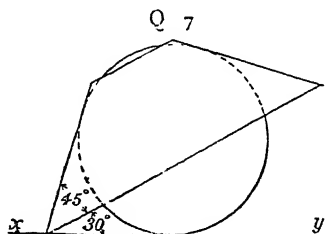
(N.B.—The diagram is not drawn to scale.)

5 Two fixed lines **AB** and **CD** of indefinite length, cross one another at an angle of  $70^\circ$ . A third line **EF**,  $3''$  long, is movable, so that the end **E** travels along the line **AB** while the end **F** travels along the line **CD**. Draw the complete curve traced by the middle point of **EF**.

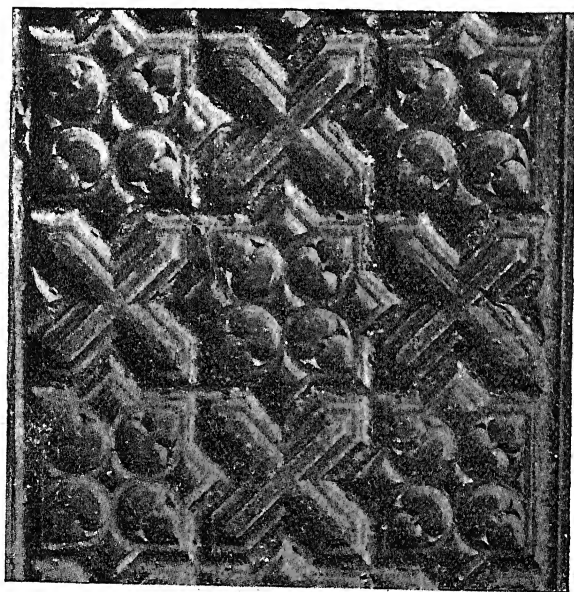
(It will be sufficient to find some 12 to 16 points on the curve.)

(18)

\*6 The diagram gives the front elevation of a regular five-pointed star, which is cut



Q. 8.



out of material  $\frac{3}{4}''$  thick. Draw the plan of the star, and also a second elevation on a vertical plane inclined at  $60^\circ$  to that of the given elevation.

\*7 An elevation is given of a sphere upon which rests a conical lamp-shade. Draw the plan of the shade.

(20)

(20.)

## SECTION B

Read the general instructions

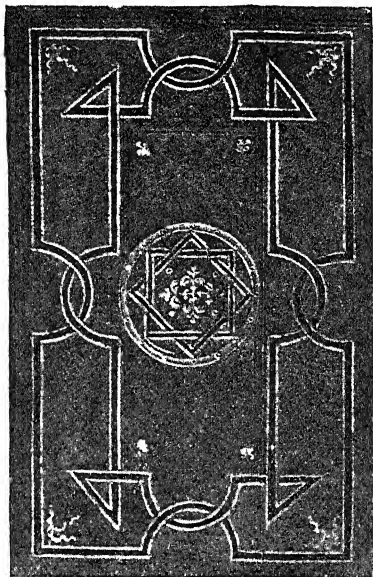
IN THIS SECTION YOU MAY ATTEMPT TWO QUESTIONS ONLY

All freehand work employed in this section must be neat and careful, and its intention must be made quite clear

*All constructions must be clearly shown*

\*8. Draw with instruments the main geometrical construction lines you would employ in setting out the given pattern. Show plainly what you consider the practical or working unit of repeat (20)

Q. 10.



9 Show how you would arrange a number of circular discs of  $\frac{1}{8}$ " radius as a diaper pattern—

- (i) When each disc touches four others
- (ii) When each disc touches six others
- (iii) When each disc touches three others

Draw the necessary constructional framework in each case, and show clearly what you consider the unit of repeat in each pattern (20)

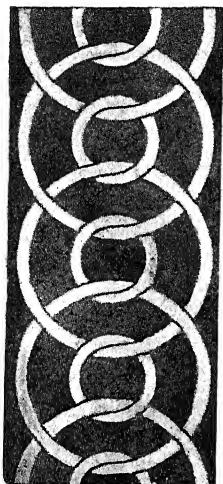
\*10 Set out carefully the lines of one quarter of the given bookbinding, making your drawing to twice the scale of the diagram (18)

\*11 Make a modified version of the border of interlacing circles, in which the radii shall be  $\frac{1}{4}$ ",  $\frac{3}{8}$ ",  $\frac{1}{2}$ ", and 1" respectively. Show at least two repeats of the unit (16)

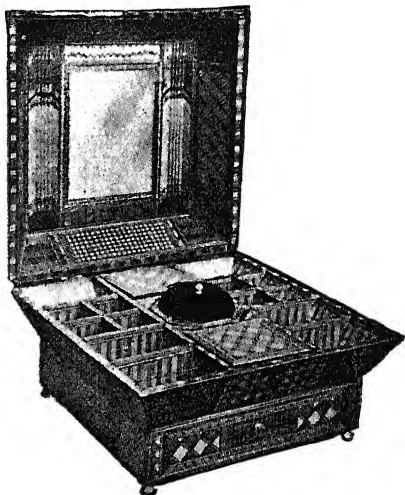
\*12 Sketch approximately, in outline, the plan and the front and side elevations of the workbook and its lid shown in the diagram, omitting all merely ornamental details. Arrange your drawings so as to show how one is projected from another (20)



Q 11



Q 12



APRIL, 1910.

6 to 7.30 p.m.

GEOMETRICAL DRAWING.

SECTION A

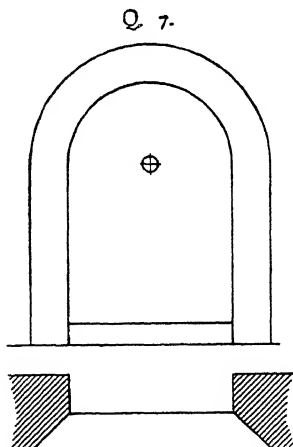
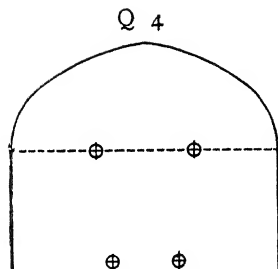
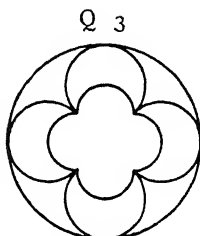
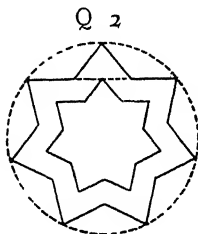
Read the general instructions

IN THIS SECTION YOU MAY ATTEMPT THREE QUESTIONS ONLY, SHOWING YOUR KNOWLEDGE BY THE USE OF INSTRUMENTS. The construction must therefore be strictly geometrical, and not the result of calculation or trial

*All lines used in the constructions must be clearly shown*

Set-squares may be used wherever convenient. Lines may be bisected by trial

1. A plan is drawn to a scale of 1" to 75'. Make a diagonal scale for the plan, by which



single feet may be measured up to 400'. Figure the scale properly, and show by two small marks on it how to take off a distance of 87'.

By means of the scale draw a quadrant of a circle of 227' radius, and write down in feet the length of its chord (20)

\*2 Copy the given figure, making the radius of the enclosing circle 1½"

(N.B.—If a protractor is used for measuring an angle, such use must be clearly shown) (18)

\*3 In the diagram the two quatrefoils are composed of semicircles and tangential arcs respectively. Copy the figure, making the diameters of the semicircles 1" (18)

\*4 The diagram shows the outline of a "four-centred" Gothic arch, the positions of the centres being marked. Draw an arch of exactly the same shape, enlarging the width or span to 4". Show clearly all points of contact. (20)

\*5 Construct a rectangle 4" long and 2½" wide. Make a point in one of the long sides 1½" from one corner. This point is to be one point of a rhombus inscribed in the rectangle. Within the rhombus inscribe a square, and within the square inscribe a circle. (16)

\*6 A rectangular packing-case 3' x 1' x 1' has its largest faces tilted at 60° to the ground. One of the longest edges is on the ground and another rests against a vertical

## Q 9



wall. Draw a plan of the case, and also an elevation on a vertical plane parallel to the wall. (Scale, 1" to 1') (20)

\*7. Plan and elevation are given of an archway with semicircular head and one step. Draw a second elevation upon a vertical plane which makes an angle of 45° with the plane of the wall. (20)

## SECTION B

Read the general instructions

IN THIS SECTION YOU MAY ATTEMPT TWO QUESTIONS ONLY

All freehand work employed in this section must be neat and careful, and its intention must be made quite clear

*All constructions must be clearly shown*

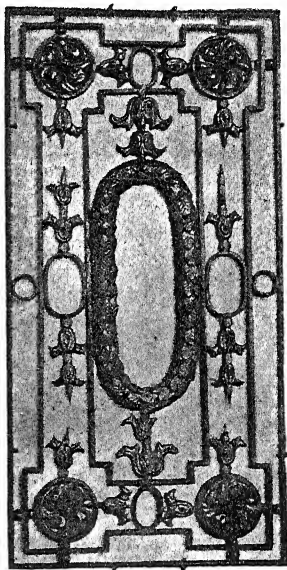
\*8 Draw, using instruments, some six or eight repeats of the given "scale" pattern. Show that either a rectangle or a rhombus could be used as a basis for the repeats, and state which you would prefer in this case. (16)

\*9 Draw, with instruments, the geometrical construction lines you would employ in setting out about four repeats of the given pattern. Show clearly what you consider the practical or working unit of repeat, sketching so much of the pattern as will suffice to explain your construction. (20)

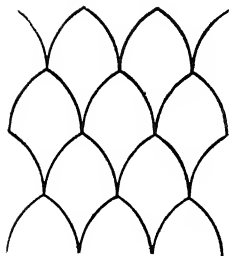
\*10 Set out, with instruments, the leading lines of the left-hand half of the given iron work panel, doubling the dimensions of the photograph. It is important that the relative proportions of the parts should be retained (20)

\*11 Draw neatly, with instruments, the geometrical part of the given stone medallion, to twice the scale of the photograph (18)

Q 10



Q 8

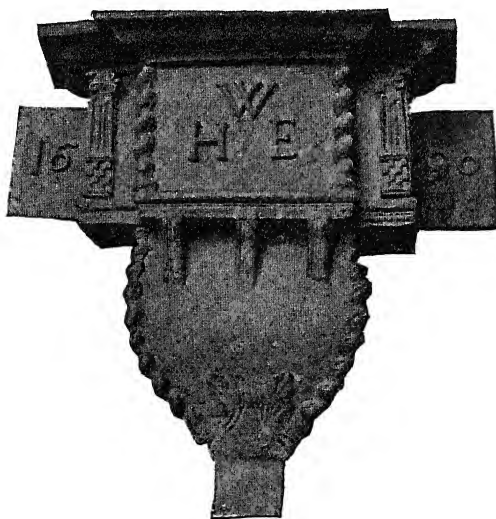


\*12 Make an approximate sketch-plan, and front and side elevations, of the given rain-pipe head, omitting merely ornamental details. Arrange your drawings so as to show how one is projected from another (18)

Q 11.



Q. 12.



MAY 6, 1911.

6 to 7 30 p m

GEOMETRICAL DRAWING

SECTION A.

Read the general instructions

IN THIS SECTION YOU MAY ATTEMPT THREE QUESTIONS ONLY, SHOWING YOUR KNOWLEDGE BY THE USE OF INSTRUMENTS. The constructions must therefore be strictly geometrical, and not the result of calculation or trial

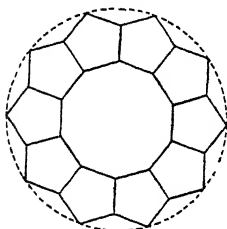
All lines used in the constructions must be clearly shown

Set-squares may be used wherever convenient. Lines may be bisected by trial

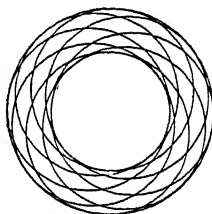
1 Make a plain scale, to show feet and inches up to 5 feet, on which a distance of 3' 6" is represented by  $4\frac{1}{4}"$ . Finish and figure the scale neatly and carefully

Draw to this scale an oblong 3' 3"  $\times$  2' 5", and in the centre of it place a second oblong of the same shape but having its longer sides 2' 9". Measure the breadth of this smaller oblong to the nearest half inch (20)

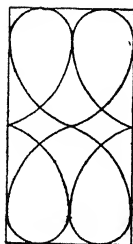
Q 2.



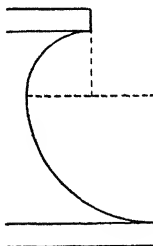
Q. 3



Q 4



Q 6



Q 7



\*2 Complete half the given figure, which is made up of regular pentagons. Make the radius of your enclosing circle 2". Show clearly any construction you employ (20)

\*3 Copy the given figure, making the radius of the outer circle  $1\frac{1}{2}"$ , and that of the inner one in proportion (18)

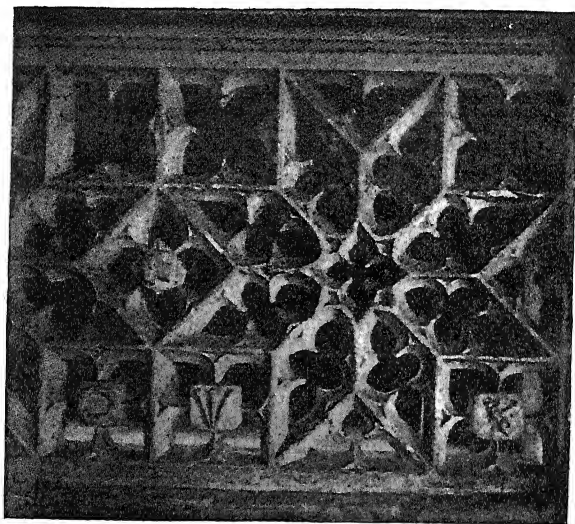
\*4 Draw the given figure. The curves are to be composed of arcs of circles of 0.5" and 1.5" radii (20)

5 Two straight lines, **AB** and **CD**, are 3" and 3.5" long respectively. **A** is 1.5" from **C**, while **B** is 1" from **D** and 3.5" from **C**. Describe two circles each touching both **AB** and **CD**, one passing through **A**, the other through **B** (16)

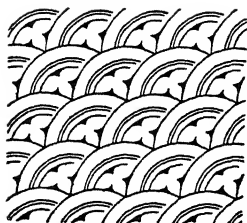
\*6 The outline of a "scotia" moulding is shown. If two lengths of this moulding are "mitred" together at right-angles, show the true form of the cut surface of the mitre. (18)

\*7, A lamp shade, in the form of a truncated regular hexagonal pyramid, is made of six

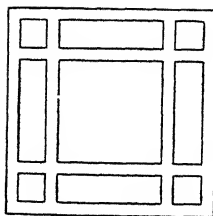
Q 8



Q 9.



Q II



pieces of card, of the exact shape and size shown. Draw its plan and elevation in any position. (20)

### SECTION B.

IN THIS SECTION YOU MAY ATTEMPT TWO QUESTIONS ONLY.  
All freehand work employed in this section must be neat and careful, and its intention must be made quite clear.  
*All constructions must be clearly shown.*

\*8 Show, using instruments, how you would set out the geometrical framework of the given openwork panel. None of the "cusping" need be drawn. (20)

\*9 Draw neatly with instruments the framework or "net" of the given pattern. Show clearly what you would intend to be the unit of repeat, and finish not less than four of these so as to show repeats both in width and height. (20)

Q. 10



\*10 Set out, as nearly as you can, the construction needed for the geometrical part of the ornament round the semicircular doorhead of which about half is shown. Only one unit of each ornament need be completed. Make your drawing twice as large as the diagram. (18)

\*11 Redraw the given figure, altering the proportions of the parts so that the sides of the four corner panels shall be exactly half those of the centre panel. Make the side of the outside square 3", and the margins throughout 0.2" wide. (16)

\*12 Draw an approximate sketch-plan with front and side elevations, of the given steps. Arrange your drawings to show how one is projected from the other. (18)



Q 12.

